$\mathcal{N} = 2$ superconformal higher-spin multiplets and their hypermultiplet couplings

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Abstract: We construct an off-shell $\mathcal{N} = 2$ superconformal cubic vertex for the hypermultiplet coupled to an arbitrary integer higher spin $\mathbf{s}$ gauge $\mathcal{N} = 2$ supermultiplet in a general $\mathcal{N} = 2$ conformal supergravity background. We heavily use $\mathcal{N} = 2, 4D$ harmonic superspace that provides an unconstrained superfield Lagrangian description. We start with $\mathcal{N} = 2$ global superconformal symmetry transformations of the free hypermultiplet model and require invariance of the cubic vertices of general form under these transformations and their gauged version. As a result, we deduce $\mathcal{N} = 2, 4D$ unconstrained analytic superconformal gauge potentials for an arbitrary integer $\mathbf{s}$. These are the basic ingredients of the approach under consideration. We describe the properties of the gauge potentials, derive the corresponding superconformal and gauge transformation laws, and inspect the off-shell contents of the thus obtained $\mathcal{N} = 2$ superconformal higher-spin $\mathbf{s}$ multiplets in the Wess-Zumino gauges. The spin $\mathbf{s}$ multiplet involves $8(2\mathbf{s} - 1)_B + 8(2\mathbf{s} - 1)_F$ essential off-shell degrees of freedom. The cubic vertex has the generic structure *higher spin gauge superfields* $\times$ *hypermultiplet supercurrents*. We present the explicit form of the relevant supercurrents.

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1 Introduction

Superconformal field theories constitute an important subclass of field theories, with numerous applications in classical and quantum field theory, gravity and string theory (see, e.g., [1], [2], [3]). For example, such theories can be treated as fixed points of the proper renormalization group flows and any quantum field theory can be recovered as a deformation of some conformal field theory (see, e.g.,[4]). One more well known application of conformal theories, especially in supergravities, is the method of conformal compensators in diverse dimensions. It allows one to derive standard Einstein gravity and the relevant non-conformal supergravities, starting from the conformal (super)gravities coupled to the appropriate matter compensating (super)fields. These compensators ensure the spontaneous breaking of conformal (super)groups to some subgroups thereof (see, e.g., [5], [6]). The compensator approach is a powerful way of constructing diverse supergravity actions.

Higher spin theories are a natural generalization of the standard (super)gauge theories and (super)gravities, and they attract vast attention due to their intimate relationships with (super)string theory [7–12]. There arises the natural task of constructing (super)conformal theories of higher spins as the basis of the whole plethora of the higher-spin theories. To know such superconformal extensions is also of high importance for constructing higher spin theories on AdS_4 and other conformally flat backgrounds. Indeed, these theories can be obtained by gauging the proper subgroups of the (super)conformal groups, like the standard $4D$ Poincaré (super)symmetry in the case of flat (super)Minkowski background.

Free higher-spin theories in $4D$ Minkowski space were pioneered by Fronsdal and Fang and Fronsdal in refs. [13, 14]. Their conformal generalizations were constructed by Fradkin and Tseytlin [15]. They introduced conformal higher spin fields and defined the corresponding gauge transformations. The actions constructed provided a higher-spin cousins of the Weyl tensor - squared actions. Since then, various generalizations of these theories, including generalizations to curved gravity backgrounds, were intensively studied (see, e.g., [16–27]). Conformal higher-spin cubic vertices were for the first time constructed in refs.

[28, 29]. Later on, they were widely discussed in the context of induced quantum actions [10, 30–32]. Using the higher-spin conformal vertices, one can construct consistent interacting higher-spin actions as the induced actions. Another approach to constructing the complete interacting higher spin conformal theories has been developed in ref. [33]. Such interacting theories are defined on a flat background and are generalizations of Weyl gravity. This means that they involve higher derivatives and so are non-unitary on their own.

$\mathcal{N} = 1, 4D$ supersymmetric generalization of conformal higher spins was considered in [29] using the component approach. The free off-shell $\mathcal{N} = 1$ superconformal theories and their couplings to chiral multiplet were constructed in refs. [34] in $\mathcal{N} = 1, 4D$ superspace. These authors, based on the earlier articles [35–37], constructed unconstrained $\mathcal{N} = 1$ higher-spin prepotentials (see also [38]), found their gauge and superconformal transformations, investigated their component structure and derived invariant actions in the flat $\mathcal{N} = 1$ superspace. A minimal extension of the actions constructed to $\mathcal{N} = 1$ conformal supergravity (for multiplets with integer higher spin) was also proposed and it was shown that these actions are gauge invariant only on conformally flat superspaces with vanishing super-Weyl tensor (in particular, on AdS superspace $AdS^{4|4}$). Some generalizations of these theories were considered in [25, 39]. In ref. [40] an off-shell formulation of $\mathcal{N} = 1$ higher-spin theories with the half-integer highest spin was given, using the appropriate compensator supermultiplets.

The $\mathcal{N} = 2, 4D$ superconformal higher-spin theories (equally as their $\mathcal{N}$ extended versions) in an arbitrary conformally flat background were elaborated in [41][1], based on the notion of $\mathcal{N} = 2$ conformal superspace [43]. The appropriate Noether couplings to an on-shell hypermultiplet were constructed there. It is worth noting, however, that the component contents of $\mathcal{N} = 2$ higher-spin superconformal multiplets and the relevant off-shell cubic vertices were not addressed in that work.

One of the ways to define gauge fields and their gauge transformations is to gauge the rigid symmetries of some free theory and to construct the corresponding cubic vertex. The simplest cubic vertex, the $(s, 0, 0)$ vertex, is the product of the higher-spin s gauge field and the Noether current bilinear in massless complex scalar fields. The vertices of this type were widely studied. The most natural questions regarding them are as to: is it possible to make the $(s, 0, 0)$ vertices gauge-invariant to all orders, and is it possible to set up such vertices on an arbitrary gravitational background?

1. Cubic $(s, 0, 0)$ vertex (plus free scalar action) can be made gauge invariant to all orders by deforming the gauge transformation laws of higher spin fields [30]. The resulting gauge transformations are generically nonabelian and nonlinear. They mix different higher-spin fields among themselves, while the scalar fields are transformed linearly and homogeneously. In ref. [32] this construction was extended to an arbitrary conformally flat background in the manifestly covariant way, as well as to $\mathcal{N} = 1$ superconformal case.

2. One can easily construct the conserved spin 1 and spin 2 currents for the conformally-

[1]The $\mathcal{N} = 2, 4D$ superconformal gravitino multiplet was described in [42].

coupled complex scalar in an arbitrary curved background. Respectively, one can build $(1,0,0)$ and $(2,0,0)$ vertices on curved backgrounds. However, starting from $s \geq 3$, the naive attempts to construct conserved currents on a curved background gave rise to the conclusion that the conservation can be achieved only for the conformally flat case [22, 32]. So the non-vanishing Weyl tensor provides an obstruction to the existence of the $(s,0,0)$ vertices for $s \geq 3$. However, it was shown in ref. [22] that, by adding the vertex $(1,0,0)$ and modifying accordingly the gauge transformation of the spin 1 field, one can achieve gauge invariance on an arbitrary curved background too. Similar conclusions were drawn in ref. [39].

