On Unified Adaptive Portfolio Management

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Abstract

This paper introduces a unified framework for adaptive portfolio management, integrating dynamic Black-Litterman (BL) optimization with the general factor model, *Elastic Net* regression, and mean-variance portfolio optimization, which allows us to generate investors' views and mitigate potential estimation errors systematically. Specifically, we propose an innovative *dynamic sliding window algorithm* to respond to the constantly changing market conditions. This algorithm allows for the flexible window size adjustment based on market volatility, generating robust estimates for factor modeling, time-varying BL estimations, and optimal portfolio weights. Through extensive ten-year empirical studies using the top 100 capitalized assets in the S&P 500 index, accounting for turnover transaction costs, we demonstrate that this combined approach leads to computational advantages and promising trading performances.

Keywords: Portfolio Management, Black-Litterman Approach, Mean-Variance Optimization *JEL Classification* : G11, C44, C63

1 Introduction

The Black-Litterman (BL) approach, first introduced by Black and Litterman (1990), incorporates investors' views to predict the expected return of underlying assets. Since then, the BL approach has been widely applied and has undergone various developments. For instance, Black and Litterman (1992) applied the BL model to global portfolio optimization, Fabozzi et al. (2006) incorporated the BL approach into trading strategies, and Martellini and Ziemann (2007) studied an extension of BL beyond the mean-variance framework to use all available information, including the equilibrium model, the investor's view, and the data.

Typically, the investors' views used in the BL model are formed *subjectively*, relying on information provided by some financial analysts or Reserve Bank statements; see Black and Litterman (1992). While some studies, such as Creamer (2015), have attempted to use *sentiment analysis* techniques to generate the views, it often requires a large amount of linguistic data. This data may not be available when timely investment decisions are needed. To address this issue, in this paper, we propose the use of general factor models, as seen in works by Asl and Etula (2012); Kolm and Ritter (2020); Giglio et al. (2022); Spears et al. (2023), to assign the BL-based views within a factor model framework systematically.

It is known that obtaining expected returns with small estimation errors is challenging; see Luenberger (2013). The difficulty is compounded when using the optimization technique to determine the portfolio weights since the resulting "optimal" portfolio may allocate significant capital

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to assets with a high estimation error in expected return, as noted by Best and Grauer (1991); Britten-Jones (1999).

To this end, recent studies, e.g., Min et al. (2021); Chen et al. (2022); Spears et al. (2023), have shown that these errors can be reduced using machine learning techniques, including ridge regression and view fusion. This paper extends the standard BL approach to involve the *Elastic Net*, a regularization technique that combines ridge and LASSO regression, see Zou and Hastie (2005), in the estimation. This combined regularization approach offers a dual advantage for stabilizing solutions in the presence of multicollinear data and mitigating potential overfitting issues. Moreover, we enhance our model by applying an Exponential Weighted Moving Average (EWMA) to dynamically estimate the covariance matrix, offering a more flexible and robust approach to error reduction.

The traditional BL approach typically employs *static* investors' views, limiting its ability to reflect the latest market information. To this end, several approaches are proposed to update an investor's view dynamically and have been shown to have better short-term trading performance in previous studies; see Guiso et al. (2018); Simos et al. (2021); Barua and Sharma (2022). To reflect the constantly changing market conditions, Prakash et al. (2021) studied a sliding window approach with capital allocation based on volatility variation. The practice of dynamically adjusting the window size has since gained traction in the field of artificial intelligence, e.g., see Ortiz Laguna et al. (2011); Haque et al. (2016); Selvin et al. (2017) and in the area of data-driven control, see Wang and Hsieh (2022). In this paper, we further build on this concept by proposing a *dynamic sliding window approach* that adapts the window size in response to fluctuations of market volatility, updates investors' views dynamically, and computes optimal weights.

1.1 Contributions of the Paper

The most salient novelty of this paper, distinguishing it from existing literature, is to provide a unified approach to the dynamic portfolio management problem. Specifically, having provided the necessary preliminaries in Section 2, we extend the Black-Litterman approach by strategically integrating Elastic Net regression and the mean-variance optimization problem, as detailed in Section 3. Our principal contribution, delineated in Section 4, introduces a novel, adaptive mechanism, see Algorithm 1, for adjusting the window size in response to market volatility. This approach generates robust estimates for the factor model and provides time-varying Black-Litterman estimates and optimal portfolio weights in a computationally efficient manner. Section 5 shows extensive ten-year empirical studies across various market conditions, demonstrating the efficacy and practical utility of our proposed methodology in practice.

2 Preliminaries

Consider a portfolio consisting of $n \ge 1$ assets. The general *factor* model, as inspired by the Arbitrage Pricing Theory (APT) with Ross (1976), states the relationship between return on the Asset *i* and factors f_j for j = 1, 2, ..., J as follows: For i = 1, 2, ..., n,

$$r_i = \alpha_i + \mathbf{F}^\top \boldsymbol{\beta}_i + \varepsilon_i \tag{1}$$

where α_i is the intercept, $\mathbf{F} := [f_1 \ f_2 \ \cdots \ f_J]^\top$ is the factor vector with J < n, and $\beta_i := [\beta_{i,1} \ \beta_{i,2} \ \cdots \ \beta_{i,J}]^\top$ are the *factor loadings*, representing the change on the return of Asset *i* per unit change in factor, and ε_i is the specific error factor for Asset *i*, which is assumed to be a white noise series and uncorrelated with the factors f_j and other factors. We assume that $\mathbb{E}[\varepsilon_i] = 0$

for all i, $\operatorname{cov}(f_j, \varepsilon_i) = 0$ for all j, i, and lastly, $\operatorname{cov}(\varepsilon_i, \varepsilon_j) = \sigma_i^2$ if i = j, and zero otherwise.¹ In finance, the typical factor models include the three-factor and five-factor models by Fama and French, see Fama and French (1992, 2015), as well as Carhart's four-factor model, see Carhart (1997). As seen later in this paper, we will adopt these models for illustrative purposes.

In general, to obtain the parameters α and β , one solves an ordinary least squares (OLS) problem; see Luenberger (2013). However, the approach can be sensitive to outliers or be prone to overfitting. To this end, we consider Elastic Net, a convex combination of LASSO and ridge penalty Zou and Hastie (2005), in the estimation to assure the flexibility and robustness of our estimates; i.e., Elastic Net regression problem:

$$\min_{\boldsymbol{\alpha},\boldsymbol{\beta}} \|\mathbf{r} - (\boldsymbol{\alpha} + \mathbf{F}^{\top}\boldsymbol{\beta})\|_{2}^{2} + \lambda_{2} \left\| \begin{bmatrix} \boldsymbol{\alpha} \\ \boldsymbol{\beta} \end{bmatrix} \right\|_{2}^{2} + \lambda_{1} \left\| \begin{bmatrix} \boldsymbol{\alpha} \\ \boldsymbol{\beta} \end{bmatrix} \right\|_{1}$$
(2)

with $\lambda_i \geq 0$ for $i \in \{1,2\}$ and $\sum_{i=1}^2 \lambda_i = 1$, where $\|\mathbf{z}\|_p$ is the ℓ_p -norm of vector \mathbf{z} which satisfies $\|\mathbf{z}\|_p = (\sum_{i=1}^n |z_i|^p)^{1/p}$ for $p \in \{1,2\}$.

