Energy self-extraction of a Kerr black hole with the force-free magnetosphere frame-dragged

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It is shown that the zero-angular-momentum-observers (ZAMOs) circulating with the frame-dragging-angular-velocity ω plays a leading part in energy extraction. When the condition $\Omega_{\rm F} < \Omega_{\rm H}$ is satisfied, where $\Omega_{\rm H}$ and $\Omega_{\rm F}$ are the horizon and field-line (FL) angular-velocities (AVs), they will see that the null surface $S_{\rm N}$ with $\omega_{\rm N} = \Omega_{\rm F}$ always exists in the force-free magnetosphere. Denoting the ZAMO-measured FLAV with $\Omega_{\rm F\omega} \equiv \Omega_{\rm F} - \omega$, this surface $S_{\rm N}$ where $\Omega_{\rm F\omega} = 0$ defines the gravito-magneto-centrifugal divider of the magnetosphere, with a kind of plasma-shed on it. The outer domain $\mathcal{D}_{\rm (out)}$ outside $S_{\rm N}$ spins forward ($\Omega_{\rm F\omega} > 0$), whereas the inner domain $\mathcal{D}_{\rm (in)}$ inside spins backward ($\Omega_{\rm F\omega} < 0$). The force-free and freezing-in conditions break down on $S_{\rm N}$, thereby allowing the particle-current sources to be set up on $S_{\rm N}$. Because the electric field $\boldsymbol{E}_{\rm p}$ reverses direction there, the Poynting flux reverses direction as well from outward to inward, though the positive angular momentum always flows outwardly. Electromagnetic self-extraction of energy will be possible only through the frame-dragged magnetosphere, with the inner domain $\mathcal{D}_{\rm (in)}$ nested between the horizon and the surface $S_{\rm N}$, in order to comply with the 1st and 2nd laws of thermodynamics.

I. INTRODUCTION

More than four decades have passed since the pioneering paper by Blandford and Znajek [1] was published on the electromagnetic extraction of energy from Kerr black holes (BHs). This controversial task, however, still remains a big challenge even in the latest new frontier in modern classical physics (see Thorne and Blandford [2]). The purpose of this paper is to venture into challenging this complicated and formidable task.

Fundamental concepts and expressions, as well as the basic formulation of general relativity, thermodynamics and electrodynamics, necessary for elucidating extraction of energy from Kerr holes, have fortunately been given in the almost complete form already four decades ago; [1], Znajek [3, 4], Blandford [5], Macdonald and Thorne [6], Thorne et al. [7]. Phinney [8, 9] developed a comprehensive model for BH-driven hydromagnetic flows or jets for active galactic nuclei (AGNs). These references presume, however, that a battery exists on the event horizon, with magnetic field lines threading the horizon. That is, a magnetized Kerr hole would possess not only a battery but also an internal resistance on the horizon. We refer to this model as the single-pulsar model with a single electric circuit. This is because the magnetosphere consists of double wind structures with a negligible violation of the force-free condition in-between for particle production and a single series circuit, with a battery on the horizon and two resistances on the horizon and infinity surface.

Uchida [10, 11] constructed a comprehensive theory of force-free electromagnetic fields, where it was pointed out that the breakdown of the force-free approximation is not a defect proper to force-freeness. Gralla and Jacobson [12] made extensive use of his theory as 'spacetime approach' to force-free magnetospheres of various objects. Barriga et al. [13] pointed out that many references on astrophysical force-free plasma leave open the issue of identifying the actual sources of the force-free electromagnetic field. We show here that the actual breakdown of the force-free condition we meet with provides rather firm foundations necessary to rebuild an active force-free magnetosphere, including the built-in particle-current sources.

We remake the pulsar force-free magnetosphere, with the dragging of inertial frames fully taken into account. The central premise is as follows: the large-scale poloidal magnetic field $\boldsymbol{B}_{\rm p}$ will be trapped (i.e., frame-dragged) by the hole into circulation around the hole with the frame-dragging-angular velocity (FDAV) ω. Magnetic FLs extend from near the horizon $S_{\rm H}$ to the infinity surface S_∞, with the field-line-angular-velocity (FLAV) $\Omega_{\rm F}$ kept constant (Ferraro's law of isorotation). The zero-angular-momentum-observers (ZAMOs) circulating with the FDAV ω will then see that Ferraro's law does not hold for the ZAMO-measured FLAV $\Omega_{\rm F\omega} = \Omega_{\rm F\omega} - \omega$ (see Eq. (III.2a)). The force-free magnetosphere consisting of charge-separated plasma is divided by the null surface $S_{\rm N}$ where $\Omega_{\rm F\omega} = 0$, i.e., $\omega = \Omega_{\rm F} \equiv \omega_{\rm N}$, into the two domains, outer semi-classical (SC) and inner general-relativistic (GR), $\mathcal{D}_{\rm (out)}$ and $\mathcal{D}_{\rm (in)}$. The ZAMOs will soon realize that the breakdown of the force-free and freezing-in conditions is inevitable on the null surface $S_{\rm N}$ with $\Omega_{\rm F\omega} \gtreqless 0$.

We think of *three* membranes in our twin-pulsar model (cf. [7]). The first *two* resistive ones on the infinity and horizon

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surfaces, $\mathcal{S}_{\rm ff\infty}$ and $\mathcal{S}_{\rm ffH}$ terminate the two force-free domains $\mathcal{D}_{\rm (out)}$ and $\mathcal{D}_{\rm (in)}$ by particle acceleration on $\mathcal{S}_{\rm ff\infty}$ and entropy production on $\mathcal{S}_{\rm ffH}$, respectively. The third *one* is the inductive membrane $\mathcal{S}_{\rm N}$ on the null surface $\rm S_N$, which hides the particle-current sources in the force-free limit.

Blandford and Globus [14] recently constructed the models for the ergo-magnetosphere, ejection disc, and magnetopause in M87. They argued that the force-free approximation is justifiable in the vicinity of the black hole. There is actually good chemistry between thermodynamics for the Kerr hole with *two* hairs (e.g., entropy S and angular momentum J) and electrodynamics for the pulsar magnetosphere with *two* conserved quantities along each FL ($\Omega_{\rm F}$ and I). This is thanks to the frame-dragging effect, which unifies force-free electrodynamics with thermodynamics. We argue that the force-free approximation is flexible and robust enough to accept the 'breakdown' of the force-free condition (as well as the freezing-in one) on the null surface $\rm S_N$, which always exists between the two light surfaces, $\rm S_{oL}$ and $\rm S_{iL}$, when the hole loses energy [1].

A brief outline of each Sec. is as follows. Sec. II points out that the Kerr holes are a rotating mass of 'entropy matter', and hence, without invoking the influx of *negative* angular momentum, it will be impossible to realize the outgoing flux of *positive* angular momentum, thereby extracting energy from the Kerr BH. In Sec. III, other than the conserved FLAV $\Omega_{\rm F}$, we newly define the ZAMO-FLAV $\Omega_{\rm F\omega}$, toward constructing gravito-thermo-electrdynamics (GTED). Then, from a thermodynamic point of view, we discuss the similarity and dissimilarity of the force-free magnetospheres between Kerr holes and pulsars in Secs. III A and III B.

In Sec. IV, in order to elucidate the KBH's elaborate mechanism of energy extraction, we use expediently the wind-, flux- and circuit-analyses. In particular, Sec. IV C argues that a pair of unipolar induction batteries with electromotive forces (EMFs), $\mathcal{E}_{\rm (out)}$ and $\mathcal{E}_{\rm (in)}$ will drive currents to flow through the circuits $C_{\rm (out)}$ and $C_{\rm (in)}$ in the outer SC domain $\mathcal{D}_{\rm (out)}$ and the inner GR domain $\mathcal{D}_{\rm (in)}$, respectively (see FIG. 2). There will be a huge voltage drop ΔV across $\rm S_N$ between the two EMFs for particle production.

Sec. V shows that the angular-momentum density of the electromagnetic field is positive ($\varepsilon_{\rm J} > 0$) in $\mathcal{D}_{\rm (out)}$ and negative ($\varepsilon_{\rm J} < 0$) in $\mathcal{D}_{\rm (in)}$. That is, the null surface $\rm S_N$ coincides with the zero-angular-momentum surface $\rm S_{ZAMD}$. The energy density $\varepsilon_{\rm E}$ changes positive to negative somewhere inward beyond the null surface $\rm S_N$.

In Sec. VI, the ZAMOs will see that reversal of the electric field $\boldsymbol{E}_{\rm p}$ and the Poynting flux $\boldsymbol{S}_{\rm EM}$ on $\rm S_N$ leads to the breakdown of the force-free and freezing-in conditions. We show how critical the severance of the current-stream lines on $\rm S_N$ due to a complete violation of the force-free condition because this surface $\rm S_N$ will be widened to such a gap $\mathcal{G}_{\rm N}$ as filled with zero-angular-momentum-particles (ZAM-particles) pair-produced due to the voltage drop ΔV (see Sec. IV C).

In Sec. VII, the structure of a gap $\mathcal{G}_{\rm N}$ under the inductive membrane $\mathcal{S}_{\rm N}$ is discussed with respect to the pinning-down of threading FLs on the pair-created plasma, resulting in magnetization of it and a magneto-centrifugal plasma-shed on the zero-angular-momentum surface $\rm S_{ZAMD}$, etc. (see also FIG. 3 for a simple model for the current function $I(\ell, \Psi)$).

In Sec. VIII, we argue the boundary condition for determining the eigenvalue of $\Omega_{\rm F} = \omega_{\rm N}$ on $\rm S_N$ in the steady axisymmetric state. Some new pieces of evidence shown will be helpful in understanding an enigmatic flow of *positive* angular momentum from the horizon membrane $\mathcal{S}_{\rm ffH}$, beyond the inductive membrane $\mathcal{S}_{\rm N}$ covering the Gap $\mathcal{G}_{\rm N}$, to the infinity membrane $\mathcal{S}_{\rm ff\infty}$ (see Sec. VIII A). Thus, the ZAM-Gap $\mathcal{G}_{\rm N}$ will allow us to impose the conservation law of angular momentum as the boundary condition determining the eigenfunction $\Omega_{\rm F}(\Psi) = \omega_{\rm N}$; this means that the eigen-magnetosphere with $\omega_{\rm N} = \Omega_{\rm F}$ is 'frame-dragged' by the hole's rotation (see Sec. VIII B).

Sec. IX attempts to explain the null surface $\rm S_N$ in terms of a new kind of rotational-tangential discontinuity (RTD) in the GR setting [15, 28]. We conjecture that this RTD involving a voltage drop ΔV between the two EMFs will bring up a new mechanism of pair-particle creation at work on $\rm S_N$ toward widening to a GR gap $\mathcal{G}_{\rm N}$. As opposed to the single-pulsar model based on BH electrodynamics with a negligible violation of the force-free condition, we propose the twin-pulsar model based on GTED because there will be 'two pulsar-type magnetospheres' coexisting, outer prograde- and inner retrograde-rotating, respectively, with the RTD spark-Gap in-between for the supply of electricity and particles.

Sec. X discusses the energetics and structure of the twin-pulsar model as opposed to that of the single-pulsar model (cf. e.g. IV D in [7]). The last Sec. XI is devoted to discussions and conclusions with some remaining issues listed.

Appendix A discusses the position and shape of the null surface $\rm S_N$ in the force-free magnetosphere for the parameter $0 \le h = a/r_{\rm H} \le 1$.

II. THE KERR BLACK HOLE AS A THERMODYNAMIC OBJECT

The no-hair theorem says that Kerr holes possess only two hairs. When one chooses S and J as two extensive variables, then all other thermodynamic quantities are expressed as functions of these two. For example, the BH's mass-energy M is expressed in terms of S and J, as follows;

$$M = \sqrt{(\hbar c S/4\pi k G) + (\pi k c J^2/\hbar G S)}. \qquad \text{(II.1)}$$

As one can, in principle, utilize a Kerr hole as a Carnot engine [16], it may be regarded as a thermodynamic object but not as an electrodynamic one because the Kerr hole by itself stores no extractable electromagnetic energy. Therefore, the Kerr hole may be regarded as a huge rotating mass of entropy matter (see, e.g., [17]), fundamentally different from the magnetized rotating NS, which consists of normal matter with magnetic field lines emanating outside. Thus, its evolutionary behaviours, such as due to the extraction of angular momentum, are strictly governed by the four laws of thermodynamics (see, e.g., [7] for a succinct summary). The mass M of the hole is divided into the irreducible and rotational masses, i.e.,

$$M = M_{\rm irr} + M_{\rm rot}, \quad \text{(II.2a)}$$

$$M_{\rm irr} = \frac{M}{\sqrt{1+h^2}} = \sqrt{c^4 A_{\rm H}/16\pi G^2} = \sqrt{\hbar c S/4\pi k G}, \quad \text{(II.2b)}$$

$$M_{\rm rot} = M[1 - 1/\sqrt{1+h^2}], \quad \text{(II.2c)}$$

where $A_{\rm H}$ is the horizon surface area, and h is defined as the ratio of $a \equiv J/Mc$ to the horizon radius $r_{\rm H}$, i.e.,

$$h = \frac{a}{r_{\rm H}} = \frac{2\pi k J}{\hbar S} = \frac{2GM\Omega_{\rm H}}{c^3}. \quad \text{(II.3)}$$

The Kerr hole's thermo-rotational state is uniquely specified by S and J, or its M and h. We see $h = 0$ for a Schwarzschild BH and $h = 1$ for an extreme-Kerr hole [18, 19, 23]. The evolutional state of the BH losing energy is then specified as the timeline of the function $h(t)$ for the 'outer horizon' in $0 \le h \le 1$.

The hole's $M_{\rm irr}$ and $A_{\rm H}$ are functions of S only, but $M_{\rm rot}$ may be a function of S as well as J. Therefore, when the hole loses angular momentum by, e.g., an influx of *negative* angular momentum ($dJ < 0$), the hole's total mass and rotational mass will decrease, i.e., $dM < 0$ and $dM_{\rm rot} < 0$, while $dM_{\rm irr} > 0$ and $T_{\rm H} dS > 0$ always hold.

Different from a magnetized NS consisting of 'normal matter', a Kerr hole with $M = M(S, J)$ in Eq. (II.1) will be the biggest rotating mass of 'entropy matter'. Then, a naive question comes to mind: "how do magnetic field lines manage to thread and survive in 'entropy matter' under the horizon?" If a battery really existed in the horizon, this might indeed necessitate the threading of FLs into the matter under $S_{\rm H}$ such as, e.g., the 'imperfect conductor' [1, 4].

The zeroth law of thermodynamics indicates that two 'intensive' variables, $T_{\rm H}$ (the surface temperature) and $\Omega_{\rm H}$, conjugate to S and J, respectively, are constant on $S_{\rm H}$, e.g., $\omega \to \Omega_{\rm H}$ for $\alpha \to 0$. In passing, the third law indicates that "by a finite number of operations, one cannot reduce the surface temperature to the absolute zero with $h = 1$." In turn, "the finite processes of mass accretion with angular momentum cannot accomplish the extreme Kerr state with $h = 1$, $T_{\rm H} = 0$ and $\Omega_{\rm H} = c^3/2GM$" [19]. Incidentally, the 'inner-horizon' thermodynamics can formally be constructed analogously to the 'outer-horizon' thermodynamics [20, 21].

It is the first and second laws that govern the extraction process of energy, i.e.,

$$c^2 dM = T_{\rm H} dS + \Omega_{\rm H} dJ, \quad \text{(II.4a)}$$

$$T_{\rm H} dS \ge 0, \quad \text{(II.4b)}$$

where $T_{\rm H}$ and $\Omega_{\rm H}$ are uniquely expressed in terms of J and S from Eq. (II.1) or M and h [18];

$$T_{\rm H} = c^2(\partial M/\partial S)_J, \quad \Omega_{\rm H} = c^2(\partial M/\partial J)_S. \quad \text{(II.5)}$$

When Kerr holes are regarded simply as a huge rotating mass of entropy matter confined by its self-gravity inside the event horizon, we conjecture that they will not allow the presence of magnetospheric or interstellar magnetic field lines threading nor being anchored in the matter inside the horizon. Therefore, the Kerr hole itself will be unable to behave like a battery (see also Punsly and Coroniti [22]). It will nevertheless be argued here that Kerr holes can acquire and keep a force-free magnetosphere by making full use of frame-dragging. It is indeed frame-dragging that acts as a bridge between (black hole) thermodynamics and (pulsar) electrodynamics across the event horizon (see [23–29] for the track of investigation so far).

III. THE FORCE-FREE APPROXIMATION AND THERMODYNAMICS

A. Kerr black hole force-free magnetospheres

The magnetospheres filled with perfectly conductive plasma around NSs and Kerr BHs are considered under the two basic presumptions of the force-free and freezing-in conditions in the stationary axisymmetric state with $\boldsymbol{E}_{\rm t} = 0$. These two conditions are given by

$$\varrho_{\rm e}\boldsymbol{E} + \boldsymbol{j}/c \times \boldsymbol{B} = 0, \quad \text{(III.1a)}$$

$$\boldsymbol{E} + \boldsymbol{v}/c \times \boldsymbol{B} = 0, \quad \text{(III.1b)}$$

where $\boldsymbol{B}$ and $\boldsymbol{E}$ are the electromagnetic fields, $\boldsymbol{j}$ is the electric current and $\boldsymbol{v}$ is the velocity of force-free charged particles. All electromagnetic quantities are measured by the ZAMOs circulating with the FDAV ω. The magnetosphere is then characterized by the two quantities, $\Omega_{\rm F}(\Psi)$ and $I(\Psi)$. The latter is the angular-momentum flux/current function. Both are conserved along each FLs, where Ψ is the stream function, and $\Omega_{\rm F}$ and ω are the AVs relative to absolute space [6].

There are then the two pivotal FLAVs; one is naturally the conserved FLAV $\Omega_{\rm F}$, and the other is the ZAMO-FLAV $\Omega_{{\rm F}\omega}$;

$$\Omega_{{\rm F}\omega} = \Omega_{\rm F} - \omega, \quad \text{(III.2a)}$$

$$\Omega_{\rm F} = \Omega_{{\rm F}\omega} + \omega. \quad \text{(III.2b)}$$

It seems that the ZAMO-FLAV $\Omega_{{\rm F}\omega}$ was not explicitly defined in the literature so far except [28]. We emphasize the need of *additionally* defining $\Omega_{{\rm F}\omega}$ for clarifying the electromagnetic extraction of energy from Kerr holes. The 3 + 1-formulation of BH electrodynamics [6] was perfectly accomplished analytically, but lamentably lacks *the finishing touches*, that is, the ZAMO-FLAV $\Omega_{{\rm F}\omega}$ was missing in [1, 6, 7] and also [10, 11], although the null surface $S_{\rm N}$ was already defined by $\omega_{\rm N} = \Omega_{\rm F}$, i.e., $\Omega_{{\rm F}\omega} = 0$, in [1] (see Sec. IV B 1 and [49]).

The null surface $S_{\rm N}$, where $\Omega_{{\rm F}\omega} \gtreqless 0$ for $\omega \lesseqgtr \Omega_{\rm F}$, divides the magnetosphere unequivocally into the two domains, i.e., the outer SC and inner GR domains. This $\Omega_{{\rm F}\omega}$ is also related to the Poynting flux (see Eqs. (IV.12b) and (IV.13a,b)), and hence plays an indispensable role in defining the efficiency of energy extraction by the 2nd law of BH thermodynamics [1]. Eqs. (III.2b,a) will lead to the 1st and the 2nd laws, respectively, on $S_{\rm H}$ where $\alpha \to 0$ and $\omega \to \Omega_{\rm H}$ (see Eqs. (III.8a,b)).