In the present paper we perform an analogous analysis for off-shell $\mathcal{N}=2$ supersymmetric generalization of the $(s,0,0)$ interactions. As we will see, already the $\mathcal{N}=2$ spin $\mathbf{3}$ superconformal multiplet simultaneously contains both spin 3 and spin 1, and this property greatly simplifies the construction of the corresponding vertices in a curved background.

Our work extends to the superconformal case some of our previous results on the off-shell $\mathcal{N}=2, 4D$ higher spins and their hypermultiplet cubic coupling [44–46] (see also reviews [47, 48]). Since the hypermultiplet has a natural off-shell formulation in harmonic superspace (HSS) [49–51] in terms of $\mathcal{N}=2$ analytic harmonic superfields, here we make use of just this formulation. We demonstrate that the harmonic analyticity imposes severe constraints on the admissible structure of the cubic interaction vertices of the hypermultiplet and higher-spin conformal $\mathcal{N}=2$ gauge superfields. We focus just on the construction of $\mathcal{N}=2$ superconformal cubic couplings with the matter hypermultiplets [2]. To set up such cubic couplings, we introduce the corresponding off-shell superconformal spin $\mathbf{s}$ gauge multiplets [3], define the corresponding minimal sets of analytic gauge potentials, derive their rigid superconformal transformation laws and, at the linearized level, their gauge transformation laws. We present the relevant Wess-Zumino gauges for the component fields. We also expound how to promote these cubic vertices to an arbitrary $\mathcal{N}=2$ conformal supergravity background: one should consider an infinite tower of $\mathcal{N}=2$ superconformal higher-spin fields interacting with a hypermultiplet. This allows one to define a nonabelian deformation of the gauge transformation algebra and demonstrate that the relevant interacting theory is gauge-invariant to all orders.

The basic novel features of superconformal couplings of $\mathcal{N}=2$ higher-spin gauge superfields to the hypermultiplet in HSS compared to the non-conformal case [45] can be schematically outlined as follows. The difference arises already in the case of the spin $\mathbf{s}=2$ multiplet (conformal $\mathcal{N}=2$ supergravity), where the analyticity-preserving harmonic derivative $\mathcal{D}^{++}$, when acting on the hypermultiplet superfields, is covariantized as

$$\mathcal{D}^{++} \;\Rightarrow\; \mathcal{D}^{++} + \kappa_2 \hat{\mathcal{H}}^{++}_{(s=2)}\,, \quad \hat{\mathcal{H}}^{++}_{(s=2)} = h^{++M}\partial_M\,, \quad M = (\alpha\dot{\alpha}\,, \alpha+\,, \dot{\alpha}+\,,\; ++)\,. \tag{1.1}$$

Here there appears a new analytic gauge superfield $h^{(+4)}$ [51]. Its necessity can be substantiated from requiring rigid conformal $\mathcal{N}=2$ invariance with respect to which only the

[2] $\mathcal{N}=2$ generalizations of Fradkin-Tseytlin action in HSS will be studied elsewhere.

[3] We use bold $\mathbf{s}$ to denote $\mathcal{N}=2$ multiplet with the highest spin s. For example, hypermultiplet corresponds to $\mathbf{s}=\frac{1}{2}$, $\mathcal{N}=2$ Maxwell multiplet to $\mathbf{s}=1$, $\mathcal{N}=2$ Weyl multiplet to $\mathbf{s}=2$.

whole set of the potentials in the operator $\hat{\mathcal{H}}_{(s=2)}$ turns out to be closed. In the spin $\mathbf{s} = 3$ case, the covariantization is accomplished by the differential operator of the second order,

$$\mathcal{D}^{++} \;\Rightarrow\; \mathcal{D}^{++} + \kappa_3 \hat{\mathcal{H}}^{++}_{(s=3)} J\,, \quad \hat{\mathcal{H}}^{++}_{(s=3)} = h^{++MN}\partial_N\partial_M + h^{++}\,, \tag{1.2}$$

where the analytic gauge potentials h^{++MN} satisfy some grading and irreducibility conditions (see below), while J is some matrix $U(1)$ generator. Once again, only the whole set of gauge potentials in (1.2) is closed under a linear realization of rigid $\mathcal{N} = 2$ superconformal group. So the latter plays the same restrictive role for cubic superconformal vertices, as the rigid $\mathcal{N} = 2$ supersymmetry for non-conformal vertices [44–46]. The radical extension of the number of gauge potentials for $\mathbf{s} \geq 3$ also gives rise to an essential extension of the gauge freedom compared to the $\mathbf{s} = 2$ case. This can be used to fully gauge away many gauge potentials,

$$\hat{\mathcal{H}}^{++}_{(s=3)} \;\Rightarrow h^{++\alpha\dot{\alpha}M}\partial_M\partial_{\alpha\dot{\alpha}}\,. \tag{1.3}$$

The rigid $\mathcal{N} = 2$ superconformal symmetry acts on this minimal set of potentials by transformations which are in general *nonlinear* in the potentials. All these notable features directly generalize to $\mathcal{N} = 2$ spins $\mathbf{s} > 3$.

The paper is organized as follows. In section 2 we recall the basic elements of harmonic superspace and describe free off-shell hypermultiplet. Section 3 contains discussion of the $\mathcal{N} = 2$ superconformal symmetry realization in harmonic superspace and expounds our strategy of construction of the off-shell superconformal $\mathcal{N} = 2$ multiplets in (curved) harmonic superspace. In sections 4 and 5 we present superconformal transformations for the spin $\mathbf{1}$ and spin $\mathbf{2}$ multiplets and the corresponding off-shell $(\mathbf{1}, \mathbf{\frac{1}{2}}, \mathbf{\frac{1}{2}})$, $(\mathbf{2}, \mathbf{\frac{1}{2}}, \mathbf{\frac{1}{2}})$ superconformal couplings. In section 5 we discuss the hypermultiplet in the background of $\mathcal{N} = 2$ conformal supergravity. Section 6 is devoted to the crucial new spin $\mathbf{3}$ case: we introduce a minimal set of analytic prepotentials, study their component structure and construct off-shell $(\mathbf{3}, \mathbf{\frac{1}{2}}, \mathbf{\frac{1}{2}})$ vertices in an arbitrary conformal supergravity background. In section 7 we generalize the spin $\mathbf{3}$ results to the general $\mathcal{N} = 2$ spin $\mathbf{s}$. In section 8 we sketch some results on nonabelian (and nonlinear) deformation of higher-spin gauge algebra in the case of infinite tower of $\mathcal{N} = 2$ conformal higher-spin fields minimally interacting with the hypermultiplet. Such a theory possesses the exact invariance with respect to these nonabelian gauge transformations. The concluding comments and the basic problems for the future study are contents of the last section 9. Appendix A contains technical details of fixing Wess-Zumino gauge in the spin $\mathbf{3}$ case. In Appendix B we discuss some interesting reparametrization freedom of free hypermultiplet. The superconformal transformation properties of the derivatives in the analytic superspace coordinates and those of some gauge potentials (for $\mathbf{s} = 2, 3$) are collected in Appendix C.