3 Problem Formulation

This section considers two main problems that are central to our subsequent development. The first involves estimating the expected return and covariance using the BL approach with Elastic Net. The second pertains to determining optimal portfolio weights using the mean-variance criterion.

3.1 Extended BL Approach with Elastic Net

The classical BL approach is driven by two key factors: market equilibrium and investor views, based on the Capital Asset Pricing Model (CAPM), see Sharpe (1964). Let Π be the *implied* returns satisfying

$$\Pi := \boldsymbol{\mu} + \varepsilon_{\Pi}, \ \varepsilon_{\Pi} \sim \mathcal{N}(0, Q),$$

where $\boldsymbol{\mu}$ is the *true* expected return vector to be determined, $\mathcal{N}(0, Q)$ is a normal distribution with zero mean and covariance matrix $Q := \tau \Sigma$ with $\tau > 0$, representing our *confidence* in estimating expected returns² and Σ is the covariance matrix of returns.

The investor views, denoted by a vector $\mathbf{q} \in \mathbb{R}^{K}$ with K views, are incorporated with the mean return μ and can be expressed with the linear equation; i.e.,

$$\mathbf{q} := P\boldsymbol{\mu} + \varepsilon_q, \ \varepsilon_q \sim \mathcal{N}(0, \Omega), \tag{3}$$

where $P \in \mathbb{R}^{K \times n}$ represents K views of n assets with $K \leq n$, and $\Omega \in \mathbb{R}^{K \times K}$ expresses the confidence (variance) of K views. Later in this paper, we shall use the factor model (1) as a proxy of the views equation (3). To incorporate the investors' views with the market equilibrium, we consider

$$\mathbf{y} := B\boldsymbol{\mu} + \varepsilon_y, \quad \varepsilon_y \sim \mathcal{N}(0, V),$$

The joint model for *n* assets is $\mathbf{r} = \boldsymbol{\alpha} + \mathbf{F}^{\top} \boldsymbol{\beta} + \boldsymbol{\varepsilon}$ where $\mathbf{r} := [r_1 \cdots r_n]^{\top}$, $\boldsymbol{\alpha} := [\alpha_1 \cdots \alpha_k]^{\top}$, $\boldsymbol{\beta} := (\beta_{ij})$ is a $n \times J$ factor-loading matrix, and $\boldsymbol{\varepsilon} := [\varepsilon_1 \cdots \varepsilon_n]^{\top}$ is the error vector with $\operatorname{cov}(\boldsymbol{\varepsilon}) := D := \operatorname{diag}(\sigma_1^2, \ldots, \sigma_k^2)$, a diagonal matrix with diagonal entries to be $(\sigma_1^2, \ldots, \sigma_k^2)$.

²The smaller τ is, the less uncertain the estimate is. A typical choice of parameter τ is between 0.01 and 0.05; see Black and Litterman (1992); Idzorek (2007); some suggest to use $\tau = 1$ directly, e.g., see Satchell and Scowcroft (2000), while some prefer the value 1 divided by the number of observations.

where $\mathbf{y} := \begin{bmatrix} \Pi \\ \mathbf{q} \end{bmatrix}$, $B := \begin{bmatrix} I_{N \times N} \\ P \end{bmatrix}$ and $V := \begin{bmatrix} Q & 0 \\ 0 & \Omega \end{bmatrix}$ and $I_{N \times N}$ is the $N \times N$ identity matrix.

Then, we seek an optimal estimator for the true expected returns, call it $\hat{\mu}$, that solves the *Elastic Net-based weighted least-squares* (WLS) problem

$$\min_{\boldsymbol{\mu}} (\mathbf{y} - B\boldsymbol{\mu})^\top V^{-1} (\mathbf{y} - B\boldsymbol{\mu}) + \lambda_2 \|\boldsymbol{\mu}\|_2^2 + \lambda_1 \|\boldsymbol{\mu}\|_1,$$
(4)

where $\lambda_1, \lambda_2 \ge 0$ and $\lambda_1 + \lambda_2 = 1$ are fixed coefficients for the regularization terms.

The key idea for incorporating the Elastic Net into the ordinary WLS regression is to address both heteroscedastic errors and potential high dimensionality on the factors, which may lead to a more robust and accurate model. If $\mathbf{q} = \Omega = \mathbf{0}$, i.e., the investor has no views or zero confidence in the views and $\lambda_i = 0$ for $i \in \{1, 2\}$, then the solution to Problem (4), call it $\hat{\boldsymbol{\mu}}$, becomes $\hat{\boldsymbol{\mu}} = \Pi$.

Remark 3.1. In addition, if $\lambda_i = 0$, one obtains the Black-Litterman estimates for expected return $\hat{\mu} = \Pi + QP^{\top}(PQP^{\top} + \Omega)^{-1}(\mathbf{q} - P\Pi)$ and for the covariance matrix

$$\widehat{\Sigma} = \Sigma + (Q^{-1} + P^{\top} \Omega^{-1} P)^{-1}.$$
(5)

A more detailed discussion can be found in Fabozzi et al. (2007); Meucci (2010). See also Kolm and Ritter (2017) for a Bayesian interpretation of the BL approach. It should be noted that our approach, including regularization terms, mitigates the potential numerical instability when computing the inverse matrix in (5).

3.2 Mean-Variance Portfolio Optimization

Let $\mathbf{w} := [w_1 w_2 \cdots w_n]^\top \in \mathbb{R}^n$ be the portfolio weights. We consider a version of Markowitz's mean-variance (MV) model to obtain the optimal portfolio weight, e.g., see Markowitz (1952, 1991). That is,

$$\max_{\mathbf{w}\in\mathcal{W}} \widehat{\boldsymbol{\mu}}^{\top} \mathbf{w} - \rho \mathbf{w}^{\top} \widehat{\boldsymbol{\Sigma}} \mathbf{w}$$
(6)

where the admissible set \mathcal{W} is given by

 $\mathcal{W} := \{ \mathbf{w} \in \mathbb{R}^n : \|\mathbf{w}\|_1 = 1, |w_i| \le W \in [0, 1] \}$

for some $W \in [0, 1]$, $\|\mathbf{w}\|_1 := \sum_{i=1}^n |w_i|$ is the ℓ^1 -norm, $\hat{\mu}$ and $\hat{\Sigma}$ are obtained via the BL approach described previously, $\rho > 0$ is a *risk aversion* coefficient, which is typically selected within an interval [1,10], see Ang (2014). It should be noted that the problem has no closed form in general if the constraint set \mathcal{W} is imposed. However, one can readily verify that the problem is a convex quadratic program, which can be solved efficiently; see Boyd and Vandenberghe (2004). In the next section, we shall present our dynamic sliding window algorithm and show how to dynamically estimate $\hat{\mu}$ and $\hat{\Sigma}$ and how to update these estimates dynamically.

