The Classical Point Particle Singularity: An Illusion in GR and Elsewhere!

Yousef Sobouti a* and Haidar Sheikhahmadi b†

^aInstitute for Advanced Studies in Basic Sciences, 444 Prof. Yousef Sobouti Blvd., Zanjan 45137-66731, Iran and ^bSchool of Astronomy, Institute for Research in Fundamental Sciences (IPM), P. O. Box 19395-5531, Tehran, Iran

Abstract

Singularities in Newton's gravity, in general relativity (GR), in Coulomb's law, and elsewhere in classical physics, stem from two ill conceived assumptions that, a) there are point-like entities with finite masses, charges, etc., packed in zero volumes, and b) the non-quantum assumption that these point-like entities can be assigned precise coordinates and momenta. In the case of GR, we argue that the classical energy-momentum tensor in Einstein's field equation is that of a collection of point particles and is prone to singularity. In compliance with Heisenberg's uncertainty principle, we propose replacing each constituent of the gravitating matter with a suitable quantum mechanical equivalent, here a Yukawa-ameliorated Klein-Gordon (YKG) field. YKG fields are spatially distributed entities. They do not end up in singular spacetime points nor predict singular blackholes. On the other hand, YKG waves reach infinity as $\frac{1}{r}e^{-(\kappa \pm ik)r}$. They create non-Newtonian and non-GR gravity forces that die out as r^{-1} as opposed to r^{-2} . This feature alone is capable of explaining the observed flat rotation curves of spiral galaxies, and one may interpret them as alternative gravities, dark matter paradigms, etc. There are ample observational data encapsulated in the Tully-Fisher relation to support these conclusions.

Keywords: Gravity beyond GR, Non-singular blackholes, Tully-Fisher relation, Einstein-Klein-Gordon equations, Flat rotation curves, Massive gravitons

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^{*}Electronic address: sobouti@iasbs.ac.ir

[†]Electronic address: h.sh.ahmadi@gmail.com;h.sheikhahmadi@ipm.ir

I. INTRODUCTION

In Newton's gravitation, GM/r, or in Coulomb's law, Q/r, one implicitly assumes there are physical entities of finite mass or charge packed in zero volumes. This assumption further implies that these point-likes can be assigned precise coordinates and momenta, and when approached infinitely closely, feel infinite gravitation or electric fields. That Newton's and Coulomb's laws have played decisive roles in setting the physics of the 17th-19th Centuries on axiomatized mathematical foundations is well acknowledged. That they are remarkably accurate to meet the everyday needs of the 21st century, from laboratory measurements to the solar system and galactic observations, is remarkable. The notion of point-like entities, however, is at odds with the foundations of quantum mechanics. Heisenberg's principle of uncertainty does not allow the assignment of precise coordinates and/or momenta to a physical entity. All one can hope for, is to find the particle, or whatever it is, in a quantum cloud of probability in a phase space.

The point particle singularity is a common feature of all classical physics, ranging from classical mechanics, classical electromagnetism, special and general relativity, to the physics of continuous media in its broadest sense. As long as the phase -space, $(\mathbf{x} \otimes \mathbf{p})$, available to a dynamical system is spacious, as is the case in everyday life, classical approximations are adequate. Deviations, however, appear when a system is forced to evolve in tighter and tighter phase volumes. Common examples are the cases of atomic and molecular spectroscopy, where electrons are forced to stay bound to the nuclei of their host atoms or molecules with minimal energies. Another example from astronomical realms is the neutron star, an aggregate of neutrons believed to be compressed into exceedingly small volumes by gravitational forces and cooled to about Fermi temperatures. Then there follows the popular conclusion that, if collapsed, the neutron star becomes a blackhole, a spacetime singularity.

This paper builds upon our ongoing effort to address the limitations of the point-particle concept. In [30], one of us removes the Coulomb singularity of the Dirac electron by proposing a mutual action-reaction partnership between the Dirac wave function and the electric field of the electron itself. And by so doing he comes up with a distributed charge and current for the electron and the correct gyromagnetic ratio for the electron, however, without recourse to QED. In the case of general relativity (GR), we argue the same. The classical energy-momentum tensor in Einstein's field equation, no matter how one conceives it, has the point particle singularity inherent to it. This inevitably entails the non-quantum act of assigning precise coordinates and momenta to the particles. In compliance with the uncertainty principle, we suggest to replace the classical energy-momentum tensor in Einstein's field equation by a quantum mechanical equivalent, a collection of bosons each represented by a *normalizable* Klein-Gordon field.