2 Harmonic superspace

We will deal with $\mathcal{N} = 2$ harmonic superspace (HSS) [49–51] parametrized by the coordinates in the analytic basis:

$$Z := (x^{\alpha\dot{\alpha}}, \theta^{+\hat{\alpha}}, \theta^{-\hat{\alpha}}, u^{\pm i}), \quad \hat{\alpha} = (\alpha, \dot{\alpha})\,. \tag{2.1}$$

In addition to the standard $4D$ superspace coordinates $(x, \theta^{\pm})$, HSS involves additional $SU(2)/U(1)$ harmonic variables $u^{\pm}_i$, $i = 1, 2$, satisfying the unitarity constraint $u^{+i}u^{-}_i = 1$.

The crucial feature of HSS is the presence of the invariant subspace with half the number of Grassmann variables. This analytic superspace is parametrized by the coordinates:

$$\zeta := (x^{\alpha\dot{\alpha}}, \theta^{+\hat{\alpha}}, u^{\pm i}). \tag{2.2}$$

For the description of massive hypermultiplet and its higher spin couplings it is also necessary to introduce an auxiliary x^5 coordinate, see, e.g., [45]. The analytic superspace is closed under the tilde-conjugation defined as:

$$\widetilde{x^{\alpha\dot{\alpha}}} = x^{\alpha\dot{\alpha}}, \qquad \widetilde{\theta^{\pm}_{\alpha}} = \bar{\theta}^{\pm}_{\dot{\alpha}}, \qquad \widetilde{\bar{\theta}^{\pm}_{\dot{\alpha}}} = -\theta^{\pm}_{\alpha}, \qquad \widetilde{u^{\pm i}} = -u^{\pm}_i, \qquad \widetilde{u^{\pm}_i} = u^{\pm i}. \tag{2.3}$$

The covariant harmonic derivatives in the analytic basis are defined by,

$$\mathcal{D}^{++} := \partial^{++} - 4i\theta^{+\rho}\bar{\theta}^{+\dot{\rho}}\partial_{\rho\dot{\rho}} + \theta^{+\hat{\rho}}\partial^{+}_{\hat{\rho}} + (\theta^{\hat{+}})^2\partial_5, \tag{2.4a}$$

$$\mathcal{D}^{--} := \partial^{--} - 4i\theta^{-\rho}\bar{\theta}^{-\dot{\rho}}\partial_{\rho\dot{\rho}} + \theta^{-\hat{\rho}}\partial^{-}_{\hat{\rho}} + (\theta^{\hat{-}})^2\partial_5, \tag{2.4b}$$

$$\mathcal{D}^{0} = \partial^{0} + \theta^{+\hat{\rho}}\partial^{-}_{\hat{\rho}} - \theta^{-\hat{\rho}}\partial^{+}_{\hat{\rho}}, \tag{2.4c}$$

and satisfy $su(2)$ algebra relations:

$$[\mathcal{D}^{++}, \mathcal{D}^{--}] = \mathcal{D}^0, \qquad [\mathcal{D}^0, \mathcal{D}^{\pm\pm}] = \pm 2\mathcal{D}^{\pm\pm}. \tag{2.5}$$

Here we used the following notations for the partial derivatives in harmonic variables:

$$\partial^{++} = u^{+i}\frac{\partial}{\partial u^{-i}}, \qquad \partial^{--} = u^{-i}\frac{\partial}{\partial u^{+i}}, \qquad \partial^{0} = u^{+i}\frac{\partial}{\partial u^{+i}} - u^{-i}\frac{\partial}{\partial u^{-i}}, \tag{2.6}$$

$$[\partial^{++}, \partial^{--}] = \partial^0. \tag{2.7}$$

Other partial derivatives are defined in the standard way, e.g., $\partial_{\alpha\dot{\alpha}} = \frac{\partial}{\partial x^{\alpha\dot{\alpha}}}$, etc [4].

The action of tilde-conjugation on various derivatives follows directly from the definitions (2.3):

$$\widetilde{\partial_{\alpha\dot{\alpha}}} = \partial_{\alpha\dot{\alpha}}, \quad \widetilde{\partial^{-}_{\alpha}} = -\partial^{-}_{\dot{\alpha}}, \quad \widetilde{\partial^{-}_{\dot{\alpha}}} = \partial^{-}_{\alpha}, \quad \widetilde{\partial^{\pm\pm}} = \partial^{\pm\pm}. \tag{2.8}$$

Harmonic superspace provides efficient tools to deal with $\mathcal{N} = 2$ supersymmetric theories, both on the classical and quantum levels. The hypermultiplet and the most general hypermultiplet self-couplings [52], $\mathcal{N} = 2$ Yang-Mills theory, different $\mathcal{N} = 2$ supergravities (see a recent review [53]), as well as $\mathcal{N} = 2$ generalizations of Fronsdal theory [44], are adequately described in $\mathcal{N} = 2$ HSS approach. The pivotal feature of the HSS approach is that all the basic $\mathcal{N} = 2$ superfields are analytic, thus manifesting the crucial role of the *harmonic Grassmann analyticity principle* in $\mathcal{N} = 2$ supersymmetric theories.

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[4] The relation with the vector notation is the same as in [51], $x^{\dot{\alpha}\beta} = x^m(\tilde{\sigma}_m)^{\dot{\alpha}\beta}$, $\partial_m = (\tilde{\sigma}_m)^{\dot{\alpha}\beta}\partial_{\dot{\alpha}\beta}$, $\partial_{\alpha\dot{\beta}} = \frac{1}{2}\sigma^m_{\alpha\dot{\beta}}\partial_m$.

2.1 Free hypermultiplet

Since our main subject will be $\mathcal{N}=2$ superconformal interactions of higher-spin superfields with hypermultiplet, we start by giving all the necessary details of the HSS formulation of hypermultiplet. It is described by an analytic unconstrained superfield $q^+(\zeta)$ with an infinite number of auxiliary fields off shell. The free hypermultiplet action reads [51]:

$$S_{free} = -\frac{1}{2}\int d\zeta^{(-4)}\, q^{+a}\mathcal{D}^{++}q^+_a = -\int d\zeta^{(-4)}\, \tilde{q}^+\mathcal{D}^{++}q^+. \tag{2.9}$$

Here we used the notation:

$$q^{+a} = (\tilde{q}^+, q^+), \qquad q^+_a = \epsilon_{ab}q^{+b} = \begin{pmatrix} q^+ \\ -\tilde{q}^+ \end{pmatrix}. \tag{2.10}$$

The superfield q^{+a} forms a doublet of the Pauli-Gürsey group $\mathrm{SU}(2)_{PG}$. The $\mathrm{SU}(2)_{PG}$ - covariant notation is useful when constructing higher-spin vertices.