4 The Dynamic Sliding Window Algorithm

This section provides our dynamic sliding window algorithm for estimating factor models, generating time-varying views, estimating expected returns and covariances, and computing optimal weights. Specifically, fix an initial window size $M \ge 1$ and set the starting time stamp $t \ge 0$. For $t - 1, t - 2, \ldots, t - M$, we first solve the Elastic Net regression problem to obtain intercept term $\boldsymbol{\alpha}$ and factor loadings $\boldsymbol{\beta}$. Using these $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ in the factor models, we generate the views \mathbf{q} :

$$\mathbf{q} = \boldsymbol{\alpha} + \mathbf{F}^{\top} \boldsymbol{\beta} + \varepsilon_q,$$

where $\varepsilon_q \sim \mathcal{N}(0, \Omega)$ is the specific error factor of views **q** defined in Equation (3) and the factor data **F** are retrieved from the database of the Wharton Research Data Service (WRDS), Wharton Research Data Services (2023). Next, we solve the Elastic Net-based WLS Problem (4) to obtain $\hat{\mu}$ and $\hat{\Sigma}$. Additionally, to incorporate the possibility of a time-varying covariance matrix, we follow Harris et al. (2017) to use the Exponentially Weighted Moving Average (EWMA) model. Specifically, let $\eta \in [0, 1]$ be the *decay* factor. The EWMA model for estimating the covariance matrix, call it $\hat{\Sigma}_{\text{EWMA}}$, is as follows:

$$\widehat{\Sigma}_{\text{EWMA}} = \eta \widehat{\Sigma} + (1 - \eta) \mathbf{r} \mathbf{r}^{\top}, \tag{7}$$

where $\widehat{\Sigma}$ is obtained from Equation (5) and $\mathbf{r} := [r_1 \ r_2 \ \cdots \ r_n]^\top$ is the return vector.

Having obtained $\hat{\mu}$ and $\hat{\Sigma}$, we solve the mean-variance portfolio optimization problems (6) to obtain the corresponding optimal portfolio weight \mathbf{w}^* . Subsequently, we then use optimal weights \mathbf{w}^* to trade in the following time stamps $[t, t+1, \ldots, t+M-1]$. If the market volatility increases, indicating a rapid change in market condition, we shorten the window size by $c_- \cdot M$ with $c_- \in (0, 1)$ to better capture recent trends. Otherwise, we increase or retain the size. Then, we reinitialize the algorithm by setting t := t + M and repeat this procedure until the terminal stage has arrived. It should be noted that Algorithm 1 is *data-driven*, which means there is no need to impose assumptions on the distribution of returns. The details of the algorithm can be found in Algorithm 1; see also Figure 1 for an illustration of the main idea of our approach.

Remark 4.1 (On Computational Complexity of Algorithm 1). The computational complexity of Algorithm 1 is about $O(M(n^3 + n^2J + nJ^2))$ where M is the window sizes, n is the number of assets in the portfolio, and J is the number of factors in the factor model. To see this, we note that the initial collecting data step involves M historical returns data and factors data, which corresponds to the complexity of O(M). Then, solving the Elastic Net regression problem involves a least square computation with L^1 and L^2 regularization, which typically has a complexity of $O(n^3 + n^2J + nJ^2)$. Computing the views vector is dominated by the matrix multiplication $\mathbf{F}^{\top}\boldsymbol{\beta}$, which has a complexity of O(nJ). Next, solving the Elastic Net-based WLS Problem is similar to the previous Elastic Net problem with complexity $O(n^3 + n^2J + nJ^2)$. Then, solving the mean-variance optimization involving quadratic programming generally has a complexity of $O(n^3)$ for n assets. The remaining transactions are about the complexity of O(Mn), and dynamic adjustment of window size involves comparison operations, which are negligible in terms of complexity. Since Algorithm 1 iterates over the window size M, the dominating factors in the computational complexity are the Elastic Net regression and the mean-variance optimization, with sizes M. Hence, the complexity of the algorithm is about $O(M(n^3 + n^2J + nJ^2))$.

4.1 Turnover Transaction Costs

In real-world trading, transaction costs are typically present. As demonstrated in dynamic portfolio optimization literature by Brown and Smith (2011); Hautsch and Voigt (2019); Wong and Hsieh (2023), such costs can significantly impact the performance of trading strategies. To better align with the dynamics of real-market conditions, we consider a percentage transaction cost TC in our empirical studies by imposing various costs from 0 to 100 basis points. Specifically, we study $TC \in \{0, 0.01, 0.1, 1\}\%,^3$ to the *turnover*, which is defined as the total value of assets added or removed from our portfolio. Some other related literature on transaction costs, such as commission fee rates, can be found in Keim and Madhavan (1998); Wang et al. (2021).

³Some brokerage services, such as Interactive Brokers, impose a maximum transaction fee rate of 1% per order on the trade value. The fee structure is outlined on their pricing page, see URL: https://www.interactivebrokers.com/en/pricing/commissions-stocks.php.



Figure 1: Idea of Generating Time-Varying Views and Weights: Dynamic Sliding Window

5 Empirical Studies: The S&P 100 Portfolio

This section provides empirical studies using Algorithm 1. We first use daily closing prices for assets comprising the top 100 market cap assets of Standard and Poor's 500 (S&P 500) constituents over ten years from January 1, 2013 to January 1, 2023.⁴ It is worth noting that during this period, the prices of S&P 500 constituents experienced fluctuations and a significant drawdown in the first half of 2020. The index price trends, as a representative of the S&P 500 and top S&P 100, are shown in Figure 5. In addition to the 100 assets, we added a Four Week U.S. Treasury bill⁵ to our portfolio, resulting in a mid-sized portfolio of a total of 101 assets. Additionally, to enhance diversification effects, in the sequel, $W \in [-0.1, 0.1]$ is imposed as an additional constraint of Problem (6); see Mohajerin Esfahani and Kuhn (2018); Hsieh (2023) for theoretical support on imposing such a constraint.

 $^{^{4}}$ The data is retrieved from CRSP and Compustat datasets, and access is authorized through the Wharton Research Data Service; see Wharton Research Data Services (2023).