Resorting to auxiliary fields in GR, for different needs and purposes, has a long history and rich literature. Pioneering work by Ruffini and Bonazzola [24] investigated systems of self-gravitating scalar bosons and spin-1/2 fermions. Based on numerical solutions, they conclude the spacetime remains non-singular. Buchdahl [8] and Virbhadra [32] study the coupled Einstein-KG equations. Moffat [21] and Moffat and Toth [20] propose TeVeS fields to have a modified gravity to explain the flat rotation curves of spiral galaxies; see also Bekenstein [5]. Jetzer et al. [18], Bezares et al. [6]. Liddle and Madsen [19] introduce scalar fields to have boson stars.

What makes this paper different from the ones quoted or not quoted above, is our emphasis on the fact that individual constituents shaping the structure of the spacetime are not point-like particles. They are extended dual wave-particle packets. We will see that the inclusion of this duality not only removes the essential and coordinate singularities from blackholes, but also alters the spacetime structure at faraway, a quantum feature at galactic distances and beyond, which to the best of our knowledge is not addressed before. We will come back to this issue in sections III D and IV.

II. DEFINITION OF THE PROBLEM

A pair of neutrons coupled through their isospin behaves like an electrically neutral boson of spin 1 or 0 (see [16], chapter 5, section 5.4, Isospin generators). To mimic a neutron star one may consider a collection of such neutron nuggets. Neutrons aside, in a much wider context, one knows that about 99% of the content of the universe is hydrogen and helium. They are either spread out in interstellar and intergalactic spaces or are in the form of ionized particles in an electrically neutral matrix of plasma in the stars. In the absence of any better choice and with some condone we contend to mimic the lump of matter we are dealing with as a collection of N bosons in which each constituent satisfies the KG equation (or a modified version of it to be explained later) in the spacetime curved by the collection itself. Thus, for the total and individual Lagrangian density of

the constituent bosons we write,

$$\mathcal{L} = \sum_{i=1}^{N} \mathcal{L}(i).$$
(1)

$$\mathcal{L}_{i} = \frac{1}{2} \left[\frac{\hbar^{2}}{m} g^{\mu\nu} \partial_{\mu} \psi^{*}(i) \partial_{\nu} \psi(i) - mc^{2} \psi^{*}(i) \psi(i) \right], \qquad (2)$$

where m is the mass of a single constituent, of the order of the mass of one or two nucleons. The Euler-Lagrange equation associated with (2) is,

$$\frac{1}{\sqrt{-g}}\partial_{\mu}\left(\sqrt{-g}\partial^{\mu}\psi\right) + \frac{m^{2}c^{2}}{\hbar^{2}}\psi = 0, \text{ for each boson.}$$
(3)

The corresponding energy-momentum tensor of each constituent boson is,

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\partial}{\partial g^{\mu\nu}} \left(\sqrt{-g} \mathcal{L} \right) = -\frac{\hbar^2}{m} \partial_{\mu} \psi^* \partial_{\nu} \psi + \frac{1}{2} g_{\mu\nu} \left[\frac{\hbar^2}{m} \partial^{\alpha} \psi^* \partial_{\alpha} \psi - mc^2 \psi^* \psi \right],$$
(4)

$$T = T^{\alpha}_{\alpha} = \frac{\hbar^2}{m} \partial^{\alpha} \psi^* \partial_{\alpha} \psi - 2mc^2 \psi^* \psi, \qquad (5)$$

$$\nabla^{\nu}T_{\mu\nu} = 0.$$
 (6)

Einstein field equation for the collection of N boson may now be written as,

$$R_{\mu\nu} = \frac{8\pi G}{c^4} N \left[T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T \right],$$

$$= \frac{4\pi G N m}{c^2} \left[g_{\mu\nu} \psi^* \psi - 2 \frac{\hbar^2}{m^2 c^2} \partial_\mu \psi^* \partial_\nu \psi \right]$$
(7)

For a standing KG field of time dependence $\exp(-i\omega t)$, the two terms $\psi^*\psi$, $\partial_\mu\psi^*\partial_\nu\psi$ are timeindependent. This in turn entails time independence of the two coupled and non-linear equations (3) and (7). Thus one may devise a static and spherically symmetric Schwarzschild-like metric,

$$ds^{2} = -B(r)c^{2}dt^{2} + A(r)dr^{2} + r^{2}d\Omega^{2}, \text{ signature:}(-, +, +, +).$$
(8)

Reduction of (3) with (8) gives,

$$\frac{d^2\psi}{dr^2} + \left(\frac{2}{r} - \frac{1}{2A}\frac{dA}{dr} + \frac{1}{2B}\frac{dB}{dr}\right)\frac{d\psi}{dr} + \frac{m^2c^2}{\hbar^2}\left(1 + \frac{1}{B}\frac{\hbar^2\omega^2}{m^2c^4}\right)\psi = 0.$$
(9)