The poloidal and toroidal components of $\boldsymbol{B}$ and the electric field $\boldsymbol{E}_{\rm p}$ are defined by

$$\boldsymbol{B}_{\rm p} = -(\boldsymbol{t}\times\boldsymbol{\nabla}\Psi/2\pi\varpi), \;\; \boldsymbol{B}_{\rm t} = -(2I(\Psi)/\varpi c\alpha)\boldsymbol{t}, \quad \text{(III.3a)}$$

$$\boldsymbol{E}_{\rm p} = -(\Omega_{\rm F\omega}/2\pi\alpha c)\boldsymbol{\nabla}\Psi, \quad \text{(III.3b)}$$

$$v_{\rm F} = \Omega_{\rm F\omega}\varpi/\alpha, \quad \text{(III.3c)}$$

where $\boldsymbol{B}_{\rm t}$ is regarded as the swept-back component of $\boldsymbol{B}_{\rm p}$ by inertial loading (i.e., particle acceleration and entropy production; see Sec. IV A 3), and $v_{\rm F}$ is the ZAMO-FLRV (field-line-rotational-velocity). Because $v_{\rm F}$ stands for the physical velocity of FLs relative to the ZAMOs, $\boldsymbol{E}_{\rm p}$ is entirely induced by the motion of the magnetic field lines, i.e., $\boldsymbol{E}_{\rm p} = -(v_{\rm F}\boldsymbol{t}/c)\times\boldsymbol{B}_{\rm p}$. Eqs. (III.3c,b) correspond to equations. (5.2) and (5.3) for v^F and $\boldsymbol{E}$ in [6], and also to equations. (4.32) and (4.33) in [7], although factor $(\Omega_{\rm F}-\omega)$ appears instead of the ZAMO-FLAV $\Omega_{\rm F\omega}$ throughout [6, 7]. When we take the physical roles of Eqs. (III.2a,b) into account, this difference is never trivial (see Eq. (IV.12b)).

The role of the FDAV ω is thus to make the ZAMO-FLAV $\Omega_{\rm F\omega}$ 'violate' Ferraro's law of iso-rotation and change sign on $\rm S_N$, and hence $\boldsymbol{E}_{\rm p}$ in Eq. (III.3b) changes direction as well. This leads to the elucidation of 'how and where' the basic conditions given in Eqs. (III.1a,b) should be broken down, thereby opening the unique path toward GTED (see Sec. IV).

The AV $\Omega_{\rm F}$ possesses two roles, i.e., as the FLAV and the electric potential gradient, and is constant along each FL. With respect to I, 'current/angular-momentum duality' also holds in the degenerate state. From equations (5.6a,b,c) in [6] for the current and charge densities, we have the poloidal and toroidal components of $\boldsymbol{j}$ and the charge density $\varrho_{\rm e}$;

$$\boldsymbol{j}_{\rm p} = -\frac{1}{\alpha}\frac{dI}{d\Psi}\boldsymbol{B}_{\rm p}, \quad \text{(III.4a)}$$

$$j_{\rm t} = \varrho_{\rm e}v_{\rm F} + \frac{2I}{\varpi\alpha^2 c}\frac{dI}{d\Psi}, \quad \text{(III.4b)}$$

$$\varrho_{\rm e} = -\frac{1}{8\pi^2 c}\nabla\cdot\left(\frac{\Omega_{\rm F\omega}}{\alpha}\boldsymbol{\nabla}\Psi\right). \quad \text{(III.4c)}$$

Because the hole's gravity produces a gravitational redshift of ZAMO clocks, their lapse of proper time $d\tau$ is related to the lapse of global time dt by the lapse function α, i.e., $d\tau/dt = \alpha$ [6].

The angular momentum and energy fluxes are given in terms of two conserved quantities $\Omega_{\rm F}(\Psi)$ and $I(\Psi)$;

$$\boldsymbol{S}_{\rm E} = \Omega_{\rm F}(\Psi)\boldsymbol{S}_{\rm J}, \quad \boldsymbol{S}_{\rm J} = (I(\Psi)/2\pi\alpha c)\boldsymbol{B}_{\rm p}, \quad \text{(III.5)}$$

where the toroidal component of $\boldsymbol{S}_{\rm E}$ (and other fluxes) is and will be omitted throughout the paper. These are apparently the same as the equations for the pulsar magnetosphere except for the redshift factor α. The output power $\mathcal{P}_{\rm E}$ and the loss rate of angular momentum $\mathcal{P}_J$ observed by distant observers are given by

$$\mathcal{P}_{\rm E} = -c^2\frac{dM}{dt} = \oint \alpha\boldsymbol{S}_{\rm E}\cdot d\boldsymbol{A}$$

$$= \frac{1}{c}\int_{\Psi_0}^{\bar{\Psi}}\Omega_{\rm F}(\Psi)I(\Psi)d\Psi, \quad \text{(III.6a)}$$

$$\mathcal{P}_J = -\frac{dJ}{dt} = \oint \alpha\boldsymbol{S}_{\rm J}\cdot d\boldsymbol{A} = \frac{1}{c}\int_{\Psi_0}^{\bar{\Psi}} I(\Psi)d\Psi \quad \text{(III.6b)}$$

(see Eqs. (3.89) and (3.90) in [7]), where $\boldsymbol{B}_{\rm p}\cdot d\boldsymbol{A} = 2\pi d\Psi$ and the integration is done over all open field lines in $\Psi_0 \le \Psi \le \bar{\Psi}$. We define the 'overall' potential gradient, calculated from $\Omega_{\rm F}(\Psi)$ weighted by $I(\Psi)$, i.e.,

$$\bar{\Omega}_{\rm F} = \int_{\Psi_0}^{\bar{\Psi}}\Omega_{\rm F}(\Psi)I(\Psi)d\Psi \bigg/ \int_{\Psi_0}^{\bar{\Psi}} I(\Psi)d\Psi = \mathcal{P}_{\rm E}/\mathcal{P}_J, \quad \text{(III.7)}$$

and then from Eqs. (III.6a,b) and (II.4a) we have

$$c^2 dM = T_{\rm H}dS + \Omega_{\rm H}dJ = \bar{\Omega}_{\rm F}dJ, \quad \text{(III.8a)}$$

$$T_{\rm H}dS = -(\Omega_{\rm H} - \bar{\Omega}_{\rm F})dJ. \quad \text{(III.8b)}$$

Then, the 2nd law $T_{\rm H}dS > 0$ requires inequality $\Omega_{\rm H} > \bar{\Omega}_{\rm F}$ to be fulfilled always for the hole to lose energy, i.e., $c^2 dM = \bar{\Omega}_{\rm F}dJ < 0$ (see Eqs. (IV.13a,b)). This situation indicates that the null surface $\rm S_N$ with $\omega_{\rm N} = \Omega_{\rm F} \approx \bar{\Omega}_{\rm F}$ exists always. This means that the outer SC domain $\mathcal{D}_{\rm (out)}$ ($\Omega_{\rm F\omega} > 0$) rotates forward, whereas the inner GR domain $\mathcal{D}_{\rm (in)}$ ($\Omega_{\rm F\omega} < 0$) rotates backward. Thus, the ZAMOs will see that each FL with the same $\Omega_{\rm F}(\Psi)$ counter-rotates between the two domains, $\mathcal{D}_{\rm (out)}$ and $\mathcal{D}_{\rm (in)}$, without losing a sense of congruity.

B. Pulsar force-free magnetospheres

It will be instructive to refer to the *adiabatic-dragging* of the pulsar force-free magnetosphere. The power and the rate of angular momentum loss in Eqs. (III.6a,b) are surely applicable to an NS with the surface AV $\Omega_{\rm NS}$. The Poynting and angular-momentum fluxes are always outwards with no reversal of the direction of flows. The FLAV $\Omega_{\rm F}$ is uniquely given by the boundary condition for FLs emanating from the surface $\rm S_{NS}$ of a magnetized NS, i.e.,

$$\Omega_{\rm F} = \Omega_{\rm NS}. \quad \text{(III.9)}$$

The current function $I(\Psi)$ is specified by the criticality condition on the fast magneto-sonic surface $\rm S_F$ near infinity, i.e.,

$$I_{\rm NS}(\Psi) = \frac{1}{2}\Omega_{\rm F}(B_{\rm p}\varpi^2)_{\rm ff\infty} \quad \text{(III.10)}$$

(see Eq. (IV.7a)), which is equivalent to Ohm's law for the surface current on the force-free infinity surface $\mathcal{S}_{\rm ff\infty}$ ($\lesssim \rm S_\infty$). Then, The total power is given by simply putting $\Omega_{\rm F} = \Omega_{\rm NS}$ and $I = I_{\rm NS}$ by Eq. (III.6a)

$$\mathcal{P}_{\rm E:NS} = \frac{1}{c}\int_{\Psi_0}^{\bar{\Psi}}\Omega_{\rm NS}I_{\rm NS}d\Psi = \frac{1}{2c}\int_{\mathcal{S}_{\rm ff\infty}}\Omega_{\rm F}^2(B_{\rm p}\varpi^2)_{\rm ff\infty}d\Psi. \quad \text{(III.11)}$$

The related EMF of the NS's surface battery is expressed in terms of potential gradient $\Omega_{\rm F}$ by

$$\mathcal{E}_{\rm NS} = -\frac{1}{2\pi c}\int_{\Psi_1}^{\Psi_2} \Omega_{\rm F}(\Psi) d\Psi \qquad \text{(III.12)}$$

[15, 27], which drives currents along FL Ψ_2 with $\boldsymbol{j}_{\rm p} > 0$ and return currents along Ψ_1 with $\boldsymbol{j}_{\rm p} < 0$, where $\Psi_0 < \Psi_1 < \Psi_{\rm c} < \Psi_2 < \bar{\Psi}$ and $\boldsymbol{j}_{\rm p} \lesseqgtr 0$ for $\Psi \lesseqgtr \Psi_{\rm c}$ (see the outer half of FIG. 2; Eqs (IV.17) and (IV.18)), where $\bar{\Psi}$ is the last limiting field line satisfying $I(\Psi_0) = I(\bar{\Psi}) = 0$ (see figure 2 in [24] for one plausible example of $I(\Psi)$). The surface return currents flow from $I(\Psi_2)$ to $I(\Psi_1)$, crossing FLs between Ψ_1 and Ψ_2 on the resistive membrane $\mathcal{S}_{\rm ff\infty}$, and the ohmic dissipation of surface current there formally represents particle acceleration taking place in the resistive membrane $\mathcal{S}_{\rm ff\infty}$ with resistivity $\mathcal{R}_\infty$ (see Eq. (IV.20a) later). That is, the conversion of field energy to kinetic energy takes place on $\mathcal{S}_{\rm ff\infty}$ in the form of the MHD particle acceleration [30, 31].

Now, we regard the toroidal field $B_{\rm t} = -(2I/\varpi c)$ as the swept-back component of $\boldsymbol{B}_{\rm p}$ due to inertial loadings on the terminating surface $\mathcal{S}_{\rm ff\infty}$ of the force-free domain. Thus, the behaviour of $I = I(\ell, \Psi)$ from the stellar surface to infinity will be described in the pulsar force-free magnetosphere as follows;

$$I(\ell,\Psi) = \begin{cases} 0 & ; \ell \lesssim \ell_{\rm NS}, \ \text{(no resistance)}, \\ I_{\rm NS}(\Psi) & ; \ell_{\rm NS} \lesssim \ell \lesssim \ell_{\rm F}, \ \text{(FF region)}, \\ \to 0 & ; \ell_{\rm F} \lesssim \ell \lesssim \ell_\infty \ \text{(particle acceleration)} \end{cases} \qquad \text{(III.13)}$$

(see Eq. (VII.3) for a Kerr hole's force-free magnetosphere). We assume simply here that $I(\ell, \Psi)$ approaches zero for $\ell \to \infty$ or $\varpi \to \infty$ along each FL. This means that all the Poynting energy is transferred eventually to the particle kinetic energy (see Eq. (IV.21) later).

NSs of normal matter are regarded as innately magnetized, and hence, their magnetospheric FLs will be anchored in the surface layer or crust. When we think of the 1st law in pulsar-thermo-electrodynamics, it is similar to the BH case, i.e.,

$$c^2 dM_{\rm NS} = T_{\rm NS} dS + \Omega_{\rm NS} dJ, \qquad \text{(III.14)}$$

we have $c^2 dM_{\rm NS} = \bar{\Omega}_{\rm F} dJ$ by Eq. (III.7), for energy loss through the force-free magnetospheres. Then, we have for any entropy generation in the boundary layer

$$T_{\rm NS} dS = -(\Omega_{\rm NS} - \bar{\Omega}_{\rm F}) dJ.$$

When $\Omega_{\rm F} \approx \bar{\Omega}_{\rm F} = \Omega_{\rm NS}$, the boundary condition (III.9) ensures $T_{\rm NS} dS = 0$, that is, no entropy production near the boundary layer on $\mathrm{S}_{\rm NS}$. This is surely *adiabatic* extraction of energy from the NS through its force-free magnetosphere. There is naturally no necessity for the breakdown of the force-free condition anywhere (except but a negligible violation for a shortage of particles). In other words, the NS can 'adiabatically' be dragging the force-free magnetosphere, through which the NS blows the magneto-centrifugal wind outward in the form of a Poynting flux.

On the other hand, Kerr BHs will consist of a huge mass of entropy matter confined within the event horizon, and there seems to be no theoretical nor observational evidence so far indicating that they are capable of being magnetized nor anchoring the magnetospheric FLs. When the adiabatic extraction of energy is excluded and $\Omega_{\rm H} > \Omega_{\rm F}$ is the case, the ZAMOs will notice that the null surface $\mathrm{S}_{\rm N}$ where $\Omega_{\rm F\omega} = 0$ always exists between the inner and outer domains, $\mathcal{D}_{\rm (in)}$ and $\mathcal{D}_{\rm (out)}$, counter-rotating each other, and the force-free condition must break down there to establish a pair of batteries with a voltage drop in between to construct the particle-current sources (see Sec. VI).

IV. TOWARD BH GRAVITO-THERMO-ELECTRODYNAMICS

In order to clarify the fundamental properties of the force-free magnetosphere of Kerr holes, we expediently use wind-, flux-, and circuit analyses, which must be complementary to each other. The key point is to keep the ZAMOs, i.e., the physical observer's viewpoint, whose index is given by the ZAMO-FLAV $\Omega_{\rm F\omega}$ in Eqs. (III.2a,b).

A. The wind-flow analysis

1. The velocity $\boldsymbol{v}$ of 'force-free' particles

Combining the two conditions (III.1a,b), with use of Eqs. (III.4a,b,c) for $\boldsymbol{j}$ and $\varrho_{\rm e}$, we have $\boldsymbol{v} = \boldsymbol{j}/\varrho_{\rm e}$ for the velocity of the force-free particles

$$\boldsymbol{v} = \frac{\boldsymbol{j}}{\varrho_{\rm e}} = -\frac{1}{\varrho_{\rm e}\alpha}\frac{dI}{d\Psi}\boldsymbol{B} + v_{\rm F}\boldsymbol{t}, \qquad \text{(IV.1)}$$

which indicates that FLs, current- and stream-lines (FCSLs) are thus parallel to each other and must be equipotential in the force-free domains. And yet, the force-free plasma must be charge-separated, and the role of force-free particles is just to carry charges, exerting no dynamical effect.

On the other hand, the axial symmetry $\boldsymbol{E}_{\rm t} = 0$ imposes $\boldsymbol{j}_{\rm p} \times \boldsymbol{B}_{\rm p} = 0$ and $\boldsymbol{v}_{\rm p} \times \boldsymbol{B}_{\rm p} = 0$, respectively, in Eqs. (III.1a,b), and hence $\boldsymbol{j}_{\rm p} = \eta \boldsymbol{B}_{\rm p}$ and $\boldsymbol{v}_{\rm p} = \kappa \boldsymbol{B}_{\rm p}$, i.e., $\boldsymbol{j}_{\rm p} = \varrho_{\rm e}\boldsymbol{v}_{\rm p} = \varrho_{\rm e}\kappa\boldsymbol{B}_{\rm p} \equiv \eta\boldsymbol{B}_{\rm p}$, where η and κ are a scalar function of the position along each FL.

The ZAMO-measured particle-velocity $\boldsymbol{v}$ is summarized as follows;

$$\boldsymbol{v} = \kappa \boldsymbol{B} + v_{\rm F}\boldsymbol{t}, \qquad \text{(IV.2a)}$$

$$\boldsymbol{v}_{\rm p} = \kappa B_{\rm p}, \quad v_{\rm t} = \kappa B_{\rm t} + v_{\rm F}, \qquad \text{(IV.2b)}$$

$$\kappa = -(1/\varrho_{\rm e}\alpha)(dI/d\Psi), \qquad \text{(IV.2c)}$$

where $v_{\rm F}$ is given by Eq. (III.3c).

2. The null surface $\mathrm{S}_{\rm N}$ between two light surfaces, $\mathrm{S}_{\rm oL}$ and $\mathrm{S}_{\rm iL}$

When the ZAMOs see along each FL in case of $\Omega_{\rm H} > \Omega_{\rm F} > 0$ from the horizon to infinity, they will find that $\Omega_{\rm F\omega}$ increases from $-(\Omega_{\rm H}-\Omega_{\rm F})$ on $S_{\rm H}$, beyond null on $S_{\rm N}$ where $\omega=\omega_{\rm N}=\Omega_{\rm F}$, to $\Omega_{\rm F}$ on S_∞ on the *same* FL, because ω decreases from $\Omega_{\rm H}$ on $S_{\rm H}$ to zero on S_∞. Thus, by putting $v_{\rm F}=\mp c$ in Eq. (III.3c), we have in the Boyer-Lindquist coordinate (see Eqs. (A.1a,b,c,d))

$$\omega_{\rm iL}=\omega_{\rm N}+c(\alpha/\varpi)_{\rm iL},\ \ \omega_{\rm oL}=\omega_{\rm N}-c(\alpha/\varpi)_{\rm oL}. \quad \text{(IV.3)}$$

That is, there are the two (inner and outer) light surfaces, $S_{\rm iL}$ and $S_{\rm oL}$, on both sides of $S_{\rm N}$ where $v_{\rm F}=0$ (see Figure 5 in [12]). For an arrangement of these characteristic surfaces, the ZAMOs will see

$$\Omega_{\rm H}\gtrsim\omega_{\rm iF}>\omega_{\rm iL}>\omega_{\rm N}=\Omega_{\rm F}>\omega_{\rm oL}>\omega_{\rm oF}\gtrsim 0 \quad \text{(IV.4)}$$

along the same FL, where $\omega_{\rm iF}$ and $\omega_{\rm oF}$ are the values of ω on the inner and outer fast-magnetosonic surfaces $S_{\rm oF}$ and $S_{\rm iF}$ (see Sec. IV A 3). We can thus coordinatize $\Omega_{{\rm F}\omega}$ as well as ω along each FL (see, e.g. the horizontal axis of FIG. 1).

Each FL prograde-rotates in the outer domain $\mathcal{D}_{\rm (out)}$ where $\omega_{\rm N}\gtrsim\omega\gtrsim 0$, i.e., $0\lesssim\Omega_{{\rm F}\omega}\lesssim\Omega_{\rm F}$, and the *same* FL retrograde-rotates in the inner domain $\mathcal{D}_{\rm (in)}$ where $\Omega_{\rm H}\gtrsim\omega\gtrsim\omega_{\rm N}$, i.e., $0\gtrsim\Omega_{{\rm F}\omega}\gtrsim(\Omega_{{\rm F}\omega})_{\rm H}=-(\Omega_{\rm H}-\Omega_{\rm F})$. This situation explains that in the former, the normal magnetocentrifugal wind of force-free particles flows passing outwardly through $S_{\rm oL}$ toward $S_{\rm oF}\lesssim S_\infty$, while in the latter, the gravito-magnetocentrifugal wind of force-free particles flows passing inwardly through $S_{\rm iL}$ towards $S_{\rm iF}\gtrsim S_{\rm H}$. This inflow is never due to gravitational accretion.