⁵The data has been sourced from the U.S. Department of The Treasury. Over the span from 2013 to 2023, the vector representing the annualized rates in percentage for ten-year Treasury bills is as follows: $r_f^{2013:2023} = [0.046\%, 0.028\%, 0.034\%, 0.249\%, 0.833\%, 1.809\%, 2.08\%, 0.347\%, 0.041\%, 1.607\%]^{\top}$.

Algorithm 1 Dynamic Sliding Window Algorithm

- **Require:** Consider a portfolio consisting of $n \ge 1$ assets, an initial window size $M \ge 1$, regularization parameters $\lambda_1, \lambda_2 \ge 0$ with $\lambda_1 + \lambda_2 = 1$, decay factor $\eta \in [0, 1]$, variation level $h \in (0, 1)$, and risk-averse constant $\rho \ge 1$.
- **Ensure:** Expected Return $\hat{\mu}$, the covariance matrix $\hat{\Sigma}$, optimal portfolio weight \mathbf{w}^* , and portfolio volatility σ .
- 1: At initial time stamp $t \ge 0$, collect M historical returns' data

$$\mathbf{r} := (\mathbf{r}(t), \mathbf{r}(t-1), \dots, \mathbf{r}(t-(M-1)))$$

and historical factors data $\mathbf{F} := (\mathbf{F}(t), \mathbf{F}(t-1), \dots, \mathbf{F}(t-(M-1))).$

- 2: Use the data in Step 1 to solve the Elastic Net regression problem (2) to obtain α and β for the factor model (1).
- 3: Having obtained (α, β) , calculate the views vector **q** by the factor model

$$\mathbf{q} := \boldsymbol{\alpha} + \mathbf{F}^{\top} \boldsymbol{\beta} + \varepsilon_q. \tag{8}$$

- 4: Solve the Elastic Net-based WLS Problem (4) to obtain $\hat{\mu}$ and use EWMA model (7) with parameter η to obtain $\hat{\Sigma}_{\text{EWMA}}$.
- 5: Use $\hat{\mu}$ and $\hat{\Sigma}_{\text{EWMA}}$ to solve the mean-variance optimization problem (6) and obtain the corresponding optimal portfolio weight \mathbf{w}^* .
- 6: Execute transactions using \mathbf{w}^* over the interval [t, t+M] and evaluate corresponding portfolio volatility σ for this interval.
- 7: Compute the previous portfolio volatility σ^* from t M to t to dynamically adjust window size M:
- 8: **if** $\sigma \ge (1+h) \cdot \sigma^*$ **then**
- 9: $M \leftarrow c_- \cdot M$ where constant $c_- \in (0, 1)$
- 10: else if $\sigma \leq (1-h) \cdot \sigma^*$ then
- 11: $M \leftarrow c_+ \cdot M$ where constant $c_+ \ge 1$;
- 12: else
- 13: Maintain M
- 14: Set t := t + M and go back to Step 1.

To evaluate the trading performance, we use the following metrics. The first one is the *excess* return of the portfolio given by $r^p := \mathbf{w}^\top \mathbf{r} - r_f$, We use $\overline{r^p}$ to denote the annualized mean excess return of the portfolio, σ to denote the annualized volatility of portfolio returns, and SR to denote the annualized Sharpe ratio. Moreover, to study the downside risks, we take d^* to be the maximum percentage drawdown. In the following sections, we compare the trading performance of an equal-weighted market-based portfolio with the mean-variance portfolios obtained by Algorithm 1.

5.1 Factor Models: FF5 and Cahart 4

To illustrate our framework, we consider the *Fama-French five-factor model* (FF5) and *Carhart four-factor model* (Carhart 4), as described in Fama and French (2015), as well as Carhart (1997), respectively. Both of the two models extend the celebrated Fama-French three-factor model in Fama and French (1993), incorporating additional factors. For example, the FF5 model for the



Figure 2: S&P 500 and S&P 100 Indexes

expected return of the ith asset is given by

$$\mathbb{E}[r_i] = r_f + \beta_i (\mathbb{E}[r_m] - r_f) + \beta_{i,SMB} SMB + \beta_{i,HML} HML + \beta_{i,RMW} RMW + \beta_{i,CMA} CMA$$

where $r_f \geq 0$ is the risk-free rate, $\beta_{i,\cdot}$ are the factor loadings, SMB stands for the size factor (small minus big), HML represents the value factor (high book-to-market ratio minus low), RMW (robust high minus weak low) is the contrast in average returns between the strong and weak operating profitability portfolios, and CMA (conservative minus aggressive) represents the difference between the average return of two conservative investment portfolios and that of two aggressive investment portfolios. On the other hand, in the Carhart Four-Factor model, RMWand CMA are replaced with UMD, representing the momentum factor (high daily momentum minus the low). That is, the Carhart 4 model for the expected return of the *i*th asset is given by

 $\mathbb{E}[r_i] = r_f + \beta_i (\mathbb{E}[r_m] - r_f) + \beta_{i,SMB} SMB + \beta_{i,HML} HML + \beta_{i,UMD} UMD.$

5.2 Out-of-Sample Trading Performance

Using an initial account with \$1,000,000 and initial window sizes of M = 50 trading days, we carry out the Dynamic Sliding Window Algorithm 1, with a variation level $h := 0.1 \in (0, 1)$. Whenever the current volatility of portfolio σ exceeds the previous volatility σ^* by $\sigma \ge (1+h)\sigma^*$, we adjust the size M by reducing the window size of $c_- = 20\%$; otherwise, we retain the original size or increase the size by the same factor. The rationale behind this dynamic window sizing is that it aims to make the optimal weights more responsive to recent market conditions. In covariance matrix estimation, we use $\eta = 0.2$ in Equation (7) to emphasize the current estimates. Increasing η prioritizes older data, potentially reducing trading performance. Adjusting the window size to $c_- = 20\%$ notably enhances trading performance. Modifying c_+ or decreasing c_- can lead to over-reliance on older data or reduced incorporation of recent information.

Figures 3 and 4 show the account value trajectories of the market-based portfolio⁶, static mean-variance (MV) portfolio without BL model, dynamic mean-variance (MV) portfolio without

⁶The market-based portfolio represents equally weighted portfolio with $w_i = \frac{1}{n}$ for all i = 1, 2, ..., n = 101.

BL model, and the dynamic MV portfolio with dynamic BL, which is generated by dynamic sliding window Algorithm 1. The gray-shaded regions signify the 95% confidence interval over the account value trajectory generated by Algorithm 1. In the figures, the red dots indicate the instance when the window size M is adjusted. Some key performance metrics, summarized in Table 1, indicate that our Algorithm 1 leads to a promising performance by attaining a lower maximum drawdown to 19.57% and reaching a higher (annualized) Sharpe ratio $SR \approx 0.953$ compared with the market-based portfolio. The detailed year-by-year performance results are included in Appendix A.2, and the effect of various transaction costs can be seen in Appendix A.3.