The free hypermultiplet equation of motions is:

$$\mathcal{D}^{++}q^{+a} = 0. \tag{2.11}$$

The discussion of the on-shell field content of hypermultiplet can be found, e.g., in section 5.1 of ref. [45]. In what follows we will merely use the superfield aspects of the HSS description of the hypermultiplet. Here we only remark that the hypermultiplet contains a doublet of complex scalars, so it can interact with both even and odd spins.

3 $\mathcal{N}=2$ superconformal symmetry of hypermultiplet

The realization of $\mathcal{N}=2$ superconformal symmetry on the HSS coordinates is given in [51, 54]. We will be interested in constructing $\mathcal{N}=2$ superconformal cubic $(\mathbf{s}, \mathbf{\frac{1}{2}}, \mathbf{\frac{1}{2}})$ vertices of the higher spin gauge superfields with the hypermultiplet. To this end, we need to introduce the appropriate set of analytic higher-spin superconformal gauge potentials and define their superconformal transformation laws. The hypermultiplet superconformal transformation law is well known, so from requiring the invariance of the interaction one can determine transformation properties of the higher spin gauge potentials. Based on the experience of dealing with the non-conformal case [45, 46], we will use, as a departure point, the most general type of interaction with higher derivatives and determine the minimal set of the analytic higher-spin potentials closed under $\mathcal{N}=2$ superconformal symmetry. In this section we first discuss $\mathcal{N}=2$ superconformal symmetry of the free massless hypermultiplet and then explain our general strategy of constructing $\mathcal{N}=2$ higher-spin superconformal couplings.

We start with the general one-derivative hypermultiplet transformations[5]:

$$\delta q^{+a} = -\hat{\Lambda}q^{+a} - \frac{1}{2}\Omega q^{+a}, \tag{3.1}$$

[5] Such transformations can be realized on the HSS coordinates, see [51]. We basically consider superfield transformations in their active form, since we are interested in their generalization to the case of higher-spin symmetries which cannot be realized on the coordinates.

where[6]:

$$\hat{\Lambda} := \lambda^M \partial_M = \lambda^{\alpha\dot{\alpha}}\partial_{\alpha\dot{\alpha}} + \lambda^{+\hat{\alpha}}\partial^-_{\hat{\alpha}} + \lambda^{++}\partial^{--} + \lambda^5\partial_5, \tag{3.2}$$

$$\Omega := (-1)^{P(M)}\partial_M\lambda^M = \partial_{\alpha\dot{\alpha}}\lambda^{\alpha\dot{\alpha}} - \partial^-_{\hat{\alpha}}\lambda^{+\hat{\alpha}} + \partial^{--}\lambda^{++}. \tag{3.3}$$

Here $\hat{\Lambda}$ is the first-order differential operator, Ω is the weight factor constructed out of the parameters λ^M, $M = \{\alpha\dot{\alpha}, +\hat{\alpha}, ++, 5\}$. Since the superfield q^{+a} is analytic, we impose the condition that the transformations (3.1) preserve the analyticity, which implies the parameters λ^M to be unconstrained analytic:

$$\partial^+_{\hat{\rho}}\lambda^{\alpha\dot{\alpha}} = 0, \qquad \partial^+_{\hat{\rho}}\lambda^{+\hat{\alpha}} = 0, \qquad \partial^+_{\hat{\rho}}\lambda^{++} = 0, \qquad \partial^+_{\hat{\rho}}\lambda^5 = 0. \tag{3.4}$$

Also we assume x^5-independence of the transformation parameters, since x^5 is an auxiliary coordinate needed merely for the description of massive hypermultiplet. Unlike the rigid symmetries considered in our previous papers [45, 46], here we allow for a nontrivial coordinate dependence of λ^M in the rigid case. This will lead to an extended algebra of rigid hypermultiplet symmetries with a larger number of independent transformation parameters.

Varying the free action (2.9) with respect to the transformations (3.1) with generic analytic parameters λ^M yields:

$$\delta S_{free} = \frac{1}{2}\int d\zeta^{(-4)}\, q^{+a}[\mathcal{D}^{++}, \hat{\Lambda}]q^+_a. \tag{3.5}$$

The precise form of the commutator in (3.5) is as follows:

$$\begin{aligned}
[\mathcal{D}^{++}, \hat{\Lambda}] = &\left(\mathcal{D}^{++}\lambda^{\alpha\dot{\alpha}} + 4i\lambda^{+\alpha}\bar{\theta}^{+\dot{\alpha}} + 4i\theta^{+\alpha}\bar{\lambda}^{+\dot{\alpha}}\right)\partial_{\alpha\dot{\alpha}} \\
&+ \left(\mathcal{D}^{++}\lambda^{+\hat{\alpha}} - \lambda^{++}\theta^{+\hat{\alpha}}\right)\partial^-_{\hat{\alpha}} + \mathcal{D}^{++}\lambda^{++}\partial^{--} + \lambda^{++}\mathcal{D}^0 \\
&+ \left(\mathcal{D}^{++}\lambda^5 - 2\lambda^{+\hat{\rho}}\theta^+_{\hat{\rho}}\right)\partial_5 + \lambda^{++}\theta^{-\hat{\alpha}}\partial^+_{\hat{\alpha}}.
\end{aligned} \tag{3.6}$$

Taking into account the relations $\mathcal{D}^0 q^{+a} = q^{+a}$ and $q^{+a}q^+_a = 0$, we derive the condition of invariance of the action (2.9) as

$$[\mathcal{D}^{++}, \hat{\Lambda}] = \lambda^{++}\mathcal{D}^0, \tag{3.7}$$

or, in terms of the parameters λ^M,

$$\begin{cases}
\mathcal{D}^{++}\lambda^{\alpha\dot{\alpha}} + 4i\lambda^{+\alpha}\bar{\theta}^{+\dot{\alpha}} + 4i\theta^{+\alpha}\bar{\lambda}^{+\dot{\alpha}} = 0, \\
\mathcal{D}^{++}\lambda^{+\alpha} - \lambda^{++}\theta^{+\alpha} = 0, \\
\mathcal{D}^{++}\lambda^{+\dot{\alpha}} - \lambda^{++}\bar{\theta}^{+\dot{\alpha}} = 0, \\
\mathcal{D}^{++}\lambda^{++} = 0, \\
\mathcal{D}^{++}\lambda^5 - 2\lambda^{+\hat{\rho}}\theta^+_{\hat{\rho}} = 0.
\end{cases} \tag{3.8}$$

[6] $M = (\alpha\dot{\alpha}, \alpha+, \dot{\alpha}+, ++, 5)$; $\quad P(\alpha\dot{\alpha}) = P(++) = P(5) = 0,\ P(\hat{\alpha}+) = 1.$