From the following combination of the components of (7) one finds

$$\frac{R_{tt}}{2B} + \frac{R_{rr}}{2A} + \frac{R_{\theta\theta}}{r^2} = \frac{1}{r^2} \left(\frac{d}{dr} \left(\frac{r}{A} \right) - 1 \right)$$

$$= -\frac{2\pi r_s}{C} \left(\left(1 - \frac{1}{B} \frac{\hbar^2 \omega^2}{m^2 c^4} \right) \psi^* \psi - \frac{1}{A} \frac{\hbar^2}{m^2 c^2} \psi'^* \psi' \right),$$
(10)

$$\frac{R_{\theta\theta}}{r^2} = \frac{1}{r^2} \left(\frac{1}{A} - 1\right) + \frac{1}{2rA} \frac{d}{dr} \ln\left(\frac{B}{A}\right) = \frac{2\pi N r_s}{C} \psi^* \psi.$$

$$r_s := 2Gm/c^2, Schwarzschild radius of each KG constituent.$$
(11)

$$C = \int \psi^* \psi d^3 r$$
, normalization const. of ψ .

In (9) and (10), $(\hbar^2 \omega^2 / m^2 c^4)$ is the square of oscillation energy to the rest mass energy of the KG constituent. And $(\frac{\hbar^2}{m^2 c^2} \approx 10^{-31} \text{m}^2)$ is the square of the Compton wavelength of the constituent. Both are extremely small. Unless one is dealing with extreme relativistic cases, one may safely ignore them without compromising the essential role of the wave nature of the KG field. Furthermore, it is preferable to work with dimensionless quantities. Thus we adopt

$$x = \frac{r}{r_s}, \quad \frac{d}{dr} = \frac{1}{r_s}\frac{d}{dx}, \quad r_s = 2Gm/c^2.$$

Applying these two simplifications to (9) and (10) one arrives at.

$$\psi'' + \left(\frac{2}{x} - \frac{1}{2}\frac{A'}{A} + \frac{1}{2}\frac{B'}{B}\right)\psi' + k^2\psi = 0, \quad k = r_s\frac{mc}{\hbar} \gg 1,$$
(12)

$$\frac{1}{x^2} \left[\frac{d}{dx} \left(\frac{x}{A} \right) - 1 \right] = \frac{2\pi N}{C} \psi^* \psi, \tag{13}$$

$$\frac{1}{x^2}\left(\frac{1}{A}-1\right) + \frac{1}{2xA}\frac{d}{dx}\ln\left(\frac{B}{A}\right) = \frac{2\pi N}{C}\psi^*\psi,\tag{14}$$

where (') denotes d/dx. The dimensionless k is the ratio of the Schwarzschild radius to the Compton wavelength. Thus $k \gg 1$.

III. SOLUTION BY ITERATION

As an initial guess to begin iteration, we use Schwarzschild's metric coefficients

$$B_s = \frac{1}{A_s} = \left(1 - \frac{1}{x}\right),$$

substitute them in (12) and solve it for ψ , substitute the result in (13) and solve it for A, substitute the results in (14) and solve it for B.

In the course of integration in different intervals of x we exercise different precautions:

- Close to the origin, $(x \to 0)$, we expand all functions as Taylor series in x.
- Close to the horizon, $(x \to 1)$, we expand all functions as Taylor series in x 1.
- At far distances, $(x \to \infty)$, we assume $(B = A^{-1} = 1)$. Thus, (12) reduces to Helmholtz's equation and along with (13) and (14) are solved analytically.

A. Near origin solutions, $x \to 0$.

Taylor expansion of initial A_s and B_s in x is

$$B_s = -\frac{1}{x} + 1, \ A_s = -\frac{x}{1-x} = -(x+x^2+\cdots).$$
$$\frac{B'}{B}\Big|_s = -\frac{A'}{A}\Big|_s = -\frac{1}{x(1-x)} = -\frac{1}{x}(1+x+x^2+\cdots),$$

Eq. (12) reduces to,

$$\psi'' + \left(x^{-1} - 1 - x - \cdots\right)\psi' + k^2\psi = 0.$$
(15)

From (13), (14) and (15) one now finds,

$$\psi(x) = 1 - \frac{1}{4}k^2x^2 + \cdots,$$
(16)

$$B(x) = \frac{1}{A(x)} = 1 + \frac{2\pi N}{C} \left(\frac{1}{3}x^2 - \frac{1}{10}k^2x^4 + \cdots\right),\tag{17}$$

The essential singularity at the origin is removed. Spacetime becomes asymptotically flat as $x \rightarrow 0$.