It was already pointed out ([1]; the footnote at p.443) that "The outer light surface corresponds to the conventional pulsar light surface and physical particles must travel radially *outwards* beyond it. Within the inner light surface, whose existence can be attributed to frame-dragging and gravitational redshift, particles must travel radially *inwards*." It is because of the counter rotation of $\mathcal{D}_{\rm (in)}$ by frame-dragging that the null surface $S_{\rm N}$ must exist between the two light surfaces, $S_{\rm oL}$ and $S_{\rm iL}$ on the same FLs. Thus we see '$\omega_{\rm oL}<\Omega_{\rm F}<\omega_{\rm iL}$', which will correspond to an inequality '$\Omega_{\rm min}<\Omega<\Omega_{\rm max}$' given below equation (15) in [14]. Also see Eqs. (A.7a,b) and (A.8) for the behaviours of $S_{\rm oL}$, $S_{\rm N}$ and $S_{\rm iL}$, and $x_{\rm iL}\to 1$, $x_{\rm M}\to 1.2599$ and $x_{\rm oL}\to\infty$ for the slow-rotation limit of $h\to 0$.

We can find a related statement in [33] that "the magnetospheric plasma is produced in the spacial region between $S_{\rm iL}$ and $S_{\rm oL}$; at the $S_{\rm iL}$ ($S_{\rm oL}$), the magnetic field rotates backwards (forwards) at the speed of light relative the plasma", and actually this is consistent with the existence between $S_{\rm iL}$ and $S_{\rm oL}$ of the null surface $S_{\rm N}$ where $\Omega_{{\rm F}\omega}\gtreqqless 0$ (also see sections 7.3.1 and 9.3, and Figure 5 in [12]).

3. *The eigen-functions $I_{\rm (out)}$ and $I_{\rm (in)}$ due to the criticality condition on the fast-magnetosonic surfaces*

Let us determine the current/ angular-momentum function $I(\Psi)$ in the two SC and GR domains. We terminate the force-free domains on the resistive membranes $\mathcal{S}_{\rm ff\infty}$ and $\mathcal{S}_{\rm ffH}$ near S_∞ and $S_{\rm H}$, respectively (exactly speaking, on the outer and inner fast-magnetosonic surfaces $S_{\rm oF}$ and $S_{\rm iF}$; see, e.g., [31–33]). We have the behaviors of $\boldsymbol{B}$ and $\boldsymbol{E}_{\rm p}$ toward S_∞ and $S_{\rm H}$ by Eq. (III.3a,b) as follows;

$$\boldsymbol{B}^2=\boldsymbol{B}_{\rm p}^2+\boldsymbol{B}_{\rm t}^2=(2I/\varpi\alpha c)^2+(B_{\rm p}\varpi)^2/\varpi^4$$

$$\simeq\begin{cases}(2I_{\rm (out)}/\varpi c)^2 & ;\mathcal{S}_{\rm ff\infty},\ \alpha\to 1,\ \varpi\to\infty,\\ (2I_{\rm (in)}/\varpi\alpha c)^2 & ;\mathcal{S}_{\rm ffH},\ \alpha\to 0,\ \omega\to\Omega_{\rm H},\end{cases} \quad \text{(IV.5)}$$

$$\boldsymbol{E}_{\rm p}^2=(\Omega_{{\rm F}\omega}\varpi/\alpha c)^2B_{\rm p}^2$$

$$\simeq\begin{cases}\left(\dfrac{\Omega_{\rm F}}{\varpi c}\right)^2(B_{\rm p}\varpi^2)^2 & ;\mathcal{S}_{\rm ff\infty},\ \alpha\simeq 1,\ \varpi\to\infty,\\ \left(\dfrac{\Omega_{\rm H}-\Omega_{\rm F}}{\alpha\varpi c}\right)^2(B_{\rm p}\varpi^2)^2 & ;\mathcal{S}_{\rm ffH},\ \alpha\to 0,\ \omega\to\Omega_{\rm H},\end{cases} \quad \text{(IV.6)}$$

where $S_{\rm oF}\lesssim\mathcal{S}_{\rm ff\infty}$ and $S_{\rm iF}\gtrsim\mathcal{S}_{\rm ffH}$. Then $(\boldsymbol{B}^2-\boldsymbol{E}^2)\to 0$ reduces to the so-called 'criticality condition' for $I_{\rm (out)}$ and $I_{\rm (in)}$ (see Eqs. (6.6b) and (6.8b) in [23]; cf. also [3, 6]), as follows;

$$I_{\rm (out)}=(1/2)\Omega_{\rm F}(B_{\rm p}\varpi^2)_{\rm ff\infty}\ \ ;\ \mathcal{S}_{\rm ff\infty}, \quad \text{(IV.7a)}$$

$$I_{\rm (in)}=(1/2)(\Omega_{\rm H}-\Omega_{\rm F})(B_{\rm p}\varpi^2)_{\rm ffH}\ \ ;\ \mathcal{S}_{\rm ffH}. \quad \text{(IV.7b)}$$

The former $I_{\rm (out)}$ expresses the external resistance of particle acceleration on the resistive membrane $\mathcal{S}_{\rm ff\infty}$ (see Eq. (III.10) for $I_{\rm NS}(\Psi)$ in the pulsar case). The latter $I_{\rm (in)}$ does another external resistance of entropy production on the resistive membrane $\mathcal{S}_{\rm ffH}$ above the horizon, but this is not an internal resistance of a horizon battery (if any) (see FIG. 2 and Sec. IV C).

By Eqs. (IV.7a,b), the energy and angular momentum fluxes in Eqs. (III.6a,b) possess different forms in the outer and inner domains along each FL with FLAV $\Omega_{\rm F}(\Psi)$;

$$\mathcal{P}_{\rm E,(out)}=\frac{1}{c}\int_{\Psi_0}^{\bar\Psi}\Omega_{\rm F}I_{\rm (out)}d\Psi,\quad \mathcal{P}_{\rm E,(in)}=\frac{1}{c}\int_{\Psi_0}^{\bar\Psi}\Omega_{\rm F}I_{\rm (in)}d\Psi \quad \text{(IV.8)}$$

$$\mathcal{P}_{\rm J,(out)}=\frac{1}{c}\int_{\Psi_0}^{\bar\Psi}I_{\rm (out)}d\Psi,\quad \mathcal{P}_{\rm J,(in)}=\frac{1}{c}\int_{\Psi_0}^{\bar\Psi}I_{\rm (in)}d\Psi. \quad \text{(IV.9)}$$

It must be on the null surface $S_{\rm N}$ that the influx of negative angular momentum in $\mathcal{D}_{\rm (in)}$ (or equivalently the outward flux of positive one) must cancel out the outward flux of positive angular momentum in $\mathcal{D}_{\rm (out)}$, i.e., $\boldsymbol{S}_{\rm J,(out)}=\boldsymbol{S}_{\rm J,(in)}=-\boldsymbol{S}_{\rm J}^{\rm (in)}$. This condition $I_{\rm (out)}=I_{\rm (in)}$ turns out to yield the boundary condition to finally determine $\omega_{\rm N}=\Omega_{\rm F}$ for the whole magnetosphere frame-dragged into rotation with $\Omega_{\rm F}=\omega_{\rm N}$ (see Sec. VIII).

Next, the surface currents $\mathcal{I}_{\rm ff\infty}$ and $\mathcal{I}_{\rm ffH}$ flowing on the resistive membranes $\mathcal{S}_{\rm ff\infty}$ and $\mathcal{S}_{\rm ffH}$ with the surface resistivity $\mathcal{R}_{\rm H}=\mathcal{R}_\infty=4\pi/c$ are defined as follows;

$$\mathcal{I}_{\rm ff\infty}=\left(\frac{I_{\rm (out)}}{2\pi\varpi}\right)_{\rm ff\infty}=\left(\frac{c}{4\pi}\frac{\Omega_{\rm F}\varpi}{c}B_{\rm p}\right)_{\rm ff\infty}=\left(\frac{E_{\rm p}}{\mathcal{R}_\infty}\right)_{\rm ff\infty}, \quad \text{(IV.10a)}$$

$$\mathcal{I}_{\rm ffH} = \left(\frac{I_{\rm (in)}}{2\pi\varpi}\right)_{\rm ffH} = \left(\frac{c}{4\pi}\frac{(\Omega_{\rm H}-\Omega_{\rm F})\varpi}{c}B_{\rm p}\right)_{\rm ffH} = \left(\frac{E_{\rm p}}{\mathcal{R}_{\rm H}}\right)_{\rm ffH}. \tag{IV.10b}$$

Ohmic dissipation of these two surface currents corresponds to particle acceleration and entropy production in each closed-circuit $C_{\rm (out)}$ and $C_{\rm (in)}$ (see Eqs. (IV.20a,b,c)).

B. The energy-flux analysis

1. Two non-conserved energy fluxes $\boldsymbol{S}_{\rm EM}$ and $\boldsymbol{S}_{\rm SD}$

As seen in Eq. (III.2b), the conserved FLAV $\Omega_{\rm F}(\Psi)$ is resolved by frame-dragging to the ZAMO-FLAV $\Omega_{{\rm F}\omega}$ and the FDAV ω (both are a 'gravito-electric potential gradient', also). Substituting $\Omega_{\rm F}$ in Eq. (III.2b) into Eq. (III.5), we have

$$\boldsymbol{S}_{\rm E} = \boldsymbol{S}_{\rm EM} + \boldsymbol{S}_{\rm SD}, \tag{IV.11}$$

where

$$\boldsymbol{S}_{\rm E} = \Omega_{\rm F}\boldsymbol{S}_{\rm J} = (\Omega_{\rm F} I/2\pi\alpha c)\boldsymbol{B}_{\rm p}, \tag{IV.12a}$$

$$\boldsymbol{S}_{\rm EM} = \Omega_{{\rm F}\omega}\boldsymbol{S}_{\rm J} = (\Omega_{{\rm F}\omega} I/2\pi\alpha c)\boldsymbol{B}_{\rm p}, \tag{IV.12b}$$

$$\boldsymbol{S}_{\rm SD} = \omega\boldsymbol{S}_{\rm J} = (\omega I/2\pi\alpha c)\boldsymbol{B}_{\rm p}, \tag{IV.12c}$$

all of which can be reproduced from equation (4.13) in [6] by defining the two non-conserved energy fluxes $\boldsymbol{S}_{\rm EM}$ and $\boldsymbol{S}_{\rm SD}$ in there (see [**?**]). The Poynting flux $\boldsymbol{S}_{\rm EM}$ can be simply derived from the vector product of $\boldsymbol{B}_{\rm t}$ and $\boldsymbol{E}_{\rm p}$ in Eqs. (III.3a,b). Just as the overall energy flux $\boldsymbol{S}_{\rm E}$ corresponds to $c^2 dM$ in Eq. (II.4a) for the 1st law, $\boldsymbol{S}_{\rm EM}$ and $\boldsymbol{S}_{\rm SD}$ correspond to $T_{\rm H} dS$ and $\Omega_{\rm H} dJ$ on the horizon.

By Eqs. (III.3b) and (IV.12b) for $\boldsymbol{E}_{\rm p}$ and $\boldsymbol{S}_{\rm EM}$, we can confirm that "Physical observers will see the electric field reverse direction on the surface $\omega_{\rm N} = \Omega_{\rm F}$. Inside this surface, they see a Poynting flux of energy going toward the hole. (..., when the hole is losing energy electromagnetically, this surface always exists.)" (see the caption of Figure 2 in [1]). Also, outside this surface ($\Omega_{{\rm F}\omega} > 0$), they will see another Poynting flux going toward infinity (see FIG. 1). They will then understand that "a sufficiently strong *in*flux of *negative* angular momentum leaving this surface $\rm S_N$ inwardly" will be equivalent to "a sufficiently strong flux of *positive* angular momentum leaving the hole". This means that the null surface $\rm S_N$ must be the zero-angular-momentum surface $\rm S_{ZAMD}$ as well (see Eqs. (III.3c) and (V.1)).

Note that $\boldsymbol{S}_{\rm E}$ is always directed outward, that is, "the direction of energy flow cannot reverse on any given field line unless the force-free condition breaks down" [1], but the ZAMOs are aware now that the Poynting flux $\boldsymbol{S}_{\rm EM}$ reverses direction on the null surface $\rm S_N$, where $\Omega_{{\rm F}\omega} \gtreqless 0$. Indeed, it will be argued that breakdown takes place on $\rm S_N$ (see Sec. VI), and a relevant pair of surface unipolar induction batteries on both sides of $\rm S_N$ will be set up there, back-to-back and yet oppositely directed, each other. In addition, a particle source related can be excavated under the null surface $\rm S_N$ between the two light surfaces $\omega_{\rm oL}$ and $\omega_{\rm iL}$ (see Secs. IV A 2 and IV C).

Gralla and Jacobson [12] also discussed the reversal of the Poynting flux in their spacetime approach. By taking into account the ZAMO-FLAV $\Omega_{{\rm F}\omega}$ lacking in [6, 7, 10, 11], it turns out that the surface of reversal must be the same as the surface of reversal of the particle velocity in their Figure 5, that is, the null surface $\rm S_N$, where $\Omega_{{\rm F}\omega} = \boldsymbol{S}_{\rm EM} = \boldsymbol{v} = 0$ (see FIG. 1 and also Figure 2 in [1]).

2. Entropy production

On entropy production on the resistive membrane $\mathcal{S}_{\rm ffH}$, utilizing $I_{\rm (in)}$ in Eqs. (IV.7b) and $\boldsymbol{S}_{\rm EM}$ in Eq. (IV.12b), we have

$$T_{\rm H}\frac{dS}{dt} = -\oint_{\rm S_{ffH}} \alpha\boldsymbol{S}_{\rm EM}\cdot d\boldsymbol{A} \tag{IV.13a}$$

$$= \frac{1}{c}\int_{\Psi_0}^{\bar{\Psi}} (\Omega_{\rm H}-\Omega_{\rm F}) I(\Psi) d\Psi \tag{IV.13b}$$

$$= -(\Omega_{\rm H}-\bar{\Omega}_{\rm F})\frac{dJ}{dt} > 0, \tag{IV.13c}$$

where $\boldsymbol{B}_{\rm p}\cdot d\boldsymbol{A} = 2\pi d\Psi$. This leads to $T_{\rm H} dS = -(\Omega_{\rm H} - \bar{\Omega}_{\rm F})dJ$ in Eq. (III.8b). This is equal to the ohmic dissipation of the surface current $\mathcal{I}_{\rm ffH}$ on the resistive membrane with the resistivity of $\mathcal{R}_{\rm H} = 4\pi/c = 377$ohm, i.e.,

$$T_{\rm H}\frac{dS}{dt} = \int_{\mathcal{S}_{\rm ffH}} \mathcal{R}_{\rm H}\mathcal{I}_{\rm ffH}^2 dA = \frac{1}{2c}\int_{\Psi_0}^{\bar{\Psi}} (\Omega_{\rm H}-\Omega_{\rm F})^2 (B_{\rm p}\varpi^2)_{\rm ffH} d\Psi \tag{IV.14}$$

(see Eq. (3.99) in [7]). The ingoing Poynting flux will not penetrate into the horizon where $I \approx 0$, with the form of a Poynting flux kept as it is. This entropy production will correspond to the amount of energy paid back to the hole as its cost of self-extraction.

3. The efficiency of energy extraction

The overall efficiency $\bar{\epsilon}_{\rm EX}$ is defined by the ratio of actual energy extracted to maximum extractable energy when unit angular momentum is removed [1], i.e. from Eqs. (II.4a), (III.6a,b) and (III.7)

$$\bar{\epsilon}_{\rm EX} = \frac{(dM/dJ)}{(\partial M/\partial J)_S} = \frac{\mathcal{P}_{\rm E}}{\Omega_{\rm H}\mathcal{P}_J} = \frac{\bar{\Omega}_{\rm F}}{\Omega_{\rm H}}. \tag{IV.15}$$

The constraints of the energy extraction and its efficiency become by Eq. (IV.13b,c)

$$\Omega_{\rm H} > \Omega_{\rm F} \approx \bar{\Omega}_{\rm F} > 0, \tag{IV.16a}$$

$$\epsilon_{\rm EX} = \frac{\Omega_{\rm F}}{\Omega_{\rm H}} \approx \bar{\epsilon}_{\rm EX} < 1. \tag{IV.16b}$$

These inequalities ensure that "when the hole is losing energy electromagnetically, the null surface $\rm S_N$ on $\omega = \Omega_{\rm F}$ always exists" (see equations (4.6) and (4.7) in [1]).

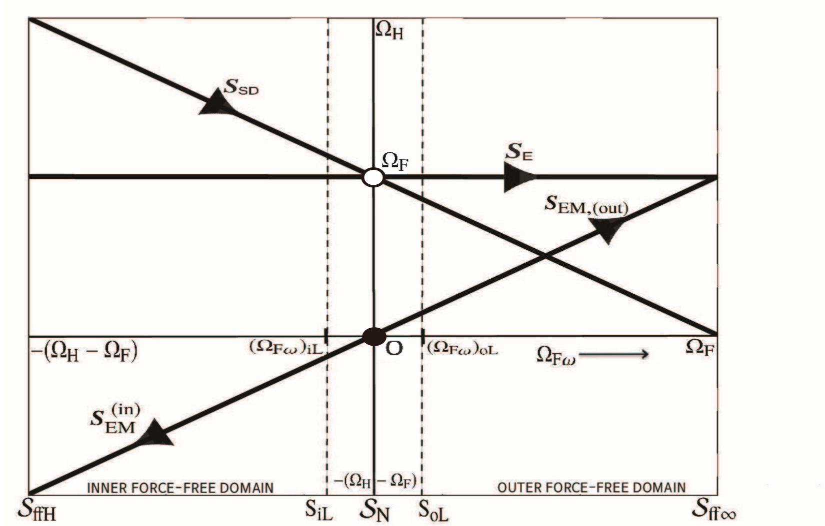

FIG. 1. The three energy fluxes $\boldsymbol{S}_{\rm E}$, $\boldsymbol{S}_{\rm EM}$ and $\boldsymbol{S}_{\rm SD}$ in Eqs. (IV.12a,b,c)) plotted against the ZAMO-FLAV $\Omega_{\rm F\omega}$ along each FCSL (modified from figure 3 in [25]). The point is to visualize the effect of the FDAV ω by coordinatizing $\Omega_{\rm F\omega}$ as well as ω (see Eq. (IV.4)). The ordinate on the null surface $\rm S_N$ ($\Omega_{\rm F\omega}=0$) divides the force-free magnetosphere into the two, outer *prograde*-rotating and inner *retrograde*-rotating domains, $\mathcal{D}_{\rm (out)}$ and $\mathcal{D}_{\rm (in)}$, with the respective light surfaces $\rm S_{oL}$ and $\rm S_{iL}$ (see Sec. IV A 2). Note that no reversal of the *conserved* flux $\boldsymbol{S}_{\rm E}=\Omega_{\rm F}\boldsymbol{S}_{\rm J}$ takes place, although $\boldsymbol{S}_{\rm E}=\boldsymbol{S}_{\rm EM}=\boldsymbol{S}_{\rm SD}=\boldsymbol{S}_{\rm J}=0$ on $\rm S_N$, due to breakdown of the force-free condition (see Sec. VI and FIG. 3). These energy fluxes are linked to the three terms of the 1st law of thermodynamics on the horizon under the resistive membrane $\mathcal{S}_{\rm ffH}$. When the hole is accepting an influx of *negative* angular momentum from the plasma-shed on $\rm S_N$ (see Sec. VII C), it looks as if the hole is launching *positive* angular momentum $\boldsymbol{S}_{\rm J}>0$ beyond $\rm S_N$ outward to $\mathcal{S}_{\rm ff\infty}$. Indeed, $\boldsymbol{S}_{\rm EM}$ reverses direction because the radiation condition for the Poynting flux is given by the sign of $\Omega_{\rm F\omega}\gtreqless 0$ for $\omega\lesseqgtr\Omega_{\rm F}$. A pair of batteries, as well as a particle source, must exist under the inductive membrane $\mathcal{S}_{\rm N}$ (see FIGs. 2 and 4).