The general solution of the system (3.8) is just the sought $\mathcal{N}=2$ **superconformal transformations**:

$$\begin{cases} \lambda^{\alpha\dot\alpha}_{sc} = & a^{\alpha\dot\alpha} - 4i\left(\epsilon^{\alpha i}\bar\theta^{+\dot\alpha} + \theta^{+\alpha}\bar\epsilon^{\dot\alpha i}\right)u^-_i + x^{\dot\alpha\rho}k_{\rho\dot\rho}x^{\dot\rho\alpha} + ax^{\alpha\dot\alpha} \\ & -4i\theta^{+\alpha}\bar\theta^{+\dot\alpha}\lambda^{(ij)}u^-_iu^-_j - 4i\left(x^{\alpha\dot\rho}\eta^i_{\dot\rho}\bar\theta^{+\dot\alpha} + \theta^{+\alpha}\eta^i_\rho x^{\rho\dot\alpha}\right)u^-_i\,, \\ \lambda^{+\alpha}_{sc} = & \epsilon^{\alpha i}u^+_i + \frac{1}{2}\theta^{+\alpha}(a+ib) + x^{\alpha\dot\beta}k_{\beta\dot\beta}\theta^{+\beta} + x^{\alpha\dot\alpha}\eta^i_{\dot\alpha}u^+_i \\ & +\theta^{+\alpha}\left(\lambda^{(ij)}u^+_iu^-_j + 4i\theta^{+\rho}\eta^i_\rho u^-_i\right), \\ \bar\lambda^{+\dot\alpha}_{sc} = & \epsilon^{\dot\alpha i}u^+_i + \frac{1}{2}\bar\theta^{+\dot\alpha}(a-ib) + x^{\dot\alpha\beta}k_{\beta\dot\beta}\bar\theta^{+\dot\beta} + x^{\alpha\dot\alpha}\eta^i_\alpha u^+_i \\ & +\bar\theta^{+\dot\alpha}\left(\lambda^{(ij)}u^+_iu^-_j - 4i\bar\theta^{+\dot\rho}\eta^i_{\dot\rho}u^-_i\right), \\ \lambda^{++}_{sc} = & \lambda^{ij}u^+_iu^+_j + 4i\theta^{+\alpha}\bar\theta^{+\dot\alpha}k_{\alpha\dot\alpha} + 4i\left(\theta^{+\alpha}\eta^i_\alpha + \eta^i_{\dot\alpha}\bar\theta^{+\dot\alpha}\right)u^+_i\,. \end{cases} \tag{3.9}$$

Respectively, the weight factor (3.3) is expressed as:

$$\Omega_{sc} = 2a + 2k_{\beta\dot\beta}x^{\beta\dot\beta} - 2\lambda^{(ij)}u^+_iu^-_j - 8i\left(\theta^{+\alpha}\eta^i_\alpha + \eta^i_{\dot\alpha}\bar\theta^{+\dot\alpha}\right)u^-_i\,. \tag{3.10}$$

It satisfies the useful relation:

$$\mathcal{D}^{++}\Omega_{sc} = -2\lambda^{++}_{sc}. \tag{3.11}$$

As follows from (3.6), the last condition in the system (3.8) appears only if $\partial_5 q^{+a} \neq 0$. So we are led to impose the constraint $\partial_5 q^{+a} = 0$, i.e. limit our consideration to the massless hypermultiplet. This is consistent with the well known fact that all theories with exact (super)conformal symmetry are massless (see, e.g., [5]).

Symmetry (3.9) extends rigid $\mathcal{N}=2$ supersymmetry of the free massless hypermultiplet, which was generalized to the higher-spin symmetries in [45, 46]:

$$\underbrace{\{a^{\alpha\dot\alpha}, \epsilon^{\hat\alpha i}\}}_{\mathcal{N}=2\text{ supersymmetry}} \quad \to \underbrace{\{a^{\alpha\dot\alpha}, \epsilon^{\hat\alpha i}, a, b, k_{\alpha\dot\alpha}, \eta^{\hat\alpha i}, \lambda^{(ij)}\}}_{\mathcal{N}=2\text{ superconformal symmetry}}. \tag{3.12}$$

The analytic parameters (3.9) are those of $\mathcal{N}=2$ superconformal symmetry in the realization on the coordinates of analytic superspace [51, 54] (here we omit Lorentz transformations). The transformation parameters can be attributed as:

- $a^{\alpha\dot\alpha}$ - global translations;
- $\epsilon^{\hat\alpha i}$ - rigid $\mathcal{N}=2$ supersymmetry;
- a - dilatations;
- b - $U(1)$ R-symmetry;
- $k_{\alpha\dot\alpha}$ - special conformal transformations;
- $\eta^{\hat\alpha i}$ - rigid $\mathcal{N}=2$ conformal supersymmetry;
- $\lambda^{(ij)}$ - $SU(2)_R$ symmetry.

One can directly check that these transformations satisfy the relations of $\mathfrak{su}(2,2|2)$ superalgebra, that is $\mathcal{N}=2$ superconformal algebra. We will require the cubic couplings to be invariant under these transformations.

For completeness, we also quote how conformal transformations are implemented on non-analytic coordinates θ^-. Using the relation

$$\theta^{+\hat{\alpha}} = \mathcal{D}^{++}\theta^{-\hat{\alpha}} \tag{3.13}$$

and the transformation laws $\delta^*\theta^{\pm\hat{\alpha}} = \lambda^{\pm\hat{\alpha}}_{sc}$, one obtains:

$$\lambda^{+\hat{\alpha}}_{sc} = \mathcal{D}^{++}\lambda^{-\hat{\alpha}}_{sc} + [\hat{\Lambda}_{sc}, \mathcal{D}^{++}]\theta^{-\hat{\alpha}} = \mathcal{D}^{++}\lambda^{-\hat{\alpha}}_{sc} + \lambda^{++}_{sc}\theta^{-\hat{\alpha}}, \tag{3.14}$$

whence

$$\begin{aligned}\lambda^{-\alpha}_{sc} = \epsilon^{\alpha i}u^-_i &+ \frac{1}{2}\theta^{-\alpha}(a+ib) + x^{\alpha\dot{\beta}}k_{\beta\dot{\beta}}\theta^{-\beta} - 2i(\theta^-)^2\bar{\theta}^+_{\dot{\beta}}k^{\dot{\beta}\alpha} \\ &+ \left(x^{\alpha\dot{\alpha}} + 4i\theta^{-\alpha}\bar{\theta}^{+\dot{\alpha}}\right)\eta^i_{\dot{\alpha}}u^-_i + 4i\eta^i_\beta\theta^{-\beta}\left(\theta^{-\alpha}u^+_i - \theta^{+\alpha}u^-_i\right) \\ &+ \lambda^{ij}u^-_i\left(u^-_j\theta^{+\alpha} - u^+_j\theta^{-\alpha}\right),\end{aligned} \tag{3.15a}$$

$$\begin{aligned}\lambda^{-\dot{\alpha}}_{sc} = \bar{\epsilon}^{\dot{\alpha}i}u^-_i &+ \frac{1}{2}\bar{\theta}^{-\dot{\alpha}}(a-ib) + x^{\dot{\alpha}\beta}k_{\beta\dot{\beta}}\bar{\theta}^{-\dot{\beta}} - 2i(\bar{\theta}^-)^2\theta^+_\beta k^{\beta\dot{\alpha}} \\ &+ \left(x^{\alpha\dot{\alpha}} + 4i\theta^{+\alpha}\bar{\theta}^{-\dot{\alpha}}\right)\eta^i_\alpha u^-_i - 4i\eta^i_{\dot{\beta}}\bar{\theta}^{-\dot{\beta}}\left(\bar{\theta}^{-\dot{\alpha}}u^+_i - \bar{\theta}^{+\dot{\alpha}}u^-_i\right) \\ &+ \lambda^{ij}u^-_i\left(u^-_j\bar{\theta}^{+\dot{\alpha}} - u^+_j\bar{\theta}^{-\dot{\alpha}}\right).\end{aligned} \tag{3.15b}$$