B. Near horizon solution, $x \to 1$

Taylor expansion of A_s and B_s in y = x - 1, $|y| \ll 1$, is

$$B_s = \frac{y}{1+y} = y - y^2 + y^3 + \dots, \quad A_s = y^{-1} + 1,$$
$$\frac{B'}{B}\Big|_s = -\frac{A'}{A}\Big|_s = y^{-1} - 1 + y - y^2 + \dots.$$

(12) reduces to,

$$\frac{d^2\psi}{dy^2} + \left(y^{-1} + 1 - y + \cdots\right)\frac{d\psi}{dy} + k^2\psi = 0,$$
(18)

To the lowest order in $y^2 = (x - 1)^2$ for ψ , and to the order (x - 1) for A and B one finds

$$\psi = 1 - \frac{1}{4}k^2(x-1)^2, \tag{19}$$

$$B = \frac{1}{A} = 1 + \frac{1}{3} \frac{2\pi N}{C} x^2 \left(\left(1 - \frac{1}{2} k^2 \right) - \frac{3}{4} k^2 \left(x - 1 \right) \right).$$
(20)

Unlike the Schwarzschild case, both A and B keep their spacelike and timelike nature and are continuous before and after (x = 1). There is hardly any justification to use the nomenclature *'horizon'*. However, despite the absence of a essential singularity at the origin and coordinate singularity at x = 1, it is still conceivable to have non-singular blackholes. This may happen if at least a fraction of the gravitating KG fields go into a bosonic degenerate state, not crushable by the gravitational compression because of Heisenberg's uncertainty principle.

C. Far distance solutions, $x \to \infty$

As x tends to infinity B and $A^{-1} \rightarrow 1$, and (12) reduces to the standard Helmholtz equation,

$$\psi'' + \frac{2}{x}\psi' + k^2\psi = 0.$$
(21)

The lowest order solutions to equation (21) are $\frac{1}{x}e^{\pm ikx}$ or any linear combination of them. However, it can be verified that none of these solutions are normalizable. To have normalizable waves we suggest a Yukawa-ameliorated Klein-Gordon (YKG) field, $\psi = \frac{1}{x}e^{-(\kappa \pm ik)x}$, where we will later find that κ is an exceedingly small positive number. Such Yukawa amelioration is a logical and perhaps the imperative way out of the dilemma; for it is inconceivable to imagine that the wave function of an single particle extends to infinity and alters the spacetime structure at universal scales.

With the inclusion of κ , the wave equation (12) changes accordingly,

$$\psi'' + \frac{2}{x}\psi' - (\kappa \pm ik)^2\psi = 0,$$
(22)

Yukawa's field, introduced in 1935, aimed to explain the strong nuclear interaction, the force that binds nucleons together within a nucleus at femtometer scales. The mediating pions were discovered in 1947. There are two masses associated with Yukawa paradigm: a) the mass of the interacting nucleons and b) the mass of the mediating bosons. Our YKG serves as the gravitational counterpart of the Yukawa paradigm, albeit operating on galactic scales. There are also two masses

in YKG: a) the mass of the gravitating constituent nucleons incorporated in $k = r_s \frac{mc}{h} = \frac{2Gm^2}{ch}$, (12). And b) the mass of the mediating graviton associated with κ . Thus,

$$\lambda_g = \frac{r_s}{2\kappa} = \frac{Gm}{\kappa c^2}, \quad Compton \text{ wavelength of the graviton,}$$
(23)

$$m_g = \frac{h}{c\lambda_g} = \frac{\kappa ch}{Gm}$$
. Compton mass of the graviton. (24)

We will come back the these Copmton parameters shortly.

Of the two lowest order solutions of (22) we choose the following real combination,

$$\psi = \frac{1}{x} e^{-\kappa x} \cos(kx + \alpha), \ \alpha \text{ mixing parameter.}$$
(25)

$$C = \int_0^\infty \psi^* \psi d^3 x = \pi \left(\frac{1}{\kappa} + \frac{\kappa \cos 2\alpha - k \sin 2\alpha}{\kappa^2 + k^2} \right) \simeq \frac{\pi}{\kappa},$$

Normalization constant. (26)

Substitution of (25) in (13) and (14) gives.

$$A^{-1} = 1 - \frac{N}{2x} e^{-2\kappa x} \left(1 - \frac{\kappa}{k} \sin 2(kx + \alpha) \right),$$
(27)

$$A = 1 + \frac{N}{2x}e^{-2\kappa x} \left(1 - \frac{\kappa}{k}\sin 2(kx + \alpha)\right),\tag{28}$$

$$B = 1 - \frac{N}{2x} e^{-2\kappa x} \left(1 + \frac{3}{4} \frac{\kappa}{k} \sin 2(kx + \alpha) \right),$$
(29)

As $x \to \infty$ the spacetime becomes asymptotically flat. Note sinusoidal undulations in A and B. We will come back and discuss the observational evidences for their existence.