C. The circuit analysis

1. The current-closure condition

We impose no net gain nor loss of charges over any closed surface in the force-free domains in the steady axisymmetric state. For the closed surface from the first open FL $\Psi=\Psi_0$ to the last open FL $\Psi=\bar\Psi$ in the poloidal plane, we have by Eq. (III.4a)

$$\oint \alpha \boldsymbol{j}_{\rm p}\cdot d\boldsymbol{A} \propto [I(\bar\Psi)-I(\Psi_0)]=0,\ \text{i.e.,}\ I(\bar\Psi)=I(\Psi_0)=0, \tag{IV.17}$$

when there is no line current at $\Psi=\Psi_0$, nor $=\bar\Psi$. This requires that function $I(\Psi)$ has at least one extremum at $\Psi=\Psi_{\rm c}$ where $(dI/d\Psi)_{\rm c}=0$, and hence

$$\boldsymbol{j}_{\rm p}=\varrho_{\rm e}\boldsymbol{v}_{\rm p}=-\frac{1}{\alpha}\frac{dI}{d\Psi}\boldsymbol{B}_{\rm p}\begin{cases}<0;\ \ \Psi_1<\Psi<\Psi_{\rm c},\\ =0;\ \ \Psi=\Psi_{\rm c},\\ >0;\ \ \Psi_{\rm c}<\Psi<\Psi_2,\end{cases} \tag{IV.18}$$

where $\Psi_0<\Psi_1<\Psi_{\rm c}<\Psi_2<\bar\Psi$ (see FIG. 2 and Sec. VII D for the two circuits $C_{\rm (out)}$ and $C_{\rm (in)}$ in the outer and inner domains).

2. A pair of batteries for the dual circuits

The breakdown of the two conditions (III.1a,b) in-between the two domains will provide an arena of setting up a pair of batteries and the voltage drop between their EMFs for particle production [28]. Let us pick up such FCSLs Ψ_1 and Ψ_2 for the circuits $C_{\rm (out)}$ and $C_{\rm (in)}$ as the two roots of an algebraic equation $I(\Psi)=I_{\overline{12}}$, i.e., $I(\Psi_1)=I(\Psi_2)\equiv I_{\overline{12}}$ in the range of $0<\Psi_1<\Psi_{\rm c}<\Psi_2<\bar\Psi$, where $(dI/d\Psi)_{\rm c}=0$ and $\boldsymbol{j}_{\rm p}\lesseqgtr 0$ for $\Psi\lesseqgtr\Psi_{\rm c}$.

The Faraday path integrals of $\boldsymbol{E}_{\rm p}$ in Eq. (III.3b) along two circuits, $C_{\rm (out)}$ and $C_{\rm (in)}$, yield

$$\mathcal{E}_{\rm (out)}=\oint_{C_{\rm (out)}}\alpha\boldsymbol{E}_{\rm p}\cdot d\boldsymbol{\ell}=-\frac{1}{2\pi c}\int_{\Psi_1}^{\Psi_2}\Omega_{\rm F}(\Psi)d\Psi, \tag{IV.19a}$$

$$\mathcal{E}_{\rm (in)}=\oint_{C_{\rm (in)}}\alpha\boldsymbol{E}_{\rm p}\cdot d\boldsymbol{\ell}=+\frac{1}{2\pi c}\int_{\Psi_1}^{\Psi_2}(\Omega_{\rm H}-\Omega_{\rm F})d\Psi. \tag{IV.19b}$$

There is no contribution to EMFs from integrating along Ψ_1 and Ψ_2 and on the null surface $\rm S_N$ because of $\boldsymbol{E}_{\rm p}\cdot d\boldsymbol{\ell}=(\boldsymbol{E}_{\rm p})_{\rm N}=0$. These batteries on both outer and inner surfaces of $\rm S_N$ with no internal resistance provide electricity to the outer resistances on the resistive membranes $\mathcal{S}_{\rm ff\infty}$ and $\mathcal{S}_{\rm ffH}$. That is, the EMFs for the two DC circuits $C_{\rm (out)}$ and $C_{\rm (in)}$ drive the volume currents of charge-separated particles $\boldsymbol{j}$ (see Eq. (IV.1)) into FCSLs in the force-free domains $\mathcal{D}_{\rm (out)}$ and $\mathcal{D}_{\rm (in)}$ and the surface membrane currents $\mathcal{I}_{\rm ff\infty}$ and $\mathcal{I}_{\rm ffH}$ on $\mathcal{S}_{\rm ff\infty}$ and $\mathcal{S}_{\rm ffH}$ (see Eqs. (IV.10a,b) and Sec. VII D). No volume current is allowed to cross the null surface $\rm S_N$ (or the Gap $\mathcal{G}_{\rm N}$) by the breakdown of the force-free condition from one circuit to another, i.e., $(\boldsymbol{j})_{\rm N}=0$ (see Eq. (VI.1b)).

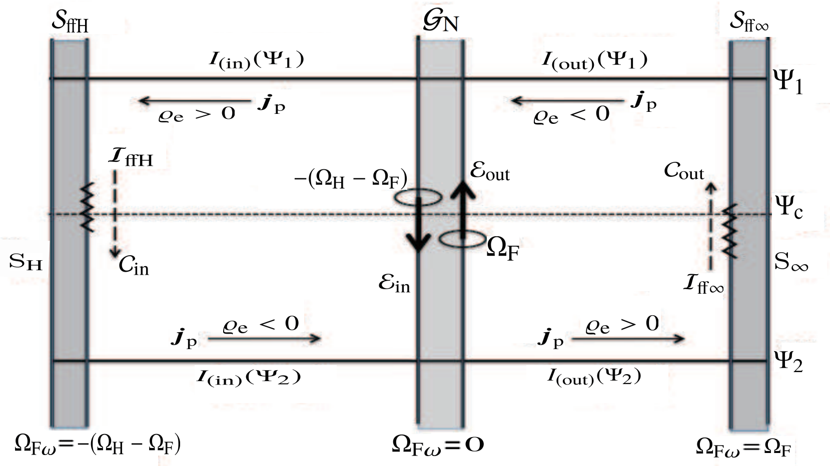

FIG. 2. A schematic diagram illustrating a pair of circuits $C_{\rm (out)}$ and $C_{\rm (in)}$ closed in the force-free domains $\mathcal{D}_{\rm (out)}$ and $\mathcal{D}_{\rm (in)}$ [28]. These domains are separated by the null surface $\rm S_N$, where current- and stream-lines are severed by the breakdown of the force-free and freezing-in conditions (see Sec. VI). There will be dual unipolar inductors with EMFs, $\mathcal{E}_{\rm (out)}$ and $\mathcal{E}_{\rm (in)}$, at work related to the magnetic spin-axes oppositely directed. The AVs of these axes are $\Omega_{\rm F}$ and $-(\Omega_{\rm H} - \Omega_{\rm F})$, respectively, and the difference is $\Omega_{\rm H}$ (see Eq. (IV.24); FIGs. 3,4). The huge voltage drop of $\Delta V \propto \Omega_{\rm H}$ (see Eq. (IV.22)) will lead to viable production of charge-neutral pair-plasma towards developing a dense Gap with the half-width $\Delta\omega$ (see FIG. 3). Along the FCSL Ψ_1 where $\boldsymbol{j}_{\rm p} < 0$, electrons flow out in $\mathcal{D}_{\rm (out)}$, and positrons flow in $\mathcal{D}_{\rm (in)}$, each charge-separated when these particles flow out from the plasma-shed to both directions (see Sec. VII C). The opposite is true along the FCSL Ψ_2 where $\boldsymbol{j}_{\rm p} > 0$; positrons flow outward in $\mathcal{D}_{\rm (out)}$ and electrons flow inward in $\mathcal{D}_{\rm (in)}$ (Sec. VII D). Note that $\boldsymbol{v}_{\rm p} = \boldsymbol{j}_{\rm p}/\varrho_{\rm e} > 0$ in the prograde-rotating $\mathcal{D}_{\rm (out)}$ and $\boldsymbol{v}_{\rm p} < 0$ in the retrograde-rotating $\mathcal{D}_{\rm (in)}$. The particles pair-created with $(\boldsymbol{v})_{\rm G} = 0$, circulating around the hole's axis with $\omega_{\rm N}$, are the ZAM-particles, dense enough to pin down magnetic field lines, to fix $\Omega_{\rm F} = \omega_{\rm N}$ and to make the Gap magnetized, thereby enabling the dual batteries to drive currents in each circuit (see FIG. 4). It is conjectured in the twin-pulsar model (see Sec. IX) that the outer half of the Gap in $0 \lesssim \Omega_{\rm F\omega} \lesssim \Delta\omega$ plays a role of a 'normal' magnetized NS spinning with $\Omega_{\rm F}$, while the inner half in $0 \gtrsim \Omega_{\rm F\omega} \gtrsim -\Delta\omega$ behaves like an 'abnormal' magnetized NS counter-spinning with $-(\Omega_{\rm H} - \Omega_{\rm F})$. Relaxation of the RTD in-between will lead to widening of the null surface $\rm S_N$ to the Gap $\mathcal{G}_{\rm N}$ with width $\sim 2\Delta\omega$ (see Sec. IX).

3. *Two outer and inner resistances*

The outer resistive membrane $\mathcal{S}_{\rm ff\infty}$ may also be interpreted as possessing the same surface resistivity $\mathcal{R}_\infty = 4\pi/c = 377$ Ohm as on the inner membrane $\mathcal{S}_{\rm ffH}$ above $\rm S_H$, and Ohm's law holds on $\mathcal{S}_{\rm ff\infty}$, i.e., $\mathcal{R}_\infty \mathcal{I}_{\rm ff\infty} = (E_{\rm p})_{\rm ff\infty}$. Thus similarly to Eq. (IV.13c), we have by Eq. (IV.7b)

$$\int_{\mathcal{S}_{\rm ff\infty}} \mathcal{R}_\infty \mathcal{I}^2_{\rm ff\infty} dA = \int_{\mathcal{S}_{\rm ff\infty}} \boldsymbol{\mathcal{I}}_{\rm ff\infty} \cdot \boldsymbol{E}_{\rm p} dA = \int_{\mathcal{S}_{\rm ff\infty}} \boldsymbol{S}_{\rm EM} \cdot d\boldsymbol{A}$$
$$= \frac{1}{2c} \int_{\mathcal{S}_{\rm ff\infty}} \Omega^2_{\rm F} (B_{\rm p} \varpi^2)_{\rm ff\infty} d\Psi = \mathcal{P}_{\rm E,(out)}, \qquad \text{(IV.20a)}$$

where $\boldsymbol{S}_{\rm EM} = \boldsymbol{S}_{\rm E}$ and $\boldsymbol{S}_{\rm SD} = 0$ on $\mathcal{S}_{\rm ff\infty}$ for $\omega \to 0$. On the other hand, from Eq. (IV.13c), one has on the horizon resistive membrane $\mathcal{S}_{\rm ffH}$

$$\int_{\mathcal{S}_{\rm ffH}} \mathcal{R}_{\rm H} \mathcal{I}^2_{\rm ffH} dA = T_{\rm H} \frac{dS}{dt} = \Omega_{\rm H} \mathcal{P}_{\rm J,(in)} - \mathcal{P}_{\rm E,(in)}. \qquad \text{(IV.20b)}$$

The above two expressions sum up to

$$\int_{\mathcal{S}_{\rm ffH}} \mathcal{R}_{\rm H} \mathcal{I}^2_{\rm ffH} dA + \int_{\mathcal{S}_{\rm ff\infty}} \mathcal{R}_{\rm H} \mathcal{I}^2_{\rm ff\infty} dA = \Omega_{\rm H} \mathcal{P}_{\rm J,(in)}, \qquad \text{(IV.20c)}$$

because $\mathcal{P}_{\rm E,(out)} = \mathcal{P}_{\rm E,(in)} = -c^2(dM/dt)$ and $\mathcal{P}_{\rm J,(in)} = \mathcal{P}_{\rm J,(out)} = -(dJ/dt)$ hold across the Gap $\mathcal{G}_{\rm N}$ with $(I)_{\rm G} = 0$ by the boundary conditions (see Eqs. (VIII.1a,b)), and hence we have $c^2 dM = T_{\rm H} dS + \Omega_{\rm H} dJ$. It turns out thus that the first law of thermodynamics participates directly in ohmic dissipation of the *surface* currents for entropy production and particle acceleration on the two resistive membranes $\mathcal{S}_{\rm ffH}$ and $\mathcal{S}_{\rm ff\infty}$ (see Eqs. (III.8a,b)).

The two EMFs in Eqs. (IV.19a,b) for circuits $C_{\rm (out)}$ and $C_{\rm (in)}$ are also responsible for launching the Poynting energy fluxes in both outward and inward directions, i.e., $\boldsymbol{S}_{\rm EM} \gtreqless 0$ for $\Omega_{\rm F\omega} \gtreqless 0$. Eq. (IV.19a) for $\mathcal{E}_{\rm (out)}$ coincides 'almost exactly' with Eq. (III.12) for a pulsar magnetosphere, because of $\omega \ll \omega_{\rm N} = \Omega_{\rm F}$ and hence $\boldsymbol{S}_{\rm EM,(out)} \approx \boldsymbol{S}_{\rm E}$ in the outer SC domain $\mathcal{D}_{\rm (out)}$.

This ohmic dissipation implies particle acceleration. The rate per unit τ time at which electromagnetic fields transfer redshifted energy to particles is by equation (4.14) in [6];

$$-\frac{1}{\alpha} \boldsymbol{\nabla} \cdot \alpha \boldsymbol{S}_{\rm E} = \alpha \boldsymbol{j} \cdot \boldsymbol{E} + (\omega/c)(\boldsymbol{j} \times \boldsymbol{B}) \cdot \boldsymbol{m}$$
$$= \frac{\Omega_{\rm F} \varpi}{c} j_\perp B_{\rm p} = -\frac{\Omega_{\rm F} \varpi}{c} \frac{B_{\rm p}}{2\pi \varpi \alpha} \left(\frac{\partial I_{\rm (out)}}{\partial \ell} \right) > 0. \qquad \text{(IV.21)}$$

This means that when the current function $I(\ell, \Psi)$ is continuously decreasing with ℓ in the resistive membrane $\mathcal{S}_{\rm ff\infty}$ from near S_{oF} to S_∞, the MHD acceleration will take place (see FIG. 3), but the force-free magnetosphere regards the 'force-free' domain with $j_\perp = 0$ formally as extending to the force-free infinity surface $\mathcal{S}_{ff\infty}$ where $|j_\perp| \gg |j_\parallel|$. By doing so, the circuit $C_{(out)}$ closes so as to fulfil the current-closure condition in the steady state.

4. Particle production due to voltage drop

The difference between the two EMFs in Eq. (IV.19a,b) across S_N is

$$[\mathcal{E}]_N = \mathcal{E}_{(out)} - \mathcal{E}_{(in)} = -(\Omega_H \Delta\Psi/2\pi c) = -\Delta V, \quad \text{(IV.22)}$$

where $\Delta\Psi = \Psi_1 - \Psi_2$, and the difference in a quantity X across S_N is denoted with

$$[X]_N = (X)_{N(out)} - (X)_{N(in)}. \quad \text{(IV.23)}$$

The difference of the ZAMO-FLAV $\Omega_{F\omega}$ between S_∞ and S_H becomes from Eq. (III.2a)

$$(\Omega_{F\omega})_\infty - (\Omega_{F\omega})_H \\ = \Omega_F - [-(\Omega_H - \Omega_F)] = \Omega_F + (\Omega_H - \Omega_F) = \Omega_H. \quad \text{(IV.24)}$$

The 'spark' models so far used for pair-production discharges in literature are based mainly on an extension from a 'negligible violation' of the force-free condition [1, 6, 8, 34–38]; also see [39, 40] for general review. It is argued here that the complete violation of the force-free condition due to frame-dragging on the null surface S_N leads to a unique gap model for the particle-current sources. It was emphasized already in [28] that "the present gap model with a pair of batteries and a strong voltage drop is fundamentally different from any existing models based on pulsar outer-gap models." The significant differences from the previous particle production mechanism come mainly from the existence of the counter-rotating inner domain $\mathcal{D}_{(in)}$ due to frame-dragging, with $\varepsilon_J \le 0$ inside S_N (see Sec. IX).

The null surface S_N dividing the force-free magnetosphere into the two (GR and SC) domains seems to be genetically endowed with the discontinuity $\Delta(\Omega_{F\omega})_N \approx (\Omega_{F\omega})_\infty - (\Omega_{F\omega})_H = \Omega_H$ and $[\mathcal{E}]_N = -\Delta V$, to widen the surface S_N to a gap $\mathcal{G}_N$ under $\mathcal{S}_N$, thereby constructing a magnetized 'zero-angular-momentum' and 'charge-neutral' Gap between the two force-free domains with $\varepsilon_J \gtreqless 0$ (see Sec. VII). Thus, the voltage drop ΔV in Eq. (IV.22) reveals that the null surface S_N will be a kind of rotational-tangential-discontinuity (RTD) due to the two (inner and outer) magnetic rotators, namely between the two, counter-rotating each other, outer and inner domains $\mathcal{D}_{(out)}$ and $\mathcal{D}_{(in)}$, although $\Omega_{F\omega}$ and $\boldsymbol{E}_p$ seem to change sign smoothly through zero (see Sec. IX; cf. [15]). It turns out thus that the maximum available voltage drop $\Delta V = (\Omega_H/2\pi c)\Delta\Psi$ will be utilizable in-between the two circuits C_{out} and C_{in}. in the steady state (see Sec. IX).

When the ZAM-Gap $\mathcal{G}_N$ is charge-neutral, i.e., $\varrho_e \approx 0$ as a result of ample pair-creation in the steady state, it will be another role of a pair of batteries that drive charge-separated particles from pair-created, charge-mixed plasma into each FCSL in the force-free domains $\mathcal{D}_{(out)}$ and $\mathcal{D}_{(in)}$ (see FIGs. 2 and 3; Sec. VII D).

V. THE ENERGY AND ANGULAR-MOMENTUM DENSITIES OF THE ELECTROMAGNETIC FIELDS

For the densities of the field energy and angular momentum, substituting $\boldsymbol{B}$ and $\boldsymbol{E}_p$ from Eqs. (III.3a,b) into equations (2.30a) and (2.31a) in [6], we have

$$\varepsilon_E = \frac{\alpha B_p^2}{8\pi}\left[1 + \frac{B_t^2}{B_p^2} + \frac{\varpi^2}{\alpha^2 c^2}(\Omega_F^2 - \omega^2)\right], \quad \text{(V.1a)}$$

$$\varepsilon_J = \frac{\varpi v_F}{c} B_p^2 = \frac{\Omega_{F\omega}(\varpi B_p)^2}{\alpha c} \quad \text{(V.1b)}$$

(also see equation (2.17a) in [23] and Eq. (55) in [41]), where ε_E and ε_J are regarded as an 'explicit' function of ω along each field FL labeled with Ψ. The angular momentum density ε_J changes sign on the null surface S_N, and we refer to the zero-angular-momentum-density surface as S_{ZAMD}, which accords with S_N.

Near the null surface S_N where $\omega = \Omega_F$ and $\varpi B_t \propto I = 0$ (see Sec. VI), we see

$$\varepsilon_E = \left(\frac{\alpha B_p^2}{8\pi}\right)_N > 0, \quad \varepsilon_J = 0. \quad \text{(V.2)}$$

The field energy will be strong enough to magnetize the plasma pair-produced with the voltage drop $[\mathcal{E}]_N = -\Delta V$ between the two EMFs in Eq. (IV.22). Conversely, the plasma density will be large enough to keep the field $\boldsymbol{B}_p$ frozen-in to ensure the magnetosphere frame-dragged by $\Omega_F = \omega_N$ (see Secs. VII and VIII)).