In the next sections, we shall consider transformations of the three types:

- δ_{sc} - rigid $\mathcal{N}=2$ superconformal transformations;
- δ_{diff} - localized $\mathcal{N}=2$ superconformal transformations, i.e. local superdiffeomorphisms (gauge group of $\mathcal{N}=2$ Weyl supergravity). The δ_{sc} transformations form a subgroup of the δ_{diff} ones, with the parameters constrained by eqs. (3.8). Using such an identification, we can study invariance with respect to the more general transformations δ_{diff}, by imposing additional constraints on the parameters in order to reduce δ_{diff} to δ_{sc}, if necessary;
- δ_λ - linearized *gauge* transformations.

3.1 The general strategy of construction of superconformal couplings and multiplets

While constructing superconformal cubic vertices, we will start with **1** singling out the minimal set of gauge superfields $h^{++M_1\ldots M_{s-1}}(\zeta)$ needed for ensuring the invariance of the most general coupling [7],

$$\begin{aligned}S^{(s)}_{int} = -\frac{\kappa_s}{2}\int d\zeta^{(-4)}\, q^{+a}h^{++M_1\ldots M_{s-1}}\partial_{M_{s-1}}\ldots\partial_{M_1}\,(J)^{P(s)}\,q^+_a \\ + \text{lower derivative terms}\,,\end{aligned} \tag{3.16}$$

[7] Here we use the projection operator $P(s) := \frac{1+(-1)^s}{2}$.

under the hypermultiplet rigid superconformal transformations[8]:

$$\delta_{diff} q^{+a} = -\hat{\Lambda} q^{+a} - \frac{1}{2}\Omega q^{+a} \tag{3.17}$$

and some appropriate transformations of the gauge superfields

$$\delta_{diff} h^{++M_1 \dots M_{s-1}} = \dots . \tag{3.18}$$

The generator J is defined as:

$$J q^{+a} := i(\tau^3)^a_b q^{+b}, \qquad (\tau^3)^a_b = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{3.19}$$

The choice of interaction as in (3.16) is largely motivated by the consideration of the non-conformal case [46][9] and is strictly constrained by the analyticity of q^{+a}.

As the next steps, we 2 analyze the gauge freedom of the superconformal action constructed

$$S^{(s)}_q = S_{free} + S^{(s)}_{int} \tag{3.20}$$

and 3 determine the set of unremovable Wess-Zumino gauge fields in the potentials $h^{++M_1 \dots M_{s-1}}(\zeta)$, and hence reveal the irreducible field contents of the full $\mathcal{N} = 2$ off-shell superconformal multiplet.

In the next sections we start with the well known spin **1** and spin **2** cubic hypermultiplet couplings in order to illustrate how the above procedure works. Then we apply the same procedure to the novel case of the superspin **3** superconformal gauge multiplet and finally generalize the results to an arbitrary integer spin **s**.

4 $\mathcal{N} = 2$ Maxwell supermultiplet and superconformal $(\mathbf{1}, \frac{\mathbf{1}}{\mathbf{2}}, \frac{\mathbf{1}}{\mathbf{2}})$ coupling

The simplest example of $\mathcal{N} = 2$ superconformal cubic interaction of the hypermultiplet is supplied by its coupling to the superspin **1** gauge multiplet [51]. The spin **1** hypermultiplet vertex $(\mathbf{1}, \frac{\mathbf{1}}{\mathbf{2}}, \frac{\mathbf{1}}{\mathbf{2}})$ has the form:

$$S^{(s=1)}_{int} = -\frac{\kappa_1}{2} \int d\zeta^{(-4)}\, q^{+a} V^{++} J q^+_a = \kappa_1 \int d\zeta^{(-4)}\, i\, V^{++} \tilde{q}^+ q^+. \tag{4.1}$$

Here $V^{++}(\zeta)$ is an arbitrary unconstrained analytic gauge superfield with the gauge transformation $\delta_\lambda V^{++}(\zeta) = \mathcal{D}^{++}\lambda(\zeta)$, where $\lambda(\zeta)$ is an arbitrary analytic superfield parameter. The gauge potential V^{++} satisfies the reality condition $\widetilde{V^{++}} = V^{++}$. According to our general strategy, this is the most general spin **1** – hypermultiplet coupling containing no derivatives.

[8]Though the analytic parameters of $\mathcal{N} = 2$ superconformal transformations are given in (3.9), in what follows we shall not stick to their specific form and deal with arbitrary analytic parameters $\lambda^M(\zeta)$ associated with the transformations δ_{diff}.

[9]As was noticed in [46], the matrix generator J perfectly well works for all odd spins $s \geq 3$. The fact that the cubic interaction of scalars with the gauge fields of higher odd spins has certain peculiarities is well known (see, e.g., [55]).

4.1 $\mathcal{N}=2$ superconformal symmetry

Next we analyze the superconformal invariance of the vertex (4.1). Under both the local superconformal hypermultiplet transformations (3.17) and still unspecified local superconformal transformation of V^{++} the vertex (4.1) transforms as:

$$\begin{aligned}\delta_{diff} S^{(s=1)}_{int} =& \frac{\kappa_1}{2} \int d\zeta^{(-4)} \left[(\hat{\Lambda} q^{+a}) V^{++} J q^+_a + q^{+a} V^{++} J (\hat{\Lambda} q^+_a) \right] \\ &+ \frac{\kappa_1}{2} \int d\zeta^{(-4)}\, \Omega q^{+a} V^{++} J q^+_a - \frac{\kappa_1}{2} \int d\zeta^{(-4)}\, q^{+a} \delta_{diff} V^{++} J q^+_a \,,\end{aligned} \tag{4.2}$$

where $\hat{\Lambda}$ and Ω were defined in (3.1) and (3.3). We will require vanishing of such a variation by choosing the appropriate spin **1** superconformal transformation law $\delta_{diff} V^{++}$.