D. Orbits in far Zone

In the weak field approximation the gravitational force is

$$\frac{1}{2}c^{2}B' = Nc^{2}e^{-2\kappa x} \left(\frac{\kappa}{x} \left(1 - \frac{3}{4}\cos 2(kx + \alpha)\right) + \frac{1}{2x^{2}} \left(1 + \frac{3}{4}\frac{\kappa}{k}\sin 2(kx + \alpha)\right)\right),$$
(30)

where terms of order κ^2 are omitted.

To appreciate the implications of equation (30), we go back and revise the primitive model of section II. There, we assumed the gravitating body consists of a collection of N bosons, each individually represented by a KG, or now a YKG, field. All were assumed to be in the ground state. This is perhaps a good approximation for the degenerate core of those galaxies that have a central

black hole. Blackholes are conceived to be degenerate systems. Their constituents are presumably arranged in an orderly manner in the lowest quantum states available to them, in a coherent and phase-tuned manner. Constituents have no freedom to wander around, for there are no empty states around. The much larger fraction of the gravitating body, however, stays non-degenerate. Its constituents reside in the spacious phase space of states with no obligation to tune themselves with neighbors. Therefore, let us rewrite the Lagrangian of (2) as the sum of two degenerate and non-degenerate components.

$$\mathcal{L} = (fN)\mathcal{L}_{deg}(ground.state) + \sum_{i}^{(1-f)N} \mathcal{L}_{non.deg}(i),$$
(31)

where we have divided the gravitating matter of a galaxy, say, into a fraction f of degenerate core in their ground state and the remaining non-degenerate fraction in the bulge and the arms of the galaxy in various $\psi(i)$ states (see [10] for blackholes at the center of spiral galaxies and the references therein. Typical values of f are $10^{-3} - 10^{-4}$). Constituents in the non-degenerate fraction will necessarily be in different non-correlated states and for all practical purposes will have random phases. In line with this division of \mathcal{L} the metric coefficients A and B divide accordingly. In particular the gravitational force $\frac{1}{2}c^2B'$ of (30) gets divided as follows,

$$\frac{1}{2}c^{2}B' = (fN)c^{2}e^{-2\kappa x} \left[\frac{\kappa}{x} \left(1 - \frac{3}{4}\cos 2(kx + \alpha)\right) + \frac{1}{2x^{2}} \left(1 + \frac{3}{4}\frac{\kappa}{k}\sin 2(kx + \alpha)\right)\right] \\
+ c^{2} \sum_{i}^{(1-f)N} e^{-2\kappa |\mathbf{x} - \mathbf{x}_{i}|} \left[\frac{\kappa}{|\mathbf{x} - \mathbf{x}_{i}|} \left(1 - \frac{3}{4}\cos 2(k|\mathbf{x} - \mathbf{x}_{i}| + \alpha_{i})\right) + \frac{1}{2|\mathbf{x} - \mathbf{x}_{i}|^{2}} \left(1 + \frac{3}{4}\frac{\kappa}{k}\sin 2(k|\mathbf{x} - \mathbf{x}_{i}| + \alpha_{i})\right)\right], (32)$$

where x and x_i are coordinates of the observation point and the source ones, respectively. Both x_i and α_i are random variables and (1 - f)N is a huge number. At far distances, $(\mathbf{x}_i \ll \mathbf{x}, \forall \mathbf{x}_i)$, one may Taylor expand terms of the form $f(|\mathbf{x} - \mathbf{x}_i|)$ as follows

$$f(|\mathbf{x} - \mathbf{x}_i|) = f(x) - \frac{df}{dx}\frac{\mathbf{x} \cdot \mathbf{x}_i}{x^2} + \frac{1}{2}\frac{d^2f}{dx^2}\left(\frac{\mathbf{x} \cdot \mathbf{x}_i}{x^2}\right)^2 + \cdots$$

Summing over *i* eliminates all odd terms in x_i due to their random nature. The next even term is an order x-2 smaller than the first term and can be dropped dropped as $x \to 0$. With the two trigonometric terms in (32), however, one should be careful. They are high frequency random undulations. The routine to deal with them is to square random terms, sum them up, and take the square root of the sum. Reducing (32) as explained gives

$$\frac{1}{2}c^{2}B' = fNc^{2}e^{-2\kappa x} \left(\frac{\kappa}{x} \left(1 - \frac{3}{4}\cos 2(kx + \alpha)\right) + \frac{1}{2x^{2}} \left(1 + \frac{3}{4}\frac{\kappa}{k}\sin 2(kx + \alpha)\right)\right) + (1 - f)Nc^{2}e^{-2\kappa x} \left(\frac{\kappa}{x} + \frac{1}{2x^{2}}\right) + \sqrt{(1 - f)N}e^{-2\kappa x} \left(\frac{3}{8}\frac{\kappa}{x} + \frac{3}{16}\frac{\kappa}{kx^{2}}\right).$$
 (33)