It is the frame-dragging term ω^2 in Eq. (V.1a) that builds a negative-energy region with $\varepsilon_E < 0$ in the inner domain $\mathcal{D}_{(in)}$. At the inner light surface S_{iL}, where $v_F = -c$ and ω_{iL} is given by Eq. (IV.3). When $(B_p\varpi^2)_{ffH}/(B_p\varpi^2)_{iL} \approx 1$ and $\Omega_F \approx 0.5\Omega_H$, we have analytically from Eq. (V.1a)

$$\varepsilon_E \approx -\left(\frac{B_p^2}{8\pi}\frac{\Omega_F\varpi}{c}\right)_{iL}\left[2 - \left(\frac{\Omega_F\varpi}{c\alpha}\right)_{iL}\right], \quad \text{(V.3)}$$

and then we see $(\varepsilon_E)_{iL} < 0$, if $(\Omega_F\varpi/c\alpha)_{iL} < 2$, i.e., $\omega_N < \omega_{iL}/2$.

Also we see in Eq. (V.1a) that there will be such a surface $S_{\varepsilon_E=0}$ that divides the inner domain $\mathcal{D}_{(in)}$ farther into the two regions by $\varepsilon_E(\omega, \Psi) \gtreqless 0$ for $\omega \lesseqgtr \omega_{\varepsilon_E=0}$ between S_N and $\mathcal{S}_{ffH} \approx S_H$, where $\omega_{\varepsilon_E=0}$ is a solution of equation for $\varepsilon_E = 0$ in Eq. (V.1a), i.e.,

$$\omega^2_{\varepsilon_E=0} = \omega_N^2 + \frac{\alpha^2 c^2}{\varpi^2}\left(1 + \frac{B_t^2}{B_p^2}\right), \quad \text{(V.4)}$$

where α/ϖ and B_t/B_p are thought of as functions of ω and Ψ. Therefore, it is the frame-dragging that produces not only the inner domain $\mathcal{D}_{(\rm in)}$ of $\Omega_{F\omega} \le 0$ with S_{iL}but also a region of the negative-energy density of $\varepsilon_E \le 0$ in $\Omega_H \ge \omega \ge \omega_{\varepsilon_E=0}$. We guess $\omega_{\varepsilon_E=0} \approx \omega_{iL}$.

Near the resistive horizon membrane $\mathcal{S}_{\rm ffH}$ where $(B_p\varpi^2)_{\rm ffH}/(B_p\varpi^2) \approx 1$ and hence $B_t^2/B_p^2 \approx (2I_{(\rm in)}/\varpi c\alpha B_p)^2 \approx ((\Omega_H - \Omega_F)\varpi/(\alpha c))^2$ by Eq. (IV.7b), we have

$$\varepsilon_E \approx -\left(\frac{B_p^2}{4\pi\alpha}\frac{\Omega_F\Omega_H\varpi^2}{c^2}\right)_{\rm ffH}\left[\left(1-\frac{\Omega_F}{\Omega_H}\right)-\left(\frac{c^2\alpha^2}{2\Omega_F\Omega_H\varpi^2}\right)_{\rm ffH}\right]$$

$$\approx -\left(1-\frac{\Omega_F}{\Omega_H}\right)\left(\frac{B_p^2}{4\pi\alpha}\frac{\Omega_F\Omega_H\varpi^2}{c^2}\right)_{\rm ffH} < 0 \qquad (V.5)$$

for $\alpha \to 0$ toward the resistive horizon membrane $\mathcal{S}_{\rm ffH}$.

For the density of angular momentum near the horizon, we have

$$\varepsilon_J = -(\Omega_H - \Omega_F)\left(\frac{(\varpi B_p)^2}{\alpha c}\right)_{\rm ffH} < 0. \qquad (V.6)$$

It will be evident in Eq. (V.4) that $\Omega_H > \omega_{\varepsilon_E=0} > \omega_N = \Omega_F$, which does not lead to any more robust condition upon Ω_F than that from the second law and the radiation condition toward the horizon.

The above result suggests that the negative-energy region will surely extend from near S_{iL} to S_H in the inner domain $\mathcal{D}_{(\rm in)}$. But the existence itself of the negative energy region near the horizon will not be an exact or direct indicator of energy extraction taking place from a Kerr hole. Important is the evidence that the force-free magnetosphere is divided into the two (prograde-rotating SC and retrograde-rotating GR) domains by the null surface S_N, i.e., the Zero-Angular-Momentum-Density surface $S_{\rm ZAMD}$ where $\varepsilon_J = \nu_F = \Omega_{F\omega} = 0$ and $\varepsilon_E > 0$ (see Eqs. (V.2)).

VI. BREAKDOWN OF THE FORCE-FREE AND FREEZING-IN CONDITIONS

The force-free condition must be broken down somewhere in the active force-free magnetosphere of a Kerr BH [10]. The ZAMOs will see that the electric field $\boldsymbol{E}_p$ and hence the Poynting flux $\boldsymbol{S}_{\rm EM} = \Omega_{F\omega}\boldsymbol{S}_J$ reverse direction on every FLs threading the null surface S_N, because of the counter-rotation of the inner GR domain ($\Omega_{F\omega} < 0$) against the outer SC domain ($\Omega_{F\omega} > 0$). Also, they will understand why a pair of unipolar-induction batteries $\mathcal{E}_{(\rm out)}$ and $\mathcal{E}_{(\rm in)}$ must be set up on the outer and inner sides of the null surface S_N, oppositely directed (see FIG. 2). These pieces of evidence obviously require the particle source as well as the current source under the null surface S_N with $\omega_N = \Omega_F$ between the outer and inner light surfaces ω_{oL} and ω_{iL} on this surface S_N with $\boldsymbol{E}_p = \Omega_{F\omega} = 0$ [**?**].

Then, when FLs thread this surface S_N, i.e., $(\boldsymbol{B}_p)_N \ne 0$ and $(\Omega_F)_N \ne 0$, the following quantities must necessarily vanish on S_N;

$$(\boldsymbol{E}_p)_N = (\boldsymbol{S}_{\rm EM})_N = (\varrho_e)_N = (\nu_F)_N = (\varepsilon_J)_N = 0, \quad (VI.1a)$$
$$(\boldsymbol{j})_N = (I)_N = (B_t)_N = (\boldsymbol{S}_J)_N = (\boldsymbol{S}_{\rm SD})_N = (\boldsymbol{S}_E)_N = 0, \quad (VI.1b)$$
$$(\mathcal{P}_E)_N = (\mathcal{P}_J)_N = 0, \quad (VI.1c)$$
$$(\boldsymbol{v})_N = (\boldsymbol{j}/\varrho_e)_N = 0, \quad (VI.1d)$$

where Eqs. (III.1a,b), (III.4a,b,c), (IV.1), (IV.12a,b,c) and (V.1b) are used, We denote the value of any function $X(\Omega_{F\omega},\Psi)$ on S_N with $\Omega_{F\omega} = 0$ as follows;

$$(X)_N = X(0,\Psi). \qquad (VI.2)$$

The above Constraints unequivocally require us to rebuild the whole force-free magnetosphere from a 'single-pulsar model' to a 'twin-pulsar model', as follows:

1. The most important of the above Constraints will be $(\boldsymbol{v})_N = (\boldsymbol{j})_N = 0$, which require current- and stream-lines to be severed on the null surface S_N. That is, the current-wind system is separated into the two (outer SC and inner GR) domains by the null surface S_N, i.e., the magneto-centrifugal divider with $\varepsilon_J = \nu_F = \Omega_{F\omega} = 0$. It is to accommodate the particle-current sources on the null surface S_N by breaking down the force-free and freezing-in conditions not negligibly but completely.

2. Constraint $(\boldsymbol{j})_N = 0$ plays the role of a perfect circuit breaker as a safety device to block acausal currents across S_N from a horizon battery (if any) to external resistances such as particle acceleration. This instead indicates the necessity of a pair of 'surface batteries' back-to-back at both sides of S_N, yet oppositely directed, with the particle source in-between, in which the voltage drop between the two EMFs will produce pair-particles (see Sec. VII and FIG. 2).

3. Constraint $(\boldsymbol{v})_N = 0$ means that the particles pair-created under S_N with $\varepsilon_J = 0$ are 'zero-angular-momentum' particles circulating with $\omega_N = \Omega_F$. Then, Constraints $(I)_N = (\boldsymbol{S}_J)_N = (\boldsymbol{S}_E)_N = 0$ mean that no angular momentum nor energy is transported across S_N, even when the FLs are continuous. It is helpful to remind that the toroidal field B_t is a swept-back component of the poloidal component $\boldsymbol{B}_p$ due to inertial loading in the resistive membranes $\mathcal{S}_{\rm ff\infty}$ and $\mathcal{S}_{\rm ffH}$ (see FIG. 3). $(I)_N = 0$ means that there must be a jump of $I(\Psi)$ from $I_{(\rm in)}$ to $I_{(\rm out)}$, just like in the NS surface (see Eqs. (III.13) and (VII.3)). Thus, we may impose the boundary condition $[I]_G = 0$ (see Eq. (VIII.1)).

4. Although the current $\boldsymbol{j}$ does not reverse direction, the velocity $\boldsymbol{v}$ does, i.e., $\boldsymbol{v} \gtreqless 0$ and $\varepsilon_J \gtreqless 0$ in Eq. (V.1b) for $\Omega_{F\omega} \gtreqless 0$ (see Sec. VII C, FIG. 4). Thus, the surface S_N will behave like a watershed in a mountain pass (i.e. 'plasma-shed') between outflows and inflows of 'force-free' and 'charge-separated' particles pair-created by the voltage drop (see Eqs. (IV.19a,b) and (IV.22)), and yet both flows are due to the magneto-centrifugal forces at

work toward the opposite directions, inward and outward by $\Omega_F \gtreqless 0$, respectively. As the outer pulsar-type magneto-centrifugal wind flows through S_{oL} in $\mathcal{D}_{(out)}$ with $v_F > 0$, the inner anti-pulsar-type wind will pass through S_{iL} in $\mathcal{D}_{(in)}$ with $v_F < 0$ (see FIG. 4; Sec. VII C).

5. Since $\varrho_e \boldsymbol{E}_p$ vanishes but does not change sign on S_N, this reacts back on the force-free condition in Eq. (III.1a), producing just $(\boldsymbol{j})_N = 0$, whereas the change in direction of $\boldsymbol{E}_p \propto \Omega_{F\omega}$ across S_N is taken over the particle velocity $\boldsymbol{v}$ as it is, i.e., $(\boldsymbol{v})_N \gtreqless 0$ for $\Omega_{F\omega} \gtreqless 0$ in the freezing-in condition (III.1b). This is because axial symmetry $\boldsymbol{E}_t = 0$ will lead to $\boldsymbol{v}_p = \kappa \boldsymbol{B}_p$ and $\kappa = -(1/\varrho_e\alpha)(dI/d\Psi) = 0$ on S_N (see Eq. (IV.2c)), and hence the ZAMOs will see that the particle velocity $\boldsymbol{v}$ behaves like $\Omega_{F\omega} \gtreqless 0$ across S_N, contrary to $\boldsymbol{j}_p$. Note that when $\boldsymbol{E}_p \propto \Omega_{F\omega}$ across S_N, this nature must straightly be succeeded to the particle velocity, i.e., $\boldsymbol{v}_p \propto \Omega_{F\omega}$ as well.

6. There will be no single circuit allowed, with such a current as crossing S_N due to a single battery at any plausible position [5–7]. Each electric circuit must close within its respective force-free domain ($\mathcal{D}_{(out)}$ or $\mathcal{D}_{(in)}$), with each EMF ($\mathcal{E}_{(out)}$ or $\mathcal{E}_{(in)}$) in Eq. (IV.19a,b), and with each eigenvalue $I(\Psi)$ ($I_{(out)}$ or $I_{(in)}$) in Eq. (IV.7a,b).

7. Two vectorial quantities $\boldsymbol{j}_p$ and $\boldsymbol{S}_J$ are closely related to each other through the current/angular-momentum function $I(\Psi)$, i.e., two-sidedness in the force-free domains. It seems that both do not reverse direction, despite that $(\boldsymbol{j}_p)_N = (\boldsymbol{S}_J)_N = 0$. This is because an outflow of negative charges means the ingoing current and an inflow of negative angular momentum means an outflow of positive one (see FIGs. 2 and 3). Despite that the null surface S_N exists always, and yet Constraints $(\boldsymbol{S}_J)_N = (\boldsymbol{S}_E)_N = 0$ hold, the overall energy flow $\boldsymbol{S}_E = \Omega_F \boldsymbol{S}_J$ seems to flow outwards all the way along each open FL, apparently as if crossing S_N, where the force-free condition 'breaks down' on S_N, i.e., $(I)_N = 0$, and indeed the Poynting flux $\boldsymbol{S}_{EM}$ reverses direction. These are quite natural tactics stemming from the supreme order of keeping the pair-producing Gap in the zero-angular-momentum and charge-neutral state.

VII. THE ZERO-ANGULAR-MOMENTUM AND CHARGE-NEUTRAL GAP $\mathcal{G}_N$

The ZAM-gap $\mathcal{G}_N$ under the inductive membrane $\mathcal{S}_N$ will play a hub of the Kerr hole's magnetospheric activities.

A. A plausible gap structure with $I = \Omega_{F\omega} = 0$

We presume that the RTD with the voltage drop $[\mathcal{E}]_N = -\Delta V$ in the force-free limit will be relaxed as a result of pair-particle production to a ZAM-Gap $\mathcal{G}_N$ with a half-width $\Delta\omega$. For the 'widened' null Gap $\mathcal{G}_N$ in $|\Omega_{F\omega}| \lesssim \Delta\omega$, we replace Constraints in Eq. (VI.1) on S_N as they are;

$$(\Omega_{F\omega})_G = (\boldsymbol{E}_p)_G = (\varrho_e)_G = (\boldsymbol{j})_G = (\boldsymbol{v})_G = (I)_G \quad \text{(VII.1a)}$$
$$= (\varepsilon_J)_G = (\boldsymbol{S}_J)_G = (\boldsymbol{S}_{EM})_G = (\boldsymbol{S}_{SD})_G = (\boldsymbol{S}_E)_G \quad \text{(VII.1b)}$$
$$= (\mathcal{P}_E)_G = (\mathcal{P}_J)_G = 0. \quad \text{(VII.1c)}$$

As before, $[X]_G$ denotes the difference of $X(\Omega_{F\omega}, \Psi)$ across the Gap width $2\Delta\omega$ (cf. $[X]_N$ in Eq. (IV.23));

$$[X]_G = X(\Delta\omega, \Psi) - X(-\Delta\omega, \Psi) \equiv (X)_{G(out)} - (X)_{G(in)}. \quad \text{(VII.2)}$$

We think of such a simple form of $I = I(\Omega_{F\omega}, \Psi)$ along a typical open FL in $\Psi_0 \le \Psi \le \bar{\Psi}$ as follows;

$$I(\Omega_{F\omega}) = \begin{cases} \to 0 & ;\ \mathcal{S}_{ff\infty}\ (\Omega_{F\omega} \to \Omega_F), \\ I_{(out)} & ;\ \mathcal{D}_{(out)}\ (\Delta\omega \lesssim \Omega_{F\omega} \lesssim \Omega_F), \\ 0 & ;\ \mathcal{G}_N\ (|\Omega_{F\omega}| \lesssim \Delta\omega), \\ I_{(in)} & ;\ \mathcal{D}_{(in)}\ (-\Delta\omega \gtrsim \Omega_{F\omega} \gtrsim -(\Omega_H - \Omega_F)), \\ \to 0 & ;\ \mathcal{S}_{ffH}\ (\Omega_{F\omega} \to -(\Omega_H - \Omega_F)) \end{cases} \quad \text{(VII.3)}$$

(see FIG. 3), where $I_{(out)}$ and $I_{(in)}$ are given by Eqs. (IV.7a,b). The behaviour of $I(\Omega_{F\omega}, \Psi)$ in the outer domain $\mathcal{D}_{(out)}$ will be similar to that of the force-free pulsar magnetosphere (see Eq. (III.13)). When the Gap $\mathcal{G}_N$ of the particle source will be situated well inside the light surfaces, we have by Eq. (IV.3)

$$|\Omega_{F\omega}| \lesssim \Delta\omega \ll c(\alpha/\varpi)_{oL} \approx c(\alpha/\varpi)_{iL}. \quad \text{(VII.4)}$$

It is not clear now how helpful or rather indispensable is the above condition in constructing a reasonable gap model, but we presume that the particle production will eventually take place intensely by the voltage drop across the Gap $\mathcal{G}_N$, $\Delta V = -(\Omega_H/2\pi c)\Delta\Psi$, almost independently of the presence of the two light surfaces in wind theory.

The Gap $\mathcal{G}_N$ under the inductive membrane $\mathcal{S}_N$ must be in the ZAM-state $(\varepsilon_J)_G = 0$, so that the particles and the field carry no angular momentum nor energy across the Gap, i.e., $(\boldsymbol{S}_J)_G = (\boldsymbol{S}_E)_G = 0$ in Eq. (VII.1b), when the Gap is threaded by poloidal field lines, i.e., $(\boldsymbol{B}_p)_G \neq 0$. Also, the Gap must be magnetized (i.e., $(\varepsilon_E)_N \neq 0$; see Eq. (V.2)), in almost the same sense as a magnetized NS because the poloidal magnetic field lines (with no toroidal component) threading the Gap will naturally be pinned down in the ZAM-particles pair-created and circulating round the hole with $\omega_N = \Omega_F$ (see Sec. VII B). Therefore, the magnetized ZAM-particles will ensure $\Omega_F = \omega_N$ (see Sec. VIII). The non-force-free magnetized ZAM-Gap $\mathcal{G}_N$ where $(\boldsymbol{j})_G = (\boldsymbol{v})_G = 0$ but $(\boldsymbol{B}_p)_G \neq 0$ will therefore be formed in the steady-state, with its 'surfaces' $S_{G(out)}$ and $S_{G(in)}$, respectively, at $\Omega_{F\omega} \approx \pm\Delta\omega$ (see FIG. 3), where $\Delta\omega \approx |(\partial\omega/\partial\ell)_N|\Delta\ell$ stands for the Gap half-width (see Eq. (VII.3)), and for $\Delta\omega \to 0$, $\mathcal{G}_N \to S_N$ (see figure 4 in [28] for the interplay of microphysics with macrophysics in the magnetized, matter-dominated Gap). Thus we conjecture that the voltage drop, $\Delta V = -[\mathcal{E}]_N$ on S_N in Eq. (IV.22), will produce pair particles copious enough, and the plasma pressure in the steady-state will expand S_N to $\mathcal{G}_N$ with a half-width $\Delta\omega$ in $|\Omega_{F\omega}| \lesssim \Delta\omega$.