The first line of (4.2), modulo a total derivative, can be rewritten as:

$$\begin{aligned}(\hat{\Lambda} q^{+a}) V^{++} J q^+_a + q^{+a} V^{++} J (\hat{\Lambda} q^+_a) &= \hat{\Lambda} \left(q^{+a} V^{++} J q^+_a \right) - q^{+a} (\hat{\Lambda} V^{++}) J q^+_a \\ &= -\Omega \left(q^{+a} V^{++} J q^+_a \right) - q^{+a} (\hat{\Lambda} V^{++}) J q^+_a .\end{aligned} \tag{4.3}$$

The first term is canceled by the first term in second line of (4.2), and so the requirement of the invariance of the coupling (4.1) implies

$$\delta_{diff} V^{++} = -\hat{\Lambda} V^{++}. \tag{4.4}$$

We observe that in the spin **1** case the $\mathcal{N}=2$ diffeomorphism transformation of potential V^{++} amounts to the transport term. Thus the vertex (4.1) is invariant with respect to the total localized $\mathcal{N}=2$ superconformal transformations, not only to the rigid form of the latter. Similar results will be found in the higher-spin case. To prevent a misunderstanding, recall that the free q^{+a} action (2.9) *is not invariant* under *general* analytic diffeomorphisms, but only with respect to the superconformal subclass of them. The same is true of course for the sum $S_{free} + S^{(s=1)}_{int}$.

4.2 Gauge freedom

At the next step we analyze the gauge freedom. The sum

$$S_{free} + S^{(s=1)}_{int} = -\frac{1}{2} \int d\zeta^{(-4)}\, q^{+a} \mathcal{D}^{++} q^+_a - \frac{\kappa_1}{2} \int d\zeta^{(-4)}\, q^{+a} V^{++} J q^+_a , \tag{4.5}$$

is invariant under the gauge $s=1$ transformations:

$$\begin{cases} \delta_\lambda V^{++} = \mathcal{D}^{++} \lambda, \\ \delta_\lambda q^+_a = -\kappa_1 \lambda J q^+_a \end{cases} \tag{4.6}$$

for an arbitrary analytic parameter $\lambda(\zeta)$. Thus the full symmetry of the action (4.5) is $\mathcal{N}=2$ superconformal symmetry and $U(1)$ gauge symmetry.

The conserved current superfield associated with (4.6) can be directly obtained by varying cubic vertex with respect to V^{++} [45]:

$$\mathcal{J}^{++} = -\frac{1}{2} q^{+a} J q^+_a , \qquad \mathcal{D}^{++} \mathcal{J}^{++} = 0 \text{ (on shell)}. \tag{4.7}$$

$\mathcal{N}=2$ superconformal transformations of $\mathcal{J}^{++}$ read:

$$\delta_{sc} \mathcal{J}^{++} = -\hat{\Lambda} \mathcal{J}^{++} - \Omega \mathcal{J}^{++}. \tag{4.8}$$

4.3 Wess-Zumino gauge: $\mathcal{N} = 2$ Maxwell multiplet

Using the gauge freedom (4.6) one can choose Wess-Zumino gauge for the analytic spin **1** potential:

$$
\begin{aligned}
V^{++}_{WZ} = &- 4i\theta^{+\alpha}\bar{\theta}^{+\dot{\alpha}}A_{\alpha\dot{\alpha}} - i(\theta^+)^2\bar{\phi} + i(\bar{\theta}^+)^2\phi \\
&+ 4(\bar{\theta}^+)^2\theta^{+\alpha}\psi^i_\alpha u^-_i - 4(\theta^+)^2\bar{\theta}^+_{\dot{\alpha}}\bar{\psi}^{\dot{\alpha}i}u^-_i + (\theta^+)^2(\bar{\theta}^+)^2 D^{ij}u^-_i u^-_j \ ,
\end{aligned}
\tag{4.9}
$$

which yields just the off-shell field content of massless $\mathcal{N} = 2$ spin **1** multiplet, *viz.* a complex scalar, a doublet of gaugini, Maxwell gauge field and a real triplet of auxiliary fields:

$$
\phi, \quad \psi^i_\alpha, \quad A_{\alpha\dot{\alpha}}\,, \quad D^{(ij)}\,. \tag{4.10}
$$

The residual gauge freedom is given by $\lambda(\zeta) = a(x)$ and it is realized as the gauge transformation of Maxwell field:

$$
\delta_\lambda A_{\alpha\dot{\alpha}} = \partial_{\alpha\dot{\alpha}}a. \tag{4.11}
$$

So the spin **1** multiplet has $\mathbf{8}_B + \mathbf{8}_F$ off-shell degrees of freedom.

For the coordinate-independent parameter a the transformations (4.6) reduce to rigid $U(1)$ symmetry of the free hypermultiplet action. This manifests the Noether nature of such an interaction. One can obtain this vertex by gauging rigid $U(1)$ symmetry.

Thus we conclude that the $(\mathbf{1}, \frac{\mathbf{1}}{\mathbf{2}}, \frac{\mathbf{1}}{\mathbf{2}})$ vertex (4.1) is invariant under general analytic $\mathcal{N} = 2$ superdiffeomorphisms realized as in (3.17) and (4.4). The special choice of parameters (3.9) yields rigid $\mathcal{N} = 2$ superconformal transformations which leave invariant the total hypermultiplet action (4.5) as well.

5 $\mathcal{N} = 2$ Weyl supermultiplet and superconformal $(\mathbf{2}, \frac{\mathbf{1}}{\mathbf{2}}, \frac{\mathbf{1}}{\mathbf{2}})$ coupling

The superconformal vertex for the $\mathcal{N} = 2$ spin **2** gauge multiplet interacting with hypermultiplet is also known [51, 53, 56, 57]. Here we reproduce it, following our general strategy. This will give insights in how to construct higher-spin interactions in non-trivial $\mathcal{N} = 2$ conformal supergravity backgrounds. Though the further generalization to higher spins will require introducing additional derivatives, the spin **2** example is still instructive for exhibiting the common features of our approach.

In the spin **2** case the most general first-derivative analytic cubic interaction with the hypermultiplet has the form:

$$
S^{(s=2)}_{int} = -\frac{\kappa_2}{2}\int d\zeta^{(-4)}\, q^{+a}h^{++M}\partial_M q^+_a = -\frac{\kappa_2}{2}\int d\zeta^{(-4)}\, q^{+a}\hat{\mathcal{H}}^{++}_{(s=2)}q^+_a. \tag{5.1}
$$

Here we have introduced the set of unconstrained analytic gauge superfields,

$$
h^{++\alpha\dot{\alpha}}(\zeta), \qquad h^{++\alpha+}(\zeta), \qquad h^{++\dot{\alpha}+}(\zeta), \qquad h^{(+4)}(\zeta)\,, \tag{5.2}
$$

and composed the first-order analytic differential operator out of them:

$$
\hat{\mathcal{H}}^{++}_{(s=2)} := h^{++M}\partial_M = h^{++\alpha\dot{\alpha}}\partial_{\alpha\dot{\alpha}} + h^{++\alpha+}\partial^-_\alpha + h^{++\dot{\alpha}+}\partial^-_{\dot{\alpha}} + h^{(+4)}\partial^{--}. \tag{5.3}
$$