To have a clear physical picture let us rewrite (33) in terms of $r = r_s x$ and $d/dr = r_s^{-1} d/dx$.

$$\frac{1}{2}c^{2}\frac{dB}{dr} = (fN)c^{2}e^{-2\kappa r/r_{s}}\left(\frac{\kappa}{r}\left(1 - \frac{3}{4}\cos 2(kr/r_{s} + \alpha)\right) + \frac{r_{s}}{2r^{2}}\left(1 + \frac{3}{4}\frac{\kappa}{k}\sin 2(kr/r_{s} + \alpha)\right)\right)$$

$$+c^{2}(1-f)Ne^{-2\kappa r/r_{s}}\left(\frac{\kappa}{r}+\frac{r_{s}}{2r^{2}}\right)+c^{2}\sqrt{(1-f)N}e^{-2\kappa r/r_{s}}\left(\frac{3}{8}\frac{\kappa}{r}+\frac{3}{16}\frac{\kappa r_{s}}{kr^{2}}\right).$$
 (34)

In (34), the sum of three $\frac{1}{r^2}$ terms, $c^2 N r_s / 2r^2 = GM/r^2$, is the familiar Newtonian gravity and plays its dominant role at close and intermediate distances. The sum of three $\frac{1}{r}$ terms, $c^2 N \kappa / r$, are non-Newtonian and non-GR forces. They die out much slower than the Newtonian forces. As we will see shortly, they are responsible for the Tully-Fisher relation, the flat rotation curves of the spiral galaxies, and can be interpreted as alternative gravities, dark matter scenarios, etc.

E. Tully- Fisher relation- Observed facts and implication

In Newtonian gravity the speed of a test object in circular orbit about a localized gravitating mass is $v^2 = \frac{1}{2}c^2r\frac{dB}{dr} = \frac{GM}{r}$. This, however, is not the case for stars and HI clouds orbiting their host galaxies at large distances. The Tully-Fisher relation (TFr), sifted from a forage of observational data, [31], states that:

- The observed rotation velocities, v^2 , of distant stars and neutral hydrogen clouds in spiral galaxies show a much gentler decline, if any, compared to the expected GM/r falloff predicted by Newtonian gravity see Figs.2 and 3.
- Beyond the visible disks of galaxies, the observed v², more often than not, has the flat asymptote v² ∝ √M, rather than ∝ M. See [3, 4, 7, 22, 23, 25–28],

In his Modified Newtonian Gravity (MOND), Milgrom interprets TFr by saying that the effective gravitational acceleration in galactic scales is,

$$g_{eff} = g_{New}$$
 if large, $= \sqrt{g_{New}a_0}$, if small, $a_0 = 1.2 \times 10^{-10} m.sec^{-2}$.

Milgrom is silent as to whether his a_0 is a universal acceleration applicable to all spirals or not. There are, however, indications that a_0 might depend on the size and/or mass of the host galaxies, see [29] for a_0 in a list of 53 spiral galaxies.

F. Determination of κ

Leveraging the Tully-Fisher Relation (TFr) and MOND, we can now determine the value of κ . Of the three κ/r terms in (34), the one with factor \sqrt{N} is small and can be neglected. The sum of the remaining two can be written as follows,

$$Nc^{2}\frac{\kappa}{r} = \sqrt{\frac{NGm}{r^{2}}\left(\frac{Nc^{4}\kappa^{2}}{Gm}\right)} = \sqrt{g_{New}\left(\frac{Nc^{4}\kappa^{2}}{Gm}\right)}$$

It only suffices to identify $\left(\frac{Nc^4\kappa^2}{Gm}\right)$ with Milgrom's a_0 and obtain,

$$\kappa = \sqrt{\frac{Gm}{Nc^4}a_0} \approx 4.06 \times 10^{-41} N^{-1/2}$$
(35)

Note the dependence of κ on $N^{-1/2}$, which signifies dependence on the inverse square root of the mass of the host galaxy. We recall that N is almost the number of nucleons in the galactic matter. For a Milky Way type galaxy of about $1.5 \times 10^{12} M_{\odot}$, one has

$$N_{MW} \approx 1.7 \times 10^{69}, \quad \sqrt{N_{MW}} \approx 4.2 \times 10^{34}, \quad \kappa_{MW} \approx 9.6 \times 10^{-76},$$