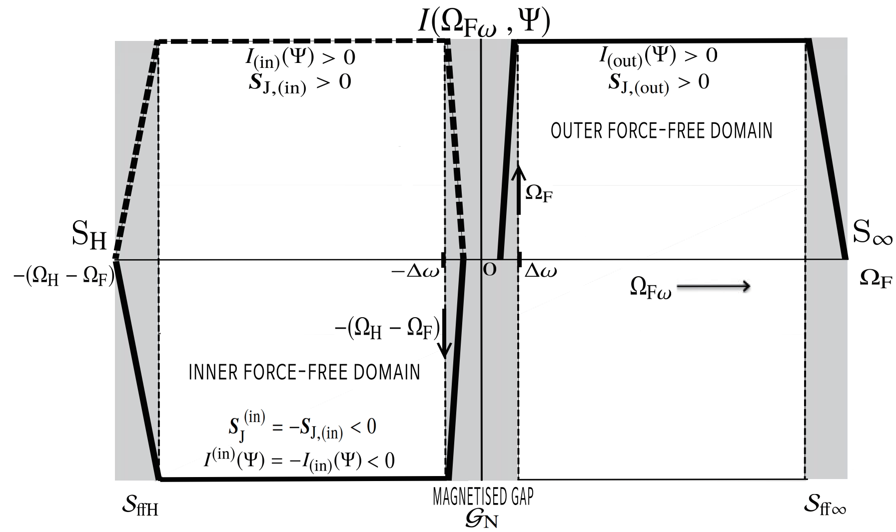

FIG. 3. A plausible behavior of the angular-momentum-flux/current function, $I(\Omega_{F\omega}, \Psi)$ (see Eq. (VII.3)). The force-free magnetosphere will be divided by the inductive membrane $\mathcal{S}_{\mathrm{N}}$ into the two domains of pro- and retro-grade rotation with $I = I_{(\mathrm{out})}(\Psi)$ and $= I_{(\mathrm{in})}(\Psi)$ in Eqs. (IV.7a,b). The voltage drop ΔV between the two EMFs in Eq. (IV.22) will produce pair-particles, thereby developing a Gap $\mathcal{G}_{\mathrm{N}}$ with $(I)_{\mathrm{G}} = 0$ in a finite zone of no internal resistance $|\Omega_{F\omega}| \lesssim \Delta\omega$ between the two domains. Current- and stream-lines are no longer allowed to thread $\mathcal{G}_{\mathrm{N}}$ by Constraints $(\boldsymbol{v})_{\mathrm{G}} = (\boldsymbol{j})_{\mathrm{G}} = 0$ in Eqs. (VII.1a). When the rate of *positive* angular momentum conveyed outwardly into $\mathcal{D}_{(\mathrm{out})}$ is equal to that of *negative* angular momentum conveyed inwardly into $\mathcal{D}_{(\mathrm{in})}$, the 'ZAM-state' of the Gap $(\varepsilon_{\mathrm{J}})_{\mathrm{G}} = 0$ will be maintained, i.e., $[I]_{\mathrm{G}} = I_{(\mathrm{out})} - I_{(\mathrm{in})} = 0$. The 'boundary condition' $[I]_{\mathrm{G}} = 0$ in Eq. (VIII.1) yields the eigenfunction $\Omega_{\mathrm{F}}(\Psi) = \omega_{\mathrm{N}}$ in Eq. (VIII.4a). The Gap filled with ZAM-particles will ensure the pinning-down of poloidal field lines $\boldsymbol{B}_{\mathrm{p}}$ with $\Omega_{\mathrm{F}} = \omega_{\mathrm{N}}$, and the pinning-down conversely ensure magnetization of ZAM-particles with $(I)_{\mathrm{G}} = 0$ inside the Gap (see Sec. VII B).

B. Pinning-down of threading field lines on ZAM-particles and magnetization of the matter-dominated Gap

When we regard the Gap surfaces $\mathrm{S}_{\mathrm{G(out)}}$ and $\mathrm{S}_{\mathrm{G(in)}}$ as being equipped with EMFs $\mathcal{E}_{(\mathrm{out})}$ and $\mathcal{E}_{(\mathrm{in})}$, respectively, these EMFs will not only drive currents in the respective circuits $C_{(\mathrm{out})}$ and $C_{(\mathrm{in})}$ but also produce a strong voltage drop $\Delta V = -[\mathcal{E}]_{\mathrm{G}}$ across the Gap, which will create copious ZAM-particles necessary to pin threading FLs down on. The ZAM-Gap filled with ZAM-particles will then circulate the hole with $\omega_{\mathrm{N}} = \Omega_{\mathrm{F}}$, and the poloidal field lines threading the Gap $\mathcal{G}_{\mathrm{N}}$ will surely be pinned down on ZAM-particles with $\Omega_{\mathrm{F}} = \omega_{\mathrm{N}}$. Thus, the ZAM-Gap will be in the perfectly magnetized state, with no electric current and no angular momentum flux allowed to cross, i.e., $(B_{\mathrm{t}})_{\mathrm{G}} = (I)_{\mathrm{G}} = (\boldsymbol{j})_{\mathrm{G}} = 0$. The physical state of the ZAM-Gap $\mathcal{G}_{\mathrm{N}}$ will be analogous to that of the NS inside, ensuring the boundary condition $\Omega_{\mathrm{F}} = \Omega_{\mathrm{NS}}$ for the FLs emanating from the NS surface.

C. Magneto-centrifugal plasma-shed on the ZAM-surface

The ZAMOs circulating with ω will see the force-free magnetosphere as follows: The outer domain $\mathcal{D}_{(\mathrm{out})}$ behaves like a pulsar-type magnetosphere rotating with Ω_{F}, whereas the inner domain $\mathcal{D}_{(\mathrm{in})}$ will behave like an anti-pulsar-type magnetosphere rotating with $-(\Omega_{\mathrm{H}} - \Omega_{\mathrm{F}})$. Then, plasma particles pair-created in-between by the voltage drop $\Delta V = -[\mathcal{E}]_{\mathrm{G}}$ circulate at $\omega = \Omega_{\mathrm{F}} = \omega_{\mathrm{N}}$ and may not behave as force-free particles with negligible inertia within the Gap. These ZAM-particles with $(\boldsymbol{v})_{\mathrm{G}} = 0$ will soon become charge-separated inside two batteries to flow from the Gap out to the two force-free domains as electric charges along current lines as well as wind particles along stream-lines (see FIGs. 2 and 4). The null surface $\mathrm{S_N}{=}\mathrm{S_{ZAMD}}$ midst the ZAM-Gap $\mathcal{G}_{\mathrm{N}}$ redefines quite a new general-relativistic type of divider due to magneto-centrifugal force modified by frame-dragging for particles pair-created in the spark ZAM-Gap, outward and inward ($\boldsymbol{v} \gtreqless 0$). That is, this surface $\mathrm{S_{ZAMD}}$ will play the role of a magneto-centrifugal plasma-shed, akin to a gravitational water-shed of a mountain ridge for heavy rainfalls on the Earth. This will be quite a natural way to launch 'magneto-centrifugal' winds from the ZAM-Gap for *both* directions toward infinity ($\boldsymbol{v} > 0$) and the horizon ($\boldsymbol{v} < 0$) (see Sec. IV A), similarly to the Poynting flux ($\boldsymbol{S}_{\mathrm{EM,(out)}} > 0$) for particle acceleration on the resistive membrane $\mathcal{S}_{\mathrm{ff}\infty}$ and the one ($\boldsymbol{S}_{\mathrm{EM,(in)}} < 0$) for entropy production on another membrane $\mathcal{S}_{\mathrm{ffH}}$ (see Sec. IV B).

D. Pair-creation, charge-separation and pair-annihilation

One of the important properties of the 'force-free' plasma will be 'charge-separatedness'. We will be able to utilize this in another way of discharging a pair of batteries on the null surface $\mathrm{S_N}$ into the two resistive membranes $\mathcal{S}_{\mathrm{ff}\infty}$ and $\mathcal{S}_{\mathrm{ffH}}$, as something like an $e^- - e^+$ collider.

Let us consider that strong enough pair-creation due to the voltage drop in the spark gap between the two EMFs will be at work in producing copious pair-particles (e.g., $\gamma + \gamma \rightarrow e^- + e^+$). These mixed charges of $e^\pm$ in the Gap $\mathcal{G}_{\rm N}$ will then be charge-separated in the presence of the battery EMF $\mathcal{E}_{\rm (out)}$ in the outer circuit $C_{\rm (out)}$, as the e^--stream flowing from under $S_{\rm N}$ outward into the 'FCS-line Ψ_1', and similarly as the e^+-stream flowing from under $S_{\rm N}$ outward into the 'FCS-line' Ψ_2 (see the arrows of $\boldsymbol{j}_{\rm p}$ in FIG. 2). This is because the wind always blows outward in the outer domain ($\Omega_{\rm F\omega} > 0$ and $\boldsymbol{v} > 0$). Also, by the effect of the battery $\mathcal{E}_{\rm (in)}$, e^+- and e^--streams from $S_{\rm N}$ will flow inward into the same FCSLs, Ψ_1 and Ψ_2, respectively, in the inner circuit $C_{\rm (in)}$ ($\Omega_{\rm F\omega} < 0$ and $\boldsymbol{v} < 0$).

These two $e^\pm$-streams in the two circuits $C_{\rm (out)}$ and $C_{\rm (in)}$ will collide with each other in the restive membranes, $\mathcal{S}_{\rm ff\infty}$ and $\mathcal{S}_{\rm ffH}$, respectively, and then pair-annihilation will take place, thereby liberating energy of the same order of magnitude as consumed when pair-particles are created in the Gap (i.e., $e^- + e^+ \rightarrow \gamma + \gamma$). This will be comparable to the amount of energy due to ohmic dissipation of the surface current $\mathcal{I}_{\rm ff\infty}$ and $\mathcal{I}_{\rm ffH}$ in order of magnitude (see Eqs. (IV.20a,b,c)). Also, the sum of the energy liberated in $\mathcal{S}_{\rm ff\infty}$ and $\mathcal{S}_{\rm ffH}$ will be similar to the amount due to the sum of the Poynting flux $\boldsymbol{S}_{\rm EM}$, inward and outward (see Eq. (X.1) later).

VIII. THE EIGEN-MAGNETOSPHERE

For a viable force-free magnetosphere, we refer to the condition by which to finally determine the eigenfunction $\Omega_{\rm F}(\Psi) = \omega_{\rm N}$ as the boundary condition, distinguishing from the criticality condition for another eigenfunction $I(\Psi)$ in Eqs. (IV.7a,b).

A. The boundary condition for the eigenfunction $\Omega_{\rm F}$

One of the vital roles of the ZAM-Gap is to anchor the poloidal field $\boldsymbol{B}_{\rm p}$ onto the ZAM-particles pair-created in there and to accomplish magnetization of the ZAM-Gap, thereby ensuring $\Omega_{\rm F} = \omega_{\rm N}$ for threading field lines. Accordingly, the ZAM-state of the Gap must always be maintained in the magnetosphere frame-dragged by the hole into circulation with $\omega_{\rm N} = \Omega_{\rm F}$.

We formulate the 'boundary condition' with Constraints (VI.1) or (VII.1) and with a postulate (VII.3) appropriately taken into account, that is, $(I)_{\rm G} = [I]_{\rm G} = 0$ at the place of the ZAM-Gap ;

$$[I]_{\rm G} = I_{\rm (out)}(\Psi) - I_{\rm (in)}(\Psi) \qquad \text{(VIII.1a)}$$

$$= I_{\rm (out)}(\Psi) + I^{\rm (in)}(\Psi) = 0. \qquad \text{(VIII.1b)}$$

This ensures the continuity of overall energy flux as well as angular momentum flow across the Gap, thereby keeping the charge-neutral ZAM-state along each FL threading the Gap (see FIG. 3; Eqs. (IV.7a,b)).

Condition (VIII.1a) shows that the outward transport rate of *positive* angular momentum leaving $S_{\rm G(out)}$ into the SC domain $\mathcal{D}_{\rm (out)}$ must be equal to that entering $S_{\rm G(in)}$ from the GR domain $\mathcal{D}_{\rm (in)}$. Note that the energy-angular momentum flow does not take place actually inside the ZAM-Gap with $(I)_{\rm G} = 0$. Condition (VIII.1b) implies equivalently that the outward rate of *positive* angular momentum leaving $S_{\rm G(out)}$ is offset by the inward rate of *negative* angular momentum leaving $S_{\rm G(in)}$ toward the hole.

Now, by Eqs. (III.5), we have

$$[\boldsymbol{S}_{\rm E}]_{\rm G} = \Omega_{\rm F}[\boldsymbol{S}_{\rm J}]_{\rm G} = 0, \qquad \text{(VIII.2)}$$

which apparently shows that the overall energy and angular momentum fluxes flow outward continuously across the Gap $\mathcal{G}_{\rm N}$, regardless of $(\boldsymbol{S}_{\rm E})_{\rm G} = (\boldsymbol{S}_{\rm J})_{\rm G} = 0$. Likewise, the 'boundary condition' (VIII.1a,b) ensures no discontinuity of the power $\mathcal{P}_{\rm E}$ and the loss rate of angular momentum $\mathcal{P}_J$ across the ZAM-Gap, i.e., as shown by Eqs. (III.6a,b) and (IV.8) and (IV.9);

$$[\mathcal{P}_{\rm E}]_{\rm G} = \mathcal{P}_{\rm E,(out)} - \mathcal{P}_{\rm E,(in)} = \mathcal{P}_{\rm E,(out)} + \mathcal{P}_{\rm E}^{\rm (in)} = 0, \quad \text{(VIII.3a)}$$

$$[\mathcal{P}_J]_{\rm G} = \mathcal{P}_{\rm J,(out)} - \mathcal{P}_{\rm J,(in)} = \mathcal{P}_{\rm J,(out)} + \mathcal{P}_J^{\rm (in)} = 0. \quad \text{(VIII.3b)}$$

Analysis of wave propagation of linear perturbations both outward and inward from the null surface $S_{\rm N}$ may be of interest with regard to the causality question of the boundary condition (VIII.1a,b) on the null surface $S_{\rm N}$ in the force-free magnetosphere [42–45].

B. The final eigenfunctions $I(\Psi)$ and $\Omega_{\rm F}(\Psi)$ in the force-free magnetosphere

From Eqs. (IV.7a,b) and (VIII.1a,b), we have [28]

$$\Omega_{\rm F}(\Psi) = \omega_{\rm N} = \frac{\Omega_{\rm H}}{1+\zeta}, \qquad \text{(VIII.4a)}$$

$$I = I_{\rm (out)} = I_{\rm (in)} = -I^{\rm (in)} = \frac{\Omega_{\rm H}}{2(1+\zeta)}(B_{\rm p}\varpi^2)_{\rm ffH}, \qquad \text{(VIII.4b)}$$

$$\zeta(\Psi) \equiv (B_{\rm p}\varpi^2)_{\rm ff\infty}/(B_{\rm p}\varpi^2)_{\rm ffH}. \qquad \text{(VIII.4c)}$$

In this eigenstate, the null surface $S_{\rm N}$=$S_{\rm ZAMD}$ will be the magneto-centrifugal plasma-shed, from which the angular momentum and the Poynting fluxes, positive and negative, flow out both ways toward $\mathcal{S}_{\rm ff\infty}$ and $\mathcal{S}_{\rm ffH}$. Their related AVs are given by $(\Omega_{\rm F\omega})_{\rm out} = \Omega_{\rm F}$ in the outer domain $\mathcal{D}_{\rm (out)}$, and by $(\Omega_{\rm F\omega})_{\rm in} = -(\Omega_{\rm H} - \Omega_{\rm F})$ in the inner domain $\mathcal{D}_{\rm (out)}$. The difference $\Omega_{\rm H}$ of the two AVs corresponds to the voltage drop between a pair of batteries ΔV in Eq. (IV.22), and this drop will lead to sustainable particle production.

Constraints $(\boldsymbol{j})_{\rm G} = (B_{\rm t})_{\rm G} = (I)_{\rm G} = (\boldsymbol{v})_{\rm G} = 0$ in Eq. (VII.1) imply that no transport of angular momentum and energy is possible within the ZAM-Gap, i.e., $(\boldsymbol{S}_{\rm J})_{\rm G} = (\boldsymbol{S}_{\rm E})_{\rm G} = 0$. These indicate a disconnection of current- and stream-lines between the two force-free domains and hence indicate the necessity of the current-particle sources and related EMFs in the Gap. It will be ensured in Eq. (VIII.1) that the copious charged ZAM-particles pair-produced in $|\Omega_{\rm F\omega}| \lesssim \Delta\omega$ serve to connect and equate both $I_{\rm (out)}$ and $I^{\rm (in)}$ across the Gap $\mathcal{G}_{\rm N}$, despite $(\boldsymbol{v})_{\rm G} = (\boldsymbol{j})_{\rm G} = 0$. Also, the overall flow of energy-angular momentum is continuous across the ZAM-Gap as seen in Eqs. (VIII.2), regardless of $(\mathcal{P}_{\rm E})_{\rm G} = (\mathcal{P}_J)_{\rm G} = 0$ as far as the boundary condition $[I]_{\rm G} = 0$ in Eq. (VIII.1) is satisfied.

The eigen-efficiency of extraction is given from Eq. (VIII.4a) by

$$\epsilon_{\rm EX} = \frac{\Omega_{\rm F}}{\Omega_{\rm H}} = \frac{1}{1+\zeta} \qquad \text{(VIII.5)}$$

(see Sec. IV B 3 and [49]). When the plausible field configuration allows us to put $\zeta \approx 1$ and hence $\epsilon_{\rm EX} \approx 0.5$, we have from Eqs. (VIII.4a),

$$\Omega_{\rm F} \approx \bar{\Omega}_{\rm F} \approx \frac{1}{2}\Omega_{\rm H}, \qquad \text{(VIII.6a)}$$

$$c^2|dM| \approx T_{\rm H} dS \approx \frac{1}{2}\Omega_{\rm H}|dJ| \qquad \text{(VIII.6b)}$$

by Eqs. (III.8a,b).

A pair of batteries' EMFs become for $\zeta \approx 1$ by Eqs. (IV.19)

$$\mathcal{E}_{\rm (out)} = -\frac{\Omega_{\rm H}}{2\pi c}\int_{\Psi_1}^{\Psi_2} \frac{1}{1+\zeta} d\Psi \approx -\frac{\Omega_{\rm H}\Delta\Psi}{4\pi c} \approx -\frac{\Delta V}{2}, \quad \text{(VIII.7a)}$$

$$\mathcal{E}_{\rm (in)} = \frac{\Omega_{\rm H}}{2\pi c}\int_{\Psi_1}^{\Psi_2} \frac{\zeta}{1+\zeta} d\Psi \approx \frac{\Omega_{\rm H}\Delta\Psi}{4\pi c} \approx \frac{\Delta V}{2}. \quad \text{(VIII.7b)}$$

Thus, the impedance matching between particle acceleration and entropy production [6, 7] yields good agreement with the result of the eigen-magnetosphere for $\zeta \approx 1$.

IX. A TWIN-PULSAR MODEL WITH ROTATIONAL-TANGENTIAL DISCONTINUITY

Path integrals of $\boldsymbol{E}_{\rm p}$ in Eq. (IV.19a,b) along the two closed circuits $C_{\rm (out)}$ and $C_{\rm (in)}$ reveal a sharp potential drop ΔV between the EMFs for the two circuits in the SC and GR domains as seen in Eq. (IV.22). This drop comes from discontinuity RTD due to the differential rotation of the outer prograde-rotating domain with $\Omega_{\rm F}$ and the inner retrograde-rotating one with $-(\Omega_{\rm H} - \Omega_{\rm F})$ (see Eq. (IV.24)) and will differ distinctly from any ordinary tangential and/or rotational discontinuities in classical magnetohydrodynamics (see, e.g., Landau et al. [15, §71]).