Figure 3: Dynamic Sliding Window with FF5 Model

5.3 Computational Efficiency

Remarkably, on a 3.50 GHz laptop with 16 GB RAM, Algorithm 1 showcases computational efficiency in the sense that it takes about a total of 15.98 seconds to compute the views, estimate the expected returns, calculate the covariance matrix, and determine the optimal MV weights. More importantly, while the empirical studies shown in this paper focus on the tailored S&P 100, preliminary scalability analysis suggests that our Algorithm 1 can be extended to the larger portfolios, such as the entire S&P 500, without a significant loss of computational efficiency. The key to such computational efficiency lies in the fact that all the optimization problems solved by Algorithm 1 are indeed convex programs; hence, efficient solvers are available; e.g., see Diamond and Boyd (2016).

5.4 Robustness Test

To validate our approach, this section provides various robustness tests.

5.4.1 An Hypothetical Flipped Scenario

To evaluate the robustness of our approach, we further conducted a nonconventional hypothetical trading scenario by *flipping* the asset prices horizontally; see Figure 5 for the hypothetical index



Figure 4: Dynamic Sliding Window with Carhart 4 Model

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	$\mathrm{FF5}$				Carhart 4				
	$\overline{r^p}\left(\%\right)$	σ	SR	$d^*~(\%)$	$\overline{r^p}$ (%)	σ	SR	$d^*~(\%)$	
Market-Based Portfolio	12.93	17.07	0.716	34.35	12.93	17.07	0.716	34.35	
Static MV	3.89	11.83	0.269	30.88	3.89	11.83	0.269	30.88	
Dynamic MV w/o BL	4.01	10.31	0.389	20.59	4.01	10.31	0.389	20.59	
Algorithm 1	14.39	15.11	0.953	19.57	14.98	20.20	0.742	37.77	

Table 1: Trading Performance Metric (2013-2023)

prices of the S&P 500 and S&P 100 index over a one-year duration from January 1, 2013 to January 1, 2023. As shown in Figures 6 and 7, Algorithm 1 demonstrates a superior performance against other benchmark portfolios, even with the reversed price conditions. Similar to the initial case, the gray-shaded regions in the figures indicate the 95% confidence interval over the account value trajectory generated by Algorithm 1. Notably, the algorithm retains its edge even in the reversed price scenarios, thereby substantiating its robustness under varying market conditions. Some key performance metrics are summarized in Table 2. It should be noted that the trading performance is relatively sensitive to the initial choice of the window size M. Here, we adopt cross-validation to fine-tune a good start and subsequently allow it to be self-tuned during the remaining trading stages. The "optimal" initial size M^* , while outside the scope of the paper, requires further research.

5.4.2 Monte-Carlo Based Robustness Test

To validate the effectiveness of our approach, we conduct extensive Monte-Carlo simulations by assuming that the underlying stock prices follow the geometric Brownian motion (GBM) with an estimated drift rate and volatility derived from historical data spanning from January 1, 2013 to January 1, 2023, used in previous empirical studies. The detailed estimates are summarized in the Appendix. During the simulation, we generate 10,000 sample paths for each asset, which



Figure 5: Hypothetical S&P 500 and S&P 100 Index Price



Figure 6: Performance for the Hypothetical Trading Test Case (FF5).

leads to a total of 1,000,000 paths.

From Table 3, we see that Algorithm 1, when incorporated with the Carhart 4 factor model, outperforms the same algorithm using the FF5 model. This may be attributed to the fact the UMD factor in the Carhart 4 model can take advantage of the prices generated by the Monte Carlo simulations. Moreover, it should be noted that both of these portfolios have a lower Sharpe ratio compared to the Market-Based portfolio. This underperformance may stem from the limited factor data; hence, a less accurate prediction is expected. To address this,



Figure 7: Performance for the Hypothetical Trading Test Case (Carhart 4).

10010 2.	Table 2. Summary of renormance in Hypothetical Scenario								
	FF5				Carhart 4				
	$\overline{r^p}(\%)$	σ	SR	$d^*~(\%)$	$\overline{r^p}$ (%)	σ	SR	$d^*~(\%)$	
Market-Based Portfolio	-11.99	17.23	-0.737	78.43	-11.99	17.23	-0.737	78.43	
Static MV	-1.468	7.307	-0.298	20.59	-1.468	7.307	-0.298	20.59	
Dynamic MV w/o BL	-4.630	3.643	-1.271	33.84	-4.630	3.643	-1.271	33.84	
Algorithm 1	10.89	19.77	0.551	45.56	4.180	14.21	0.294	23.91	

 Table 2: Summary of Performance in Hypothetical Scenario

future work could involve collecting extensive historical factor data and modeling factor dynamics through suitable stochastic differential equations (SDEs). With this approach, one may be able to simulate the factor data, e.g., see Ammann and Verhofen (2008).

Table 3: Robustness Test Via Monte-Carlo Simulations									
	FF5				Carhart 4				
	$\overline{r^{p}}\left(\% ight)$	σ	SR	$d^*~(\%)$	$\overline{r^p}$ (%)	σ	SR	$d^*~(\%)$	
Market-Based Portfolio	16.40	11.48	1.396	16.79	16.40	11.48	1.396	16.79	
Static MV	9.356	6.751	1.338	11.55	9.356	6.751	1.338	11.55	
Dynamic MV w/o BL	6.936	4.974	1.400	7.447	6.936	4.974	1.400	7.447	
Algorithm 1	6.517	10.12	0.645	22.22	7.402	10.20	0.725	27.43	

Hyperparameters Selection 5.4.3

This section studies the impact of regularization parameters used in the Elastic Net regression via a cross-validation technique. In the special case where both $\lambda_1 = \lambda_2$ are set to zero, the Elastic Net regression reduces to the ordinary least squares (OLS) regression. Some of the

empirical results, as shown in Tables 4 and 5, indicate that when regularization terms $\lambda_1 = \lambda_2 = 0.5$, Algorithm 1 leads to a performance surpassing its non-regularized counterpart in terms of the Sharpe ratio (SR) and daily mean excess returns $(\overline{r^p})$ in both the FF5 and Carhart 4 factor models. Similar findings hold for other combinations of $\lambda_i > 0$ with $i \in \{1,2\}$ and $\sum_{i=1}^{2} \lambda_i = 1$. Consequently, Elastic Net regularization effectively mitigates estimation errors, thereby enhancing the overall performance and stability of the portfolio.