G. Massive Graviton

There is a common consensus that gravitons mediate gravitational interactions. In the context of GR, graviton is massless and propagate with the speed of light. This also means that gravitational waves as time dependent perturbations of the spacetime propagate with the speed of light. The gravitation developed here, however, is non-GR. It is caused by massive nucleons all right; but it is Yukawa-massive as well on account of κ . From (22) and (35) we now find

$$\lambda_g = \frac{r_s}{2\kappa} = \frac{Gm}{c^2\kappa} = 3.04 \times 10^{-14} N^{1/2} \text{m}, \quad m_g = \frac{h}{c\lambda_g} = 7.2 \times 10^{-29} N^{-1/2} \text{ kg}.$$
(36)

Note dependence of the Compton parameters of our graviton on the mass of the host gravitating body. The heavier the host the longer the Compton wavelength, and the lighter the graviton. For a Milky Way type galaxy one gets

$$\lambda_g(MW) \approx 1.2 \times 10^{21} \text{m} \approx 38 \text{ kpc}, \quad m_g(MW) \approx 1.7 \times 10^{-63} \text{ kg} \approx 95 \times 10^{-29} \text{ ev}.$$
 (37)

Two interesting features: a) Transition from the Newtonian to non-Newtonian regime occurs where the two forces become equal,

$$\frac{GNm}{r_{tr}^2} = \frac{Nc^2\kappa}{r_{tr}} = \sqrt{\frac{GNm}{r_{tr}^2}} a_0 \implies r_{tr} = \frac{Gm}{c^2\kappa} = \lambda_g.$$
(38)

b) One may also ask how steep or gentle the exponential falloff $e^{-2\kappa r/r_s}$ is? We note that at $2\kappa r/r_s = 1$ the exponential falloff reduces to e^{-1} . The answer is again at $r_{tr} = \frac{Gm}{\kappa c^2} = \lambda_g$.

To appreciate the significance of the 38 kpc (37), let us recall that the radius of the optically visible Milky Way is about 26.8 kpc. That is the bulk of the mass of the galaxy is within that radius, and transition from the Newtonian gravitation to the non-Newtonian one takes place at far outside of the visible galaxy, confirming TFr and MOND.

In Table (I) we have calculated the mass and wavelength of graviton for the Milky Way, the Sun and the Earth. Evidently, up to the Kuiper belt at ≈ 30 to 50 AU the gravity is dominantly Newtonian, r^{-2} . Oort's clouds is believed to have a spread of 2000 to 5000 AU. Its outer reaches might just be in or about the transition from r^{-2} to r^{-1} gravity.

	MW		Sun	Earth
Mass(kg)	$1.5 \times$	10^{42}	2×10^{30}	$6 imes 10^{24}$
\sqrt{N}	$4.2 \times$	10^{34}	3.5×10^{28}	5.9×10^{25}
$r_{tr} = \lambda_g(m)$	$1.2 \times$	10^{21}	1.03×10^{15}	1.7×10^{11}
$m_g(ev)$	$0.95 \times$	10^{-27}	1.2×10^{-21}	6.8×10^{-19}

TABLE I:

There is a rich literature on the mass of graviton depending on how the assumed gravitation differs from the standard GR [2, 11, 12, 14, 17], or how they are deduced from astronomical observations.

Closest to what we obtain here is [13]. There a Yukawa-ameliorated gravitation is used to deduce the mass of graviton from observation on Abell 1689 galaxy cluster. They report

$$m_q(Abell1689) < 1.37 \times 10^{-29} \text{ ev}$$
 (39)

This graviton mass is a factor of about 69 lighter than our $\approx 95 \times 10^{-29}$ ev for the Milky Way (37). Accordingly their Compton wavelength should be larger than ours by the same factor,

namely $\lambda_g(Abell\ 1689) \approx 2635$ kpc. This should be attributed to the much larger mass of the Abell 1689 cluster. Noting the dependence of m_g on \sqrt{N} , this in turn means that the mass of Abell 1689 cluster should be $69^2 \simeq 4800$ times greater than the mass of Milky Way. The mass of Abell 1689 cluster as reported in [1] is about 1000 galaxies. We would not be surprised if the mass of these 1000 galaxies where normalized to the mass of the Milky Way, the agreement of (37) with (39) would come much closer one to another.