We attempt to briefly analyze a fundamental feature of this RTD on this surface $S_{\rm N}$ in the force-free limit. Important is the evidence that the above results, including the voltage drop ΔV across $S_{\rm N}$ comes from the 'continuous' function of the FDAV ω, and yet all these results seem also to be obtainable by assuming a 'discontinuous' step function $\overline{\omega}$ for the FDAV ω;

$$\overline{\omega} = \begin{cases} 0 & ;\ \mathcal{D}_{\rm (out)}\ (\Omega_{\rm F\omega} > 0), \\ \omega_{\rm N} \equiv \Omega_{\rm F} & ;\quad S_{\rm N}\quad (\Omega_{\rm F\omega} = 0), \\ \Omega_{\rm H} & ;\ \mathcal{D}_{\rm (in)}\ (\Omega_{\rm F\omega} < 0), \end{cases} \qquad \text{(IX.1)}$$

which means that $\omega \approx 0$ in the SC domain and $\approx \Omega_{\rm H}$ in the GR domain, but $\omega = \omega_{\rm N} = \Omega_{\rm F}$ on the null surface $S_{\rm N}$, i.e., the ZAM-dividing surface $S_{\rm ZAMD}$. Likewise, $\Omega_{\rm F\omega}$, $v_{\rm F}$ and $\varepsilon_{\rm J}$ are also replaced by the following step-functions ($\overline{\Omega}_{\rm F\omega} \equiv \Omega_{\rm F} - \overline{\omega}$);

$$\overline{\Omega}_{\rm F\omega} = \begin{cases} \Omega_{\rm F} \equiv (\overline{\Omega}_{\rm F\omega})_{\rm (out)} & ;\Uparrow \mathcal{D}_{\rm (out)}\ (\Omega_{\rm F\omega} > 0), \\ 0 \equiv (\overline{\Omega}_{\rm F\omega})_{\rm (N)} & ;\quad S_{\rm N}\quad (\Omega_{\rm F\omega} = 0), \\ -(\Omega_{\rm H} - \Omega_{\rm F}) \equiv (\overline{\Omega}_{\rm F\omega})^{\rm (in)} & ;\Downarrow \mathcal{D}_{\rm (in)}\ (\Omega_{\rm F\omega} < 0), \end{cases} \qquad \text{(IX.2)}$$

$\overline{v}_{\rm F} = \overline{\Omega}_{\rm F\omega}\varpi/\alpha$ and $\overline{\varepsilon}_J = \overline{v}_{\rm F}(\varpi B_{\rm p}^2/c)$, where $\Uparrow$ and $\Downarrow$ show that the $\mathcal{D}_{\rm (out)}$ prograde-rotates with $\Omega_{\rm F}$, while the $\mathcal{D}_{\rm (in)}$ retrograde-rotates with $-(\Omega_{\rm H} - \Omega_{\rm F})$, respectively (see the two arrows in FIGs. 2, 3, and 4). The differences of $\overline{\Omega}_{\rm F\omega}$ and the EMFs across $S_{\rm N}$ become

$$[\overline{\Omega}_{\rm F\omega}]_{\rm N} = (\overline{\Omega}_{\rm F\omega})_{\rm (out)} - (\overline{\Omega}_{\rm F\omega})^{\rm (in)} = \Omega_{\rm H}, \qquad \text{(IX.3a)}$$

$$[\mathcal{E}]_{\rm N} = -\frac{[\overline{\Omega}_{\rm F\omega}]_{\rm N}}{2\pi c}\Delta\Psi = -\Delta V. \qquad \text{(IX.3b)}$$

The related electric field $\overline{\boldsymbol{E}_{\rm p}}$ and its discontinuity at $S_{\rm N}$ become from Eq. (III.3b)

$$\overline{\boldsymbol{E}_{\rm p}} = -\frac{\overline{\Omega}_{\rm F\omega}}{2\pi\alpha c}\boldsymbol{\nabla}\Psi, \qquad \text{(IX.4a)}$$

$$[\overline{\boldsymbol{E}_{\rm p}}]_{\rm N} = -\frac{[\overline{\Omega}_{\rm F\omega}]_{\rm N}}{2\pi c}\left(\frac{\boldsymbol{\nabla}\Psi}{\alpha}\right)_{\rm N} = -\frac{\Omega_{\rm H}}{2\pi c}\left(\frac{\boldsymbol{\nabla}\Psi}{\alpha}\right)_{\rm N}. \qquad \text{(IX.4b)}$$

Eq. for $\overline{\boldsymbol{E}_{\rm p}}$ naturally reproduces the same results for $\mathcal{E}_{\rm (out)}$ and $\mathcal{E}_{\rm (in)}$ in Faraday path integrals of $\boldsymbol{E}_{\rm p}$ along the circuits $C_{\rm (out)}$ and $C_{\rm (in)}$, respectively, as given in Eqs. (IV.19a,b). Also, the discontinuity of the EMF is given already in Eq. (IX.3b).

The energy fluxes $\boldsymbol{S}_{\rm EM}$ and $\boldsymbol{S}_{\rm SD}$ are also replaced with the step-functions $\overline{\boldsymbol{S}}_{\rm EM}$ and $\overline{\boldsymbol{S}}_{\rm SD}$, respectively, i.e.,

$$\overline{\boldsymbol{S}}_{\rm EM} = \overline{\Omega}_{\rm F\omega}\boldsymbol{S}_{\rm J}, \quad \overline{\boldsymbol{S}}_{\rm SD} = \bar{\omega}\boldsymbol{S}_{\rm J} \qquad \text{(IX.5)}$$

(cf. $\boldsymbol{S}_{\rm EM,(out)}$ and $\boldsymbol{S}_{\rm EM}^{\rm (in)}$ in FID. 1). There is naturally no discontinuity in the overall energy and angular momentum fluxes across $S_{\rm N}$ with $[\boldsymbol{B}_{\rm p}]_{\rm N} = 0$, i.e., similarly to Eq. (VIII.2)

$$[\boldsymbol{S}_{\rm E}]_{\rm N} = [\overline{\boldsymbol{S}}_{\rm EM} + \overline{\boldsymbol{S}}_{\rm SD}]_{\rm N} = \Omega_{\rm F}[\boldsymbol{S}_{\rm J}]_{\rm N} = 0. \qquad \text{(IX.6)}$$

We compute likewise the differences of the Poynting flux $\overline{\boldsymbol{S}}_{\rm EM}$ and the spin-down flux $\overline{\boldsymbol{S}}_{\rm SD}$ across $S_{\rm N}$ from Eqs. (IX.3a) and (IX.5);

$$[\overline{\boldsymbol{S}}_{\rm EM}]_{\rm N} = \overline{\boldsymbol{S}}_{\rm EM,(out)} - \overline{\boldsymbol{S}}_{\rm EM}^{\rm (in)}, \qquad \text{(IX.7a)}$$

$$[\overline{\boldsymbol{S}}_{\rm SD}]_{\rm N} = -\overline{\boldsymbol{S}}_{\rm SD,(in)}, \qquad \text{(IX.7b)}$$

where $\overline{\boldsymbol{S}}_{\rm SD,(out)} = 0$ and $\overline{\boldsymbol{S}}_{\rm EM,(out)} = \boldsymbol{S}_{\rm E,(out)}$, because ω is regarded as negligible in the outer SC domain. Then, by Eq. (IX.6), i.e. $[\overline{\boldsymbol{S}}_{\rm EM} + \overline{\boldsymbol{S}}_{\rm SD}]_{\rm N} = 0$, we have

$$\overline{\boldsymbol{S}}_{\rm EM}^{\rm (in)} + \overline{\boldsymbol{S}}_{\rm SD,(in)} = \overline{\boldsymbol{S}}_{\rm EM,(out)} = \Omega_{\rm F}\boldsymbol{S}_{\rm J,(out)} = \Omega_{\rm F}\boldsymbol{S}_{\rm J,(in)}, \quad \text{(IX.8)}$$

which is equivalent to Eq. (IV.11). So, the 'overall' energy flux becomes

$$\boldsymbol{S}_{\rm E,(in)} = \overline{\boldsymbol{S}}_{\rm EM}^{\rm (in)} + \overline{\boldsymbol{S}}_{\rm SD,(in)} = -\Omega_{\rm F}\boldsymbol{S}_{\rm J}^{\rm (in)} = \Omega_{\rm F}\boldsymbol{S}_{\rm J,(in)} > 0, \quad \text{(IX.9)}$$

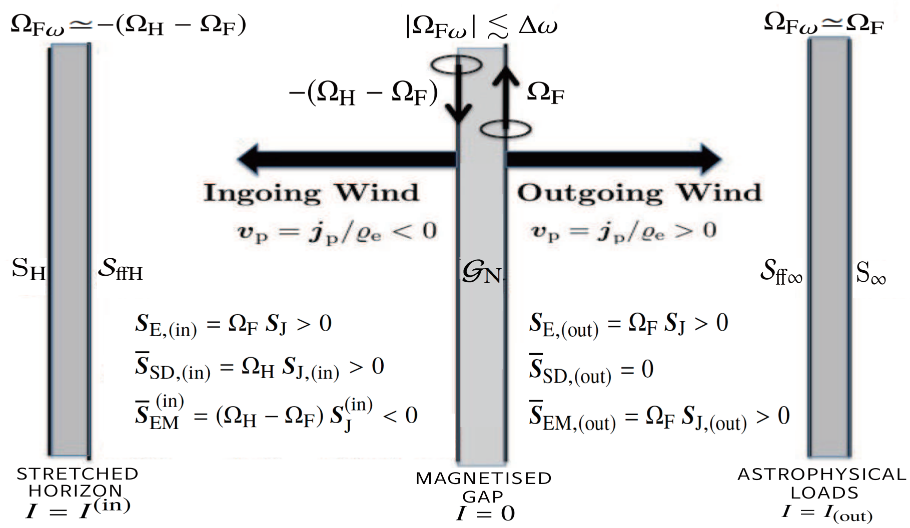

FIG. 4. A twin-pulsar model with three (two *resistive* and one *inductive*) membranes with $\overline{\Omega}_{F\omega}$ in Eq. (IX.2). The two domains are anti-symmetric to each other with respect to the ZAM-surface $S_{ZAMD}= S_N$; the outer one $\mathcal{D}_{(out)}$ behaves like a normal pulsar-type magnetosphere rotating with the FLAV $(\overline{\Omega}_{F\omega})_{(out)} = \Omega_F$, whereas the inner one $\mathcal{D}_{(in)}$ does like an anti-pulsar-type magnetosphere counter-rotating with the FLAV $(\overline{\Omega}_{F\omega})^{(in)} = -(\Omega_H - \Omega_F)$. The inductive membrane $\mathcal{S}_N$ covers the magnetized ZAM-Gap in $|\Omega_{F\omega}| \lesssim \Delta\omega$ at $(\overline{\Omega}_{F\omega})_G = 0$. The ingoing Poynting flux of negative energy in $\mathcal{D}_{(in)}$ from the Gap is equivalent to the outgoing Poynting flux of positive energy from the hole, just as the ingoing flux of negative angular momentum is so to the outgoing flux of positive angular momentum. A steady pair-production mechanism due to the voltage drop ΔV will be at work to supply ZAM-particles dense enough to anchor threading field lines, thereby ensuring $\Omega_F = \omega_N$. The two batteries will supply electricity to 'external resistances', where Joule heating implies particle acceleration and entropy production, respectively, in the resistive membranes $\mathcal{S}_{ff\infty}$ and $\mathcal{S}_{ffH}$ (see FIG. 2, Sec. VII D).

which will be equal to $\boldsymbol{S}_{E,(out)} = \Omega_F \boldsymbol{S}_{J,(out)}$ across the Gap (see Eqs. (IX.8) and (VIII.2); Sec. X A).

In the pulsar force-free magnetosphere, the conserved energy flux $\boldsymbol{S}_E = \Omega_F \boldsymbol{S}_J$ alone flows outward from S_{NS} to S_∞. In contrast, in the hole's force-free magnetosphere, the ZAMOs will see that the FD effect ω forcibly split the *conserved* energy flux $\boldsymbol{S}_E$ into the two *non-conserved* fluxes $\boldsymbol{S}_{EM}$ and $\boldsymbol{S}_{SD}$ in $\mathcal{D}_{(in)}$, to comply with the 1st and 2nd laws of thermodynamics. But these energy fluxes flow along the same equipotential FCSLs so that the Kerr hole will be unable to discriminate between the sum of $\boldsymbol{S}_{EM} = -(\omega - \Omega_F)\boldsymbol{S}_J$ and $\boldsymbol{S}_{SD} = \omega \boldsymbol{S}_J$ and that of $\overline{\boldsymbol{S}}_{EM} = -(\Omega_H - \Omega_F)\boldsymbol{S}_J$ and $\overline{\boldsymbol{S}}_{SD} = \Omega_H \boldsymbol{S}_J$ in the inner GR domain, while $\overline{\boldsymbol{S}}_{EM} = \boldsymbol{S}_E$ and $\overline{\boldsymbol{S}}_{SD} = 0$ hold in the outer SC domain (see Eqs. (IX.1), (IX.2) and (IX.5)). Therefore, the basic properties of the energy fluxes in the curved space with ω and $\Omega_{F\omega}(\omega, \Psi)$ will be fully taken over into the pseudo-flat space with $\overline{\omega}$ and $\overline{\Omega}_{F\omega}(\overline{\omega}, \Psi)$ [28] (see FIG. 4).

It is conjectured in the above that a kind of inevitable relaxation of the RTD due to the pair-creation by the voltage drop ΔV will lead to widening from the ZAM-surface to a ZAM-Gap $\mathcal{G}_N$ with a finite thickness. This Gap may be regarded as effectively consisting of two halves of 'fictitious' magnetized NSs of 'shell-like structures', e.g., the outer one forward-rotating with $(\Omega_{F\omega})_\infty = \Omega_F \approx (\Omega_H/2)$ and the inner one backward-rotating with $(\Omega_{F\omega})_H = -(\Omega_H - \Omega_F) \approx -(\Omega_H/2)$, and yet the two structures are reversely packed together, and threaded by the poloidal field $(\boldsymbol{B}_p)_G \neq 0$ with no toroidal component and pinned down in the ZAM-particles pair-produced in the Gap, to ensure $\Omega_F = \omega_G$.

X. ENERGETICS AND STRUCTURE IN THE TWIN-PULSAR MODEL

A. Energetics of the hole's self-extraction of energy

A variant of the first law, $\Omega_H |dJ| = T_H dS + c^2 |dM|$, seems to indicate that the energy extracted through the spin-down energy flux will be shared at the inductive membrane $\mathcal{S}_N$ between the two Poynting fluxes toward the two resistive membranes $\mathcal{S}_{ffH}$ and $\mathcal{S}_{ff\infty}$. Actually, by integrating Eq. (IX.8) over all the open field lines from Ψ_0 to $\bar{\Psi}$, we have

$$\int_{\Psi_0}^{\bar{\Psi}} \alpha \overline{\boldsymbol{S}}_{SD,(in)} \cdot d\boldsymbol{A} = -\int_{\Psi_0}^{\bar{\Psi}} \alpha \overline{\boldsymbol{S}}_{EM}^{(in)} \cdot d\boldsymbol{A} + \int_{\Psi_0}^{\bar{\Psi}} \alpha \overline{\boldsymbol{S}}_{EM,(out)} \cdot d\boldsymbol{A}, \tag{X.1}$$

which explains that the power $\Omega_H \mathcal{P}_J$ self-extracted from the horizon is allocated between entropy production in $\mathcal{S}_{ffH}$ and particle acceleration in $\mathcal{S}_{ff\infty}$ (cf. [6], Section 7.3; [7], Ch. IV D). The two terms of the right-hand side of Eq. (X.1) become, by Eqs. (IV.7a,b), (IV.8) and (IV.13c), respectively;

$$T_H \frac{dS}{dt} = \frac{1}{2c} \int_{\Psi_0}^{\bar{\Psi}} (\Omega_H - \Omega_F)^2 (B_p \varpi^2)_{ffH} d\Psi, \tag{X.2a}$$

and

$$-c^2\frac{dM}{dt} = \mathcal{P}_{\rm E,(out)} = \frac{1}{2c}\int_{\Psi_0}^{\bar\Psi} \Omega_{\rm F}^2 (B_{\rm p}\varpi^2)_{\rm ff\infty} d\Psi \qquad (X.2b)$$

$$= \mathcal{P}_{\rm E,(in)} = \frac{1}{2c}\int_{\Psi_0}^{\bar\Psi} \Omega_{\rm F}(\Omega_{\rm H}-\Omega_{\rm F})(B_{\rm p}\varpi^2)_{\rm ffH} d\Psi \qquad (X.2c)$$

(see Eq. (IV.20a)), where $c^2 dM = \bar\Omega_{\rm F} dJ = -\mathcal{P}_{\rm E,(in)}dt = -\mathcal{P}_{\rm E,(out)}dt$. The left hand-side of Eq. (X.1) reduces by Eqs. (IV.9) and (IV.7) to

$$-\Omega_{\rm H}\frac{dJ}{dt} = \Omega_{\rm H}\mathcal{P}_{\rm J,(out)} = \frac{\Omega_{\rm H}}{2c}\int_{\Psi_0}^{\bar\Psi} \Omega_{\rm F}(B_{\rm p}\varpi^2)_{\rm ff\infty} d\Psi \quad (X.3a)$$

$$= \Omega_{\rm H}\mathcal{P}_{\rm J,(in)} = \frac{\Omega_{\rm H}}{2c}\int_{\Psi_0}^{\bar\Psi} (\Omega_{\rm H}-\Omega_{\rm F})(B_{\rm p}\varpi^2)_{\rm ffH} d\Psi, \quad (X.3b)$$

(see Eq. (27) in [14]). Summing up Eqs. (X.2a) and (X.2b) or (X.2c) with use of the boundary condition $I_{\rm (out)} = I_{\rm (in)}$ yields $-\Omega_{\rm H}(dJ/dt) = \Omega_{\rm H}\mathcal{P}_J$. Also, Eqs. (X.2) and (X.3) show $[\mathcal{P}_{\rm E}]_{\rm G} = [\mathcal{P}_J]_{\rm G} = 0$, despite of the RTD of the EMFs $[\mathcal{E}]_{\rm G} = -\Delta V$ existent in the Gap $\mathcal{G}_{\rm N}$ (see Eq. (IV.22)).

The point is that the ZAM-particles created inside the Gap $\mathcal{G}_{\rm N}$ are spinning with $\omega_{\rm N} = \Omega_{\rm F}$ dragged by the hole's rotation, literally with no angular momentum. This means that the particles will easily flow out of the Gap, flung outwards or inwards from the surfaces $S_{\rm G(out)}$ or $S_{\rm G(in)}$ on the 'plasma-shed', with positive or negative angular momenta by the respective magneto-centrifugal forces, thus keeping the ZAM-state of the Gap. This corresponds to the situation where the outgoing Poynting flux $\boldsymbol{S}_{\rm EM} > 0$ is related to the outer EMF $\mathcal{E}_{\rm (out)}$, whereas the ingoing Poynting flux $\boldsymbol{S}_{\rm EM} < 0$ is related to the inner EMF $\mathcal{E}_{\rm (in)}$. Then, the distant observers may think as if the spin-down energy 'self-extracted' through the resistive horizon membrane $\mathcal{S}_{\rm ffH}$ were shared between the out- and ingoing Poynting fluxes reaching the two resistive membranes $\mathcal{S}_{\rm ff\infty}$ and $\mathcal{S}_{\rm ffH}$, respectively, to dissipate in particle acceleration and entropy generation as seen in Eq. (X.1). It seems that Kerr holes can play two roles: an acceptor of *negative* angular momentum and an emitter of *positive* angular momentum simultaneously and consistently.