Table 4: Effect of Elastic Net (FF5)									
	$\lambda_1 = \lambda_2 = 0$					$\lambda_1 = \lambda_1$	$\lambda_1 = \lambda_2 = 0.5$		
	$\overline{r^p}\left(\%\right)$	σ	SR	$d^*~(\%)$	$\overline{r^p}$ (%)	σ	SR	$d^*~(\%)$	
Market-Based Portfolio	12.93	17.07	0.716	34.35	12.93	17.07	0.716	34.35	
Static MV	3.89	11.83	0.269	30.88	3.89	11.83	0.269	30.88	
Dynamic MV w/o BL	4.01	10.31	0.389	20.59	4.01	10.31	0.389	20.59	
Algorithm 1	6.19	21.23	0.292	47.40	14.39	15.11	0.953	19.57	

Table 5: Effect of Elastic Net (Carhart 4)									
		$\lambda_1 = \lambda_2 = 0$					$\lambda_1 = \lambda_2 = 0.5$		
	$\overline{r^p}\left(\% ight)$	σ	SR	$d^*~(\%)$	$\overline{r^p}$ (%)	σ	SR	$d^*~(\%)$	
Market-Based Portfolio	12.93	17.07	0.716	34.35	12.93	17.07	0.716	34.35	
Static MV	3.89	11.83	0.269	30.88	3.89	11.83	0.269	30.88	
Dynamic MV w/o BL	4.01	10.31	0.389	20.59	4.01	10.31	0.389	20.59	
Algorithm 1	5.26	22.45	0.234	39.18	14.98	20.20	0.742	37.77	

6 Concluding Remarks

This paper presents an innovative unification to adaptive portfolio management by integrating the Black-Litterman model with time-varying views. To mitigate potential estimation errors, we incorporated the Elastic Net regression. The use of a dynamic sliding window algorithm allows for a time-varying estimation of mean returns and covariance. These estimates are then used as inputs to solve a series of mean-variance portfolio optimization problems, resulting in time-varying optimal weights. Our results show, by extensive empirical studies using a portfolio with S&P 100 assets, a great potentiality when compared to standard trading strategies, such as the equal-weight buy-and-hold strategy.

As for future research directions, finding an "optimal" dynamically adjusted window size is a promising direction to pursue; see initial research along this line can be found in Ortiz Laguna et al. (2011); Wang and Hsieh (2022). In addition, as seen in Section 5.4.2, Algorithm 1 relies heavily on the factor data and price data. Therefore, to further examine the effectiveness, it might be interesting to explore methods for generating artificial factor data in future research, as suggested by Ammann and Verhofen (2008). Lastly, in the context of high-frequency trading, the fundamental business-related factor does not work in a much shorter time scale. Therefore, alternative nonlinear or dynamic factor models might be an option, e.g., Martellini and Ziemann (2007) with the fourth-moment CAPM or recursive neural network techniques might be worth pursuing.

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Data Availability Statement (DAS)

Raw data for this study were obtained from the Wharton Research Data Services (WRDS) database Wharton Research Data Services (2023). The derived data supporting the findings of this study are available from the authors on request.

Disclosure of Interest

No potential conflict of interest was reported by the author(s).

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A Appendix

A.1 S&P100 Portfolio Data for Monte-Carlo Simulation

The following table summarizes the estimated annualized drift rates and volatility, used in the Monte-Carlo simulation, of the S&P 100 portfolio. We have identified 47 Tickers, sorted alphabetically, that remain in our portfolio:

Ticker	Drift Rate	Volatility	Ticker	Drift Rate	Volatility
AAPL	18.69~%	28.50~%	MA	17.54~%	26.25~%
ABT	10.88~%	22.50~%	MCD	9.85~%	19.02~%
AMGN	11.51~%	24.34~%	MDLZ	8.42~%	20.68~%
AMZN	18.38~%	32.46~%	MO	5.14~%	21.32~%
AXP	8.42~%	28.68~%	MRK	9.74~%	20.57~%
BA	5.44~%	36.84~%	MS	12.00~%	31.48~%
BAC	11.08~%	31.18~%	MSFT	20.32~%	26.41~%
BLK	11.18~%	26.56~%	NEE	12.26~%	22.21~%
BMY	2.96~%	23.67~%	PEP	8.64~%	17.49~%
\mathbf{C}	1.16~%	32.03~%	PFE	3.50~%	21.49~%
CAT	8.12~%	28.29~%	\mathbf{PG}	7.56~%	17.69~%
CMCSA	9.28~%	23.89~%	\mathbf{PM}	3.11~%	21.94~%
CVS	4.38~%	24.10~%	QCOM	$4.75 \ \%$	33.32~%
CVX	2.16~%	27.84~%	SBUX	10.05~%	25.85~%
DHR	17.14~%	26.07~%	Т	1.84~%	21.26~%
DIS	4.74~%	25.87~%	TXN	13.18~%	26.85~%
GS	9.22~%	27.69~%	UNH	17.70~%	24.72~%
HON	10.88~%	21.97~%	UNP	11.43~%	24.68~%
IBM	0.12~%	22.57~%	UPS	6.12~%	23.13~%
INTC	3.53~%	31.00~%	V	17.02~%	24.17~%
JNJ	8.20~%	17.08~%	VZ	1.73~%	18.37~%
JPM	12.11~%	26.47~%	WFC	2.88~%	28.86~%
KO	5.54~%	17.49~%	XOM	1.43~%	25.85~%
LLY	21.23~%	25.21~%			

Throughout the trading period, other than the 47 Tickers mentioned above, there are additional 125 Tickers that were either removed or added from our asset pool, based on their market capitalization:

Ticker	Drift Rate	Volatility	Ticker	Drift Rate	Volatility
AAL	-3.49 %	49.46 %	ICE	12.48 %	22.60 %
ACN	14.70~%	23.70~%	ILMN	3.63~%	41.78~%
ADBE	20.58~%	31.00~%	INTU	17.26~%	29.16~%
ADI	12.03~%	28.93~%	ISRG	10.50~%	31.79~%
ADP	12.54~%	22.60~%	ITW	13.59~%	22.40~%
ADSK	10.86~%	35.80~%	JCI	7.65~%	25.15~%
AIG	5.25~%	31.51~%	KMB	5.58~%	18.91~%
AMAT	17.38~%	36.85~%	KMI	-5.54 %	29.95~%
AMD	11.53~%	56.52~%	\mathbf{KR}	9.38~%	26.96~%
AMT	9.51~%	23.91~%	LIN	10.54~%	21.71 %
APA	-22.25 %	55.62~%	LMT	14.72~%	21.35~%
APD	9.74~%	23.70~%	LOW	15.62~%	28.14~%
AVGO	26.63~%	34.40~%	LRCX	19.23~%	38.12~%
BAX	1.60~%	23.37~%	LYB	7.93~%	35.06~%
BDX	9.38 %	20.69~%	MAR	13.07~%	30.72~%
BEN	-2.54 %	30.18 %	MCK	11.84 %	26.77%
BIIB	-1.00 %	39.70 %	MCO	17.46 %	27.22 %
BK	6.12 %	26.53 %	MDT	5.70 %	22.16 %
BKNG	11 56 %	31.81 %	MET	5 24 %	29.95 %
BSX	16 14 %	27 29 %	MMC	14 87 %	19 29 %
CB	9.95 %	22 21 %	MMM	2 21 %	22.03 %
CCI	7.44 %	23.46 %	MPC	13.09 %	39.56%
CI	12.23%	28.69 %	MU	11.24 %	44.58 %
CL	4.99 %	18.16 %	NEM	-8.13 %	35.92 %
CME	12.77%	23.82 %	NFLX	20.22 %	48.94 %
COF	4.96 %	33.72 %	NKE	10.39 %	27.61 %
COP	2.47%	35.69 %	NOC	16.39 %	23.03 %
COST	17.32 %	20.40 %	NOV	-18.03 %	43.67 %
CRM	13.66 %	34.36 %	NSC	8.07 %	26.92 %
CSCO	7 60 %	25.12 %	NVDA	32 61 %	43 45 %
CSX	10.73%	27.31%	ORCL	9.69 %	25.61 %
CTSH	3 20 %	28.95 %	OXY	-11 50 %	46 51 %
D	0.20 %	21.24 %	PGR	18.31 %	22.18 %
DAL	6 56 %	39 71 %	PLD	12.02%	25 10 %
DD	4 16 %	29 47 %	PNC	6 50 %	27.59%
DE	11 46 %	27.70%	PRU	4 10 %	31.32 %
DUK	5 27 %	19 40 %	PSA	7.33%	21.39 %
EBAY	5.98 %	29.36 %	REGN	16 84 %	34 88 %
ECL	8 40 %	23.69 %	BTX	3 69 %	25.40 %
EL	4 83 %	28.00%	SCHW	10.40 %	33.02 %
ELV	12.23%	28.70%	SHW	17.99%	25.02%
EMB	5 18 %	26.10 %	SLB	-7.06 %	37 18 %
EOG	0.89 %	39 11 %	SO	5.63 %	20 10 %
EOIX	15 39 %	27 11 %	SPG	-1.05 %	34 08 %
LWIN	10.00 /0	21.11 /0	DI U	-1.00 /0	04.00 /0
ETN	16.00%	30.86 %	SPGI	17 20 %	25 37 %
ETN EW	$16.00\ \%$	30.86 % 32.66 %	SPGI STT	17.20 % 2.09 %	25.37 % 31 49 %

Ticker	Drift Rate	Volatility	Tick	er Drift Rate	Volatility
F	-1.59~%	33.20~%	TFC	0.88~%	30.82~%
FCX	-10.56~%	51.01~%	TGT	6.81 %	28.64~%
FDX	5.41~%	29.83~%	TJX	12.91~%	25.68~%
FIS	4.16~%	27.84~%	TMO) 17.74 %	24.05~%
GD	10.94~%	21.66~%	TMU	JS 14.60%	31.76~%
GE	-3.50~%	31.87~%	TRV	9.29~%	22.36~%
GILD	9.15~%	27.49~%	TSL	A 24.92 %	56.42~%
GIS	5.03~%	19.63~%	USB	2.32~%	27.29~%
GM	0.91~%	34.02~%	VLO	11.70 %	38.63~%
GOOGL	14.13~%	26.98~%	VRT	X 12.43 %	41.82~%
GPN	9.35~%	30.63~%	WBA	A -4.12 %	29.08~%
HAL	-7.69~%	43.83~%	WM	14.75~%	18.03~%
HCA	17.97~%	32.55~%	WM	B -0.92 %	39.45~%
HD	16.71~%	23.35~%	WM	T $8.02~\%$	19.88~%
HPQ	4.16~%	34.36~%	YUN	I 8.18 %	24.14~%
HUM	10.08~%	29.51~%			

A.2 Performance Results by Year

Tables 8 and 9 summarize yearly performance metrics spanning from 2013 to 2023 with the FF5 model and the Carhart 4-factor model, respectively.

A.3 Different Transaction Costs

Tables 10 and 11 summarize performance metrics under various transaction costs $TC \in \{0, 0.01, 0.1, 1\}$ in percentage with the FF5 and the Carhart 4 factor model, respectively.

			2013			2014			
	$\overline{r^p}\left(\% ight)$	σ	SR	$d^*~(\%)$	$\overline{r^p}$ (%)	σ	SR	$d^*~(\%)$	
Market-Based Portfolio	30.46	11.17	2.727	5.390	15.20	11.34	1.340	7.470	
Static MV	4.853	8.255	0.588	5.789	2.368	8.844	0.268	8.424	
Dynamic MV w/o BL	4.673	7.973	0.586	6.637	1.404	8.269	0.170	8.428	
Algorithm 1	18.64	10.56	1.765	5.725	-0.816	15.66	-0.052	16.99	
			2015	18 (04)		20	016 G D	1* (07)	
	r^{p} (%)	σ	SR	d^* (%)	r^p (%)	σ	SR	d^{*} (%)	
Market-Based Portfolio	-0.01	15.55	-0.001	11.94	13.85	13.00	1.065	11.92	
Static MV	19.27	9.413	2.047	5.851	-4.166	7.571	-0.550	8.635	
Dynamic MV w/o BL	4.997	7.908	0.632	6.233	-4.045	8.225	-0.492	11.49	
Algorithm 1	21.30	13.44	1.584	10.90	10.30	14.14	0.728	12.98	
			2017			20	018		
	$\overline{r^p}\left(\% ight)$	σ	SR	$d^*~(\%)$	$\overline{r^p}$ (%)	σ	SR	$d^{st}~(\%)$	
Market-Based Portfolio	18.35	6.580	2.789	3.268	0.119	15.14	0.008	10.54	
Static MV	7.483	7.753	0.965	6.414	-6.329	9.601	-0.659	11.31	
Dynamic MV w/o BL	14.04	7.891	1.779	3.800	5.851	11.66	0.502	7.733	
Algorithm 1	19.14	13.42	1.426	6.033	21.82	20.77	1.051	11.20	
			2019			20	020		
	$\overline{r^p}(\%)$	σ	SR	d^{*} (%)	$\overline{r^p}$ (%)	σ	SR	d^{*} (%)	
Market-Based Portfolio	19.91	13.66	1.458	9.113	20.85	34.58	0.603	34.35	
Static MV	24.94	10.66	2.339	5.321	6.393	17.28	0.370	11.48	
Dynamic MV w/o BL	-10.30	9.011	-1.143	10.61	18.89	17.44	1.084	10.79	
Algorithm 1	15.99	12.15	1.316	5.944	52.61	21.54	2.443	9.028	
			2021	* (~)		2	2022		
	r^{p} (%)	σ	SR	d^{*} (%)	r^p (%)	σ	SR	d^{*} (%)	
Market-Based Portfolio	24.42	11.82	2.066	5.288	-5.379	21.04	-0.256	21.87	
Static MV	1.243	12.98	0.096	12.99	-12.05	19.22	-0.627	22.73	
Dynamic MV w/o BL	15.00	10.59	1.417	6.105	-4.397	9.988	-0.440	13.47	
Algorithm 1	3.186	12.74	0.250	14.41	-7.493	12.26	-0.611	13.48	