A recent paper, [9], employs a Yukawa cosmology to constrain the graviton mass from observations of the Milky Way and M31. Although their starting point differs from ours, the fact that both they and we discuss Yukawa in galactic and cosmic contexts, suggests commonalities in our conclusions. For example, both studies address the deduction of graviton mass from observations of rotation curves, or propose alternative gravities and dark matter paradigms are the two faces of the same coin, or both maintain rotation curves should be considered as quantum manifestations of galactic matters. More importantly their graviton mass $m_g \approx 1.54 \times 10^{-62}$ kg, is strikingly close to what we have in (37), as $m(MW)_g \approx 1.7 \times 10^{-63}$ kg.

IV. CONCLUSION

We argue that matter in classical physics and GR is a collection of discrete point-like entities to which, in defiance of Heisenberg's principle of uncertainty, one assigns precise coordinates and momenta. We propose to replace that discrete collection with a collection of distributed wave-like entities.

In the course of the analysis, we find that the conventional KG field does not fall off steeply enough to be normalizable. We impose a Yukawa-type falloff to suppress divergences, and arrive at YKG field. The task reduces to solving a set of coupled non-linear Einstein-YKG equations.

The coupled equations allow spherically symmetric and static solutions. In an iteration scheme, we find no singular solutions at r = 0 nor at the Schwarzschild horizon. That is we find no blackholes with essential or coordinate singularity.

The existence of non-singular blackholes, however, is not ruled out. This happens if a fraction of YKG bosons (isospin coupled neutron pairs, say) at the center of galaxies go into degenerate states. Perhaps the blackholes detected at the center of most of the spiral galaxies, including the Milky Way, are of this type.

The far zone solutions are the most interesting. YKG fields reach infinity as $\frac{1}{r}\exp(-\kappa/r)\cos(kr+\alpha)$ if degenerate, and as $\frac{1}{r}\exp(-\kappa/r)$ if non-degenerate. At intermediate distances the gravitational force is dominantly Newtonian. But becomes non-Newtonian and non-GR as $r \to \infty$.

The conclusions above are supported by the Tully-Fisher relation, abstracted from an immense pool of observations of rotation curves of spiral galaxies. This in turn can be interpreted as alternative gravities, e.g. MOND, dark matter etc.

The sinusoidal undulation of the degenerate core can be seen in some rotation curves. See Fig.1 for the rotation curve of our toy galaxy and Fig.3 for the rotation curves of M31 and NGC 6503.

We recall that the Yukawa field was introduced in 1935 to mediate strong nuclear interactions within femtometer nuclear sizes. The mediators were pions, discovered in 1947.

Our YKG is the gravitational copy of Yukawa's field, albeit on galactic scales. In a Milky Way size galaxy, YKG is mediated by massive gravitons of Compton wavelength of about 38 kpc and mass 95×10^{-29} ev. These figures are in accord with the findings of other investigators reported in, e.g. [1, 9, 13], say.

Nowadays scientists and technologists can transmit and receive quantum messages across distances of tens and hundreds of kilometers through the science and technology of quantum entanglement. Do astronomers have sufficient evidence to claim they are observing quantum effects across galaxies through rotation curves? Couldn't TFr be claimed as a quantum manifestation of the galactic matter? We think it could.

Two birds with one stone? At one extreme blackholes with no essential and no coordinate singularity. On the other extreme flat rotation curves, interpretable as alternative gravity, dark matter, etc. All on account of setting aside the notion of point-like entities and paying due respect to the quantum nature of the matter, even if they are located at galactic and perhaps cosmic distances.

Last but not least: Spacetime is a continuum. The classical matter is the world of discrete point-likes. In the formulation of GR, one chooses Einstein's tensor from the continuum and the energy-momentum tensor from the discretes. One equates them and arrives at the celebrated GR. But GR turns out to be singularity prone. To ensure greater consistency between the two quantities being equated, it would have been appropriate if both originated either from the continuum or from the discretes. That is what we have tried to do in this paper. Besides Einstein's tensor from the continuum, we have selected the energy-momentum tensor from the continuum of

quantum waves.

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FIG. 1: Rotation curve of our toy galaxy. In the right hand panel we have scaled up the periodic undulations. They are caused by the degenerate core of the model and are quantum effects.

Free parameters: $N = 10^{70}, \kappa = 10^{-3}, k = 2, f = 10^{-2}, \alpha = 0.$



FIG. 2: A sample of rotation curves of spiral galaxies - Reference: Carroll & Ostlie, Ch 24; Rohlf, Ch. 19; Kaufmann, Ch. 28.



FIG. 3: Left panel: Rotation curves of M31 - Reference: Tamm et al., Astronomy & Astrophysics 546,2,A4, 2012. Right panel: Rotation curve of NGC 6503-Reference: Freese K, arXiv:0812.4005, [15].Note periodic undulations on the observed dotted points and compare them with those in Fig 1.