B. The stream equation for the twin-pulsar model

We derive two expressions for $j_{\rm t}$; firstly, from Eqs. (III.4b,c)

$$j_{\rm t} = -\frac{\Omega_{{\rm F}\omega}\varpi}{8\pi^2\alpha c}\boldsymbol{\nabla}\cdot\left(\frac{\Omega_{{\rm F}\omega}}{\alpha}\boldsymbol{\nabla}\Psi\right) + \frac{1}{\varpi\alpha^2 c}\frac{dI^2}{d\Psi}, \qquad (X.4)$$

and secondly, from equation (10.4) in [6]

$$j_{\rm t} = -\frac{\varpi c}{8\pi^2\alpha}\left[\boldsymbol{\nabla}\cdot\left(\frac{\alpha}{\varpi^2}\boldsymbol{\nabla}\Psi\right) + \frac{\Omega_{{\rm F}\omega}}{\alpha c^2}(\boldsymbol{\nabla}\Psi\cdot\boldsymbol{\nabla})\omega\right]. \qquad (X.5)$$

Equating the above two expressions for $j_{\rm t}$ leads to the 'stream equation' in the force-free limit for the FCS-line structure in terms of the FLAV $\Omega_{\rm F}(\Psi)$ (also ZAMO-FLAV $\Omega_{{\rm F}\omega}$) and the current/angular-momentum function $I(\Psi)$ as follows;

$$\nabla\cdot\left\{\frac{\alpha}{\varpi^2}\left[1-\frac{\Omega_{{\rm F}\omega}^2\varpi^2}{\alpha^2c^2}\right]\nabla\Psi\right\} + \frac{\Omega_{{\rm F}\omega}}{\alpha c^2}\frac{d\Omega_{\rm F}}{d\Psi}(\nabla\Psi)^2 + \frac{16\pi^2}{\alpha\varpi^2c^2}I\frac{dI}{d\Psi} = 0 \qquad (X.6)$$

(see equation (6.4) in [6]). This reduces to the 'pulsar equation' in the flat space for $\alpha \to 1$ and $\omega \to 0$ [30], whereas this contains not only the two light surfaces $S_{\rm oL}$ and $S_{\rm iL}$but also the null surface $S_{\rm N}$ in-between, by $v_{\rm F} = \pm c$ and $= 0$. The breakdown of the force-free and freezing-in conditions appears in a complicated form of severance of both current- and stream-lines, $\boldsymbol{j} = \boldsymbol{v} = 0$, and the emergence of a spark-gap $\mathcal{G}_{\rm N}$, $I = \Omega_{{\rm F}\omega} = 0$, on the null surface $S_{\rm N}$ (Secs. VI and VII; FIG 3). All these complications will allow the particle-current sources to be inserted into the force-free magnetosphere by particle production due to the voltage drop between a pair of batteries under the null surface $S_{\rm N}$.

When the null surface $S_{\rm N}$ develops into a gap $\mathcal{G}_{\rm N}$ with any finite width $|\Omega_{{\rm F}\omega}| \lesssim \Delta\omega$ where $I(\ell,\Psi) = 0$, then the stream equation (X.6) will relevantly be modified. We conjecture now that the poloidal component $\boldsymbol{B}_{\rm p}$ with no toroidal one, $B_{\rm t} = I(\ell,\Psi) = 0$, will be robust enough to thread the particle-production Gap $\mathcal{G}_{\rm N}$ due to the voltage drop ΔV, with the ZAM-state of the Gap maintained to keep circulation with $\Omega_{\rm F} = \omega_{\rm N}$ around the hole. Probably, this situation will be compatible with the solution of the stream equation $\nabla\cdot((\alpha/\varpi^2)\nabla\Psi) = 0$ for the particle production Gap within $|\Omega_{{\rm F}\omega}| \lesssim \Delta\omega$ where $\Omega_{{\rm F}\omega} = I = 0$.

XI. DISCUSSION AND CONCLUSIONS

A. Astrophysical roles of frame-dragging in energy extraction

The FD effect plays an indispensable role in reforming the pulsar force-free magnetosphere into the specifications suitable for that of a Kerr hole, in particular, so as to be adaptive to the 1st and 2nd laws of thermodynamics and to include the current-particle sources. The observance of the two laws demands a breakdown of the force-free condition. But the issue, "where does the breakdown take place in the force-free magnetosphere?", seems to have remained almost untouched for more than four decades since Blandford and Znajek [1], probably because the astrophysical roles of frame-dragging remain so far ill-understood (see, e.g., [10, 46, 47]). This may explain why the topic of extracting energy from Kerr holes has continued to be a big challenge so far [2].

The ZAMOs circulating with ω around the hole will be certain that the Kerr hole can self-extract energy if and only if frame-dragging is correctly taken into account. The coupling of FDAV ω with FLAV $\Omega_{\rm F}$ begins with the ZAMO-FLAV, $\Omega_{{\rm F}\omega} = \Omega_{\rm F} - \omega$. Then, they will see for $0 < \Omega_{\rm F} < \Omega_{\rm H}$ that the coupling necessarily leads to nesting the inner domain $\mathcal{D}_{\rm (in)}$ counter-rotating ($\Omega_{{\rm F}\omega} < 0$) inside the outer domain $\mathcal{D}_{\rm (out)}$.

The magnetic sling-shot effect works inwardly through S_{iL} towards the hole in the former domain, oppositely to in the latter domain through S_{oL}, and hence the ZAMOs will see "a sufficiently strong flux of *negative* angular momentum leaving the null surface S_N", and this does not contradict with "a Poynting flux going towards the hole" [1]. They will understand that the overall energy flux $\boldsymbol{S}_E = \Omega_F \boldsymbol{S}_{J,(in)}$ always flows outward. It is on the null surface S_N=S_{ZAMD} that the spin-axis of the hole's force-free magnetosphere changes from positive in $\mathcal{D}_{(out)}$ to negative in $\mathcal{D}_{(in)}$, and the Poynting flux emitted changes direction from outward to inward, and hence the *complete* violation *must* take place of freezing-in-ness and force-freeness.

The inner domain $\mathcal{D}_{(in)}$ with the negative-angular-momentum density ($\varepsilon_J < 0$) may be referred to as the 'effective ergosphere' [23], because FCSLs there represent not only negative-angular-momentum orbits but also *negative-energy orbits* in the ergosphere in the Penrose process. The ingoing Poynting flux entering into the horizon leads to the hole's entropy increase (see Eq. (IV.13a,c)) and instead allows the hole to lose positive energy.

The counter-rotating inner domain will be designed so that the electrodynamical process of self-extraction of energy can surely obey the thermodynamic laws. The null surface S_N is then the key surface where the eigen-FLAV Ω_F and the eigen-FDAV ω_N can simultaneously be determined uniquely, thereby dragging the force-free magnetosphere into circulation around the hole with the FDAV $\omega_N = \Omega_F$ non-adiabatically. It is recently pointed out that "what is dragged by the Kerr hole are the ZAMOs and the compass of inertia" [48]. The force-free magnetospheres circulating with $\omega_N = \Omega_F$ around the Kerr hole may be included among them as well.

Thanks to frame-dragging, "a physical observer will see not only a Poynting flux of energy from S_N entering the hole" [1], but also he will see another Poynting flux from S_N outward, to transform into kinetic energy through the particle acceleration zone in the outer resistive membrane $\mathcal{S}_{ff\infty}$, and then to evolve to a high-energy gamma-ray jet beyond [29]. The force-free theory, inclusive of the 'complete violation' of its force-freeness, will yield a self-consistent, unique theory for extracting energy from Kerr holes. These ingenious actions of frame-dragging may seem to be due to a string puller manoeuvring behind the scenes, and we may so far never have seen them as his real astrophysical effects. The ZAMOs will not think of these rather modest actions of frame-dragging as spooky, for they are the 'physical observers' [1] and the 'fiducial observers' [7].

B. Concluding remarks

If observed large-scale high-energy γ-ray jets from AGNs really originate from quite near the event horizon of the central super-massive BHs, it appears to be plausible that these jets are a magnificent manifestation of the trinity of general relativity, thermodynamics, and electrodynamics (GTED); precisely speaking, frame-dragging, the first and second laws, and unipolar induction. The heart of the black hole's central engine may lie in the Gap $\mathcal{G}_N$ between the two (outer and inner) light surfaces just above the horizon, and the embryo of a jet will be born in the Gap $\mathcal{G}_N$ under the inductive membrane $\mathcal{S}_N$. The confirmation of this postulate awaits further illumination of the BH Gap physics.

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Appendix A: The place and shape of the null surface S_N

The absolute space around a Kerr hole with mass M and angular momentum per unit mass $a = J/Mc$ is described in Boyer-Lindquist coordinates as follows:

$$ds^2 = (\rho^2/\Delta)dr^2 + \rho^2 d\theta^2 + \varpi^2 d\phi^2, \quad \text{(A.1a)}$$

$$\rho^2 \equiv r^2 + a^2\cos^2\theta,\ \Delta \equiv r^2 - 2GMr/c^2 + a^2, \quad \text{(A.1b)}$$

$$\Sigma^2 \equiv (r^2 + a^2)^2 - a^2\Delta\sin^2\theta,\ \ \varpi = (\Sigma/\rho)\sin\theta, \quad \text{(A.1c)}$$

$$\alpha = \rho\Delta^{1/2}/\Sigma.\ \ \omega = 2aGMr/c\Sigma^2 \quad \text{(A.1d)}$$

(see [6]). It will be the two parameters α and ω that prepare the comfortable surrounding spacetime of a Kerr hole with the two hairs for its force-free magnetosphere with the two functions $\Omega_F(\Psi)$ and $I(\Psi)$.

It is the final eigenvalue $\Omega_F(\Psi)$ that determines not only the efficiency $\epsilon_{EX}(\Psi)$ of energy extraction but the place and shape of the null surface S_N, which hides a magnetized ZAM-Gap $\mathcal{G}_N$ under it in the force-free limit. Some basic properties of the structure of force-free eigen-magnetospheres have already been clarified in some detail (see [25, 26] for the monopolar 'exact' solution in the slow-rotation limit). For a tractable expression of FDAV ω, we deduce

$$\frac{\omega}{\Omega_H} = \frac{(1+h^2)^2 x}{(x^2+h^2)^2 - h^2(x-1)(x-h^2)\sin^2\theta} \quad \text{(A.2)}$$

from Eq. (A.1), where $x \equiv r/r_H$, $h = a/r_H$ and $\Omega_H = (c^3/2GM)h$. When we use $\omega_N = \Omega_H/(1+\zeta(\Psi))$ from Eq. (VIII.4a), the expression of $x_N = x_N(\theta)$ for the shape of S_N with the parameters h and ζ reduces to an algebraic equation;

$$\begin{aligned} F_N(x,\theta,\zeta;h) &= (x^2+h^2)(x^2+h^2\cos^2\theta) \\ &-(1+h^2)[(1+h^2\cos^2\theta) + (1+h^2)\zeta]x = 0. \quad \text{(A.3)} \end{aligned}$$

It will be helpful to define a 'mid-surface' S_M with $\omega_M = 0.5\Omega_H$ [23] to examine topological features of $S_M \approx S_N$. When $\zeta(\Psi) \simeq 1$, we have

$$\begin{aligned} F_M(x,\theta;h) &= (x^2+h^2)(x^2+h^2\cos^2\theta) \\ &-(1+h^2)[(2+h^2(1+\cos^2\theta)]x = 0. \quad \text{(A.4)} \end{aligned}$$

For comparison, we consider the static-limit surface S_E as the surface limiting the ergosphere from $g_{tt} = -(\Delta - a^2 \sin^2\theta)/\rho^2 = 0$,

$$F_E(x,\theta;h) = (x-1)(x-h^2) - h^2\sin^2\theta, \quad (A.5)$$

and its solution is expressed as

$$x_E(\theta,h) = \frac{1}{2}\left((1+h^2) + \sqrt{(1-h^2)^2 + 4h^2\sin^2\theta}\right). \quad (A.6)$$

From Eqs. (A.4) and (A.6), for $h \ll 1$ we have

$$x_M = 2^{1/3}\left[1 + \frac{h^2}{6}\left(2(2-2^{1/3}) + (2^{1/3}-1)\sin^2\theta\right)\right], \quad (A.7a)$$

$$x_E = 1 + h^2\sin^2\theta, \quad (A.7b)$$

while the two light surfaces, S_{oL} and S_{iL}, become for $h \ll 1$ (see Eqs. (7.7a,b) in [23])

$$x_{oL} = \frac{2}{h}\left(1 - \frac{\sin\theta}{4}\right), \quad x_{iL} = 1 + \frac{h^2}{4}\sin^2\theta. \quad (A.8)$$

Then, we see that $x_{iL} < x_E < x_M < x_{oL}$. For $h \to 0$, it turns out that both of x_{iL} and $x_E \to 1$ and $x_{oL} \to \infty$, while $x_M \to 2^{1/3} = 1.2599$. Therefore, when $\zeta \simeq 1$ and hence $S_M \simeq S_N$, S_N will interestingly keep a position of $x_N \to 2^{1/3}$ above the horizon between $x_{iL} = x_E = 1$ and $x_{oL} \to \infty$ (i.e., $S_E \leftarrow S_{iL} < S_M \approx S_N < S_{oL} \rightarrow S_\infty$), even for $h \to 0$.

There is a certain surface S_{Mc}, which contacts with S_E from the outside at the equator, i.e., $x_M = x_E$. This occurs when $h_c = \sqrt{\sqrt{2}-1} = 0.6436$, and then $x_M = 1.3960$ at $\theta = 0$ and $x_E = x_M = 1 + h_c^2 = \sqrt{2}$ at $\theta = \pi/2$ (see Figs. 1 and 2 in [25]).

For the extreme-Kerr state with $h \to 1$, Eq. (A.4) reduces to

$$F_M(x,\theta;1) = (x^2+1)(x^2+\cos^2\theta) - 2(3+\cos^2\theta)x = 0, \quad (A.9)$$

which yields $x_M = 1.6085$ for $\theta = 0$ at the pole and $x_M = 1.6344$, while by $F_E(x,\pi/2;1) = 0$, we have $x_E = 2$ at the equator (see figure 3 in [23]).

When $\zeta \simeq 1$, from the above analysis, one can read such interesting features at $\theta = \pi/2$ for $0 \le h \le 1$ that

$$\begin{array}{ll} 1 \le x_E(h) \le 2, & \text{for } S_E, \\ 2^{1/3} = 1.2599 \le x_N(h) \le 1.6433, & \text{for } S_N, \end{array} \quad (A.10)$$

and that $x_N \gtreqless x_E$ for $h \lesseqgtr h_c = (2^{1/2}-1)^{1/2} = 0.6436$. This shows that, for $1 \ge h \ge h_c$, the equatorial portion of the null surface S_N lies within the ergosphere S_E, while, for $h < h_c$, the whole of the ergosphere S_E lies within the null surface. It turns out that the ergosphere changes from a spherical shape at $h = 0$ to a spheroidal one at $h = 1$, while when $\zeta \simeq 1$, the null surface keeps an almost spherical shape from $h = 0$ to $h = 1$. In any case, it appears that mechanical properties in the ergosphere have no direct connection with electrodynamic properties of the null surface S_N and the inner domain $\mathcal{D}_{(in)}$.

[1] R. D. Blandford and R. L. Znajek, Mon. Not. R. Astron. Soc. **179**, 433 (1977).

[2] K. S. Thorne and R. Blandford, *Modern Classical Physics* (Princeton University Press, 2017).

[3] R. L. Znajek, Mon. Not. R. Astron. Soc. **179**, 457 (1977).

[4] R. L. Znajek, Mon. Not. R. Astron. Soc. **185**, 833 (1978).

[5] R. D. Blandford, *Accretion disc and black hole electrodynamics* in *Active Galactic Nuclei*, eds. C. Hazard and S. Mitton (Cambridge University Press, Cambridge p.241, 1979).

[6] D. A. Macdonald and K. Thorne , Mon. Not. R. Astron. Soc. **198**, 345 (1982)

[7] K. S. Thorne, R. H., Price and D. A. Macdonald, *Black Holes: The Membrane Paradigm* (Yale University Press, New Haven, 1986).

[8] Phinney S., *Proc. Torino Workshop on Astrophysical Jets*, ed. A. Ferrari & A. Pacholczyk (Reidel, Dordrecht, 1983a).

[9] Phinney S., *A theory of radio sources* (Ph.D. thesis, Univ. of Cambridge, 1983b).

[10] Uchida T., Phys. Rev. E **56**, 2181 (1997a)

[11] Uchida T., Phys. Rev. E **56**, 2198 (1997b).

[12] S.E. Gralla and T. Jacobson, Mon. Not. R. Astron. Soc. **445**, 2500 (2014).

[13] G. Barriga, F. Canfora, M. Torres and A. Vera, Phys. Rev. D **103**, 096023 (2021).

[14] R. D. Blandford and N. Globus, Mon. Not. R. Astron. Soc. **514**, 5141 (2022).

[15] L. D. Landau, E. M. Lifshitz and L. P. Pitaevskii, *Electrodynamics of Continuous Media*, second edition (Butterworth-Heinemann, Oxford, 1984).

[16] O. Kaburaki and I. Okamoto, Phys. Rev. D **43**, 340 (1991).

[17] B. Basu and D. Lynden-Bell, Q. Jl. Roy. astron. Soc. **31**, 359 (1990).

[18] I. Okamoto and O. Kaburaki, Mon. Not. R. Astron. Soc. **247**, 244 (1990).

[19] I. Okamoto and O. Kaburaki, Mon. Not. R. Astron. Soc. **250**, 300 (1991).

[20] I. Okamoto and O. Kaburaki, Mon. Not. R. Astron. Soc. **225**, 539 (1990).

[21] M. Cvetič, G.W. Gibbons, H. Lü and C.N. Pope, Phys. Rev. D **98**, 106015 (2018).

[22] B. Punsly and F.V. Coroniti, Phys. Rev. D **40**, 3834 (1989).

[23] I. Okamoto, Mon. Not. R. Astron. Soc. **254**, 192 (1992).

[24] I. Okamoto, Publ. Astron. Soc. Jpn **58**, 1047 (2006).

[25] I. Okamoto, Publ. Astron. Soc. Jpn **61**, 971 (2009).

[26] I. Okamoto, Publ. Astron. Soc. Jpn **64**, 50 (2012a).

[27] I. Okamoto, *How does unipolar induction work in Kerr black holes?* (Proceedings of the Ginzburg Conference on physics, Lebedev Physical Institute, Moscow, Russia, May 28 - June 2, 2012, 2012b).

[28] I. Okamoto, Publ. Astron. Soc. Jpn **67**, 89 (2015a).

[29] I. Okamoto, *Frame dragging, unipolar induction and jet source* (Proceedings of the Texas Symposium on Relativistic Astrophysics, 2015.12.13–18, Geneva, Switzerland, 2015b).

[30] I. Okamoto, Mon. Not. R. Astron. Soc. **167**, 457 (1974).

[31] I. Okamoto, Mon. Not. R. Astron. Soc. **167**, 457 (1978).
[32] C. F. Kennel, F.S. Fujimura and I. Okamoto, Geophys. Ap. Fluid Dyn. **26**, 147 (1983).
[33] B. Punsly and F. V. Coroniti, ApJ **350**, 518 (1990).
[34] V. S. Beskin, Ya.N. Istomin and V. I. Par'ev, Sov. Astron. **36(6)**, 642 (1992).
[35] K. Hirotani and I. Okamoto, ApJ **497**, 563 (1998).
[36] Y. Song, H.-Y. Pu, K. Hirotani, S. Matsushita, A. K. H. Kong and H.-K. Chang, Mon. Not. R. Astron. Soc. **471**, L135 (2017).
[37] K. Hirotani, H.-Y. Pu, S. Outmani, H. Huang, D. Kim, Y. Song, S. Matsushita and A. K. H. Kong, ApJ, **867**, 120 (2018b).
[38] M. C. Sitarz, M. V. Medvedev and Ford A. L., ApJ, 960.4 (2024)
[39] R. Ruffini, G. Vereshchagin and She-Schen Xue,, Physics Reports **487**, Issues 1- 4, 1-140 (2010).
[40] H. Chen and F. Fiuza, Phys. Plasmas **30**, 020601 (2023).
[41] S. S. Komissarov, J. Korean Phys. Soc. **54**, 2503 (2009).
[42] T. Uchida, Mon. Not. R. Astron. Soc. **286**, 931 (1997c).
[43] T. Uchida, Mon. Not. R. Astron. Soc. **291**, 125 (1997d).
[44] B. Punsly, ApJ **583**, 842 (2003).
[45] B. Punsly, *Black Hole Gravitohydromagnetics*, 2nd Ed. (Springer, New York, 2008).
[46] B. Punsly, ApJ **467**, 105 (1996).
[47] R. D. Blandford, *To the Lighthouse*, in *Lighthouse of the Universe*, (eds. Gilfanov, M. et al. Springer: Berlin, 2002).
[48] L.F.O. Costa and J. Natário, Universe, **7**, 388 (2021).