 Table 8: Trading Performance Metrics (FF5)

			2013			2014			
	$\overline{r^p}\left(\%\right)$	σ	SR	$d^*~(\%)$	$\overline{r^p}$ (%)	σ	SR	$d^*~(\%)$	
Market-Based Portfolio	30.46	11.17	2.727	5.390	15.20	11.34	1.340	7.470	
Static MV	4.853	8.255	0.588	5.789	2.368	8.844	0.268	8.424	
Dynamic MV w/o BL	4.673	7.973	0.586	6.637	1.404	8.269	0.170	8.428	
Algorithm 1	29.64	15.15	1.956	7.272	3.768	14.70	0.256	9.441	
			2015		2016				
	r^{p} (%)	σ	SR	d^{*} (%)	r^p (%)	σ	SR	d^{*} (%)	
Market-Based Portfolio	-0.01	15.55	-0.001	11.94	13.85	13.00	1.065	11.92	
Static MV	19.27	9.413	2.047	5.851	-4.166	7.571	-0.550	8.635	
Dynamic MV w/o BL	4.997	7.908	0.632	6.233	-4.045	8.225	-0.492	11.49	
Algorithm 1	36.60	16.87	2.170	7.832	13.87	13.75	1.008	7.205	
						_			
	— (64)		2017		- (~)	2	018	** (04)	
	r^{p} (%)	σ	SR	d^* (%)	r^p (%)	σ	SR	d^{*} (%)	
Market-Based Portfolio	18.35	6.580	2.789	3.268	0.119	15.14	0.008	10.54	
Static MV	7.483	7.753	0.965	6.414	-6.329	9.601	-0.659	11.31	
Dynamic MV w/o BL	14.04	7.891	1.779	3.800	5.851	11.66	0.502	7.733	
Algorithm 1	24.36	12.59	1.934	8.088	3.133	22.75	0.138	20.77	
			2010			9	020		
	$\overline{r^p}(\%)$	σ	SR	d^{*} (%)	$\overline{r^p}$ (%)	σ^{2}	SR	d^{*} (%)	
	10.01	10.00	1.450	a (70)	(70)	0	0.000	a (70)	
Market-Based Portfolio	19.91	13.66	1.458	9.113	20.85	34.58	0.603	34.35	
Static MV	24.94	10.00	2.339	5.321	0.393	17.28	0.370	11.48	
Algorithm 1	-10.30	9.011 12.69	-1.143	10.01	18.89	17.44	1.084	10.79	
Algorithm 1	-2.078	13.08	-0.190	8.817	44.40	38.79	1.140	25.89	
			2021			2	2022		
	$\overline{r^p}(\%)$	σ	SR	$d^{*}~(\%)$	$\overline{r^p}$ (%)	σ	SR	$d^*~(\%)$	
Market-Based Portfolio	24.42	11.82	2.066	5.288	-5.379	21.04	-0.256	21.87	
Static MV	1.243	12.98	0.096	12.99	-12.05	19.22	-0.627	22.73	
Dynamic MV w/o BL	15.00	10.59	1.417	6.105	-4.397	9.988	-0.440	13.47	
Algorithm 1	32.63	18.73	1.743	10.06	-17.82	21.07	-0.845	25.31	

Table 9: Trading Performance Metrics (Carhart 4)

						· · · · · ·		
		Т	C = 1%		TC = 0.1%			
	$\overline{r^p}\left(\%\right)$	σ	SR	$d^*~(\%)$	$\overline{r^p}$ (%)	σ	SR	$d^*~(\%)$
Market-Based Portfolio	12.93	17.07	0.716	34.35	12.93	17.07	0.716	34.35
Static MV	3.89	11.83	0.269	30.88	3.89	11.83	0.269	30.88
Dynamic MV w/o BL	4.01	10.31	0.389	20.59	4.32	10.31	0.419	20.10
Algorithm 1	14.39	15.11	0.953	19.57	14.76	15.11	0.977	19.36
		TC	r = 0.01%)		TC	= 0%	
	$\overline{r^p}\left(\% ight)$	σ	SR	$d^*~(\%)$	$\overline{r^p}$ (%)	σ	SR	$d^*~(\%)$
Market-Based Portfolio	12.93	17.07	0.716	34.35	12.93	17.07	0.716	34.35
Static MV	3.89	11.83	0.269	30.88	3.89	11.83	0.269	30.88
Dynamic MV w/o BL	4.35	10.31	0.422	20.05	4.36	10.31	0.423	20.04
Algorithm 1	14.80	15.11	0.980	19.33	14.81	15.11	0.980	19.33

Table 10: Performance Under Different Transaction Costs (FF5)

Table 11: Performance Under Different Transaction Costs (Carhart 4)

		Т	C = 1%					
	$\overline{r^p}(\%)$	σ	SR	$d^*~(\%)$	$\overline{r^p}$ (%)	σ	SR	$d^*~(\%)$
Market-Based Portfolio	12.93	17.07	0.716	34.35	12.93	17.07	0.716	34.35
Static MV	3.89	11.83	0.269	30.88	3.89	11.83	0.269	30.88
Dynamic MV w/o BL	4.01	10.31	0.389	20.59	4.32	10.31	0.419	20.10
Algorithm 1	14.98	20.20	0.742	37.77	15.26	20.19	0.756	37.47
		TC	t = 0.01%)		TC	= 0%	
	$\overline{r^p}(\%)$	σ	SR	$d^*~(\%)$	$\overline{r^p}$ (%)	σ	SR	$d^*~(\%)$
Market-Based Portfolio	12.93	17.07	0.716	34.35	12.93	17.07	0.716	34.35
Static MV	3.89	11.83	0.269	30.88	3.89	11.83	0.269	30.88
Dynamic MV w/o BL	4.35	10.31	0.422	20.05	4.36	10.31	0.423	20.04
Algorithm 1	15.29	20.19	0.757	37.44	15.29	20.19	0.757	37.44