As compared to the analogous operator in the non-conformal case [46], here we have added the new analytic potential $h^{(+4)}$ entering with the partial harmonic derivative ∂^{--}. The necessity of such a modification will become clear later. Due to the reality of the action (5.1), the operator (5.3) should also satisfy the reality condition:

$$\widetilde{\hat{\mathcal{H}}^{++}_{(s=2)}} = \hat{\mathcal{H}}^{++}_{(s=2)} \quad \Rightarrow \quad \widetilde{h^{++\alpha\dot{\alpha}}} = h^{++\alpha\dot{\alpha}}, \qquad \widetilde{h^{++\alpha+}} = h^{++\dot{\alpha}+}, \qquad \widetilde{h^{(+4)}} = h^{(+4)}. \tag{5.4}$$

5.1 $\mathcal{N} = 2$ superconformal symmetry

To start with, we require invariance of the cubic vertex (5.1) under rigid $\mathcal{N} = 2$ superconformal transformations. The variation of the $(\mathbf{2}, \frac{\mathbf{1}}{\mathbf{2}}, \frac{\mathbf{1}}{\mathbf{2}})$ vertex with respect to local $\mathcal{N} = 2$ superconformal transformations with *arbitrary analytic superfield parameters* reads:

$$\delta_{diff} S^{(s=2)}_{int} = \frac{1}{2} \int d\zeta^{(-4)}\, q^{+a} [\hat{\mathcal{H}}^{++}_{(s=2)}, \hat{\Lambda}] q^{+}_{a} - \frac{1}{2} \int d\zeta^{(-4)}\, q^{+a} \delta_{diff} \hat{\mathcal{H}}^{++}_{(s=2)} q^{+}_{a}. \tag{5.5}$$

The condition of invariance under local $\mathcal{N} = 2$ superconformal transformations gives rise to the following transformation law for $\hat{\mathcal{H}}^{++}_{(s=2)}$:

$$\delta_{diff} \hat{\mathcal{H}}^{++}_{(s=2)} = [\hat{\mathcal{H}}^{++}_{(s=2)}, \hat{\Lambda}], \tag{5.6}$$

or, in terms of the analytic potentials,

$$\delta_{diff} h^{++M} = -\hat{\Lambda} h^{++M} + h^{++N} \partial_N \lambda^M. \tag{5.7}$$

The resulting $\mathcal{N} = 2$ superconformal transformation laws for the spin **2** analytic potentials (corresponding to the choice (3.9)) are inhomogeneous, so that various gauge potentials transform through each other. For example, the transformations of $h^{++\alpha+}$ under rigid special conformal transformations (parameter $k_{\alpha\dot{\alpha}}$ in (3.9)) and rigid conformal supersymmetry (parameter $\eta^{i}_{\dot{\alpha}}$) are:

$$\delta_{k_{\alpha\dot{\alpha}}} h^{++\alpha+} = -\hat{\Lambda} h^{++\alpha+} + h^{++\alpha\dot{\rho}} k_{\rho\dot{\rho}} \theta^{+\rho} + h^{++\rho+} x^{\alpha\dot{\rho}} k_{\rho\dot{\rho}}, \tag{5.8a}$$

$$\delta_{\eta^{i}_{\dot{\alpha}}} h^{++\alpha+} = -\hat{\Lambda} h^{++\alpha+} + h^{++\alpha\dot{\alpha}} \eta^{i}_{\dot{\alpha}} u^{+}_{i} - 4i h^{++\beta+} \theta^{+}_{\beta} \eta^{\alpha i} u^{-}_{i} + h^{(+4)} x^{\alpha\dot{\alpha}} \eta^{i}_{\dot{\alpha}} u^{-}_{i}. \tag{5.8b}$$

From the transformation

$$\delta_{k_{\alpha\dot{\alpha}}} h^{(+4)} = -\hat{\Lambda} h^{(+4)} + 4i h^{++\alpha+} \bar{\theta}^{+\dot{\alpha}} k_{\alpha\dot{\alpha}} + 4i \theta^{+\alpha} h^{++\dot{\alpha}+} k_{\alpha\dot{\alpha}} \tag{5.9}$$

it is obvious that it is impossible to avoid introducing the extra potential $h^{(+4)}$ in addition to the potentials of $\mathcal{N} = 2$ Einstein's supergravity. Indeed, equating it to zero would inevitably break rigid $\mathcal{N} = 2$ superconformal symmetry (only $\mathcal{N} = 2$ rigid Poincarè supersymmetry would survive).

5.2 Gauge freedom

Now we shall analyze the gauge freedom of the action:

$$S_{free} + S^{(s=2)}_{int} = -\frac{1}{2} \int d\zeta^{(-4)}\, q^{+a} \mathcal{D}^{++} q^{+}_{a} - \frac{\kappa_2}{2} \int d\zeta^{(-4)}\, q^{+a} h^{++M} \partial_M q^{+}_{a}. \tag{5.10}$$

It is well known that $\mathcal{N}=2$ Weyl multiplet is produced as a result of gauging $\mathcal{N}=2$ superconformal transformations, so in this case one should identify $\delta_\lambda^{(s=2)} = \kappa_2 \delta_{diff}$. So we start with the hypermultiplet transformation of the form:

$$\delta_\lambda^{(s=2)} q^{+a} = -\kappa_2 \hat{\Lambda} q^{+a} - \frac{\kappa_2}{2} \Omega q^{+a}. \tag{5.11}$$

Here we treat all gauge parameters

$$\lambda^{\alpha\dot{\alpha}}(\zeta), \qquad \lambda^{+\alpha}(\zeta), \qquad \lambda^{+\dot{\alpha}}(\zeta), \qquad \lambda^{++}(\zeta) \tag{5.12}$$

as arbitrary unconstrained analytic functions.

In (3.5) we have derived the variation of the free hypermultiplet action under such transformations:

$$\delta_\lambda^{(s=2)} S_{free} = \frac{\kappa_2}{2} \int d\zeta^{(-4)}\, q^{+a} [\mathcal{D}^{++}, \hat{\Lambda}] q^+_a. \tag{5.13}$$

There we required it to vanish in order to derive the constraints on the parameters (5.12) yielding the rigid superconformal symmetry of the free hypermultiplet. Now, instead of nullifying this term, we cancel it in the sum (5.10) by picking up the special gauge transformation of the $s=2$ operator:

$$\delta_\lambda \hat{\mathcal{H}}^{++}_{(s=2)} = [\mathcal{D}^{++}, \hat{\Lambda}] - \lambda^{++}\mathcal{D}^0. \tag{5.14}$$

This transformation law amounts to the linearized gauge transformations of the potentials $h^{++M}(\zeta)$:

$$\begin{cases} \delta_\lambda h^{++\alpha\dot{\alpha}} & = \mathcal{D}^{++}\lambda^{\alpha\dot{\alpha}} + 4i\lambda^{+\alpha}\bar{\theta}^{+\dot{\alpha}} + 4i\theta^{+\alpha}\bar{\lambda}^{+\dot{\alpha}}, \\ \delta_\lambda h^{++\alpha+} & = \mathcal{D}^{++}\lambda^{+\alpha} - \lambda^{++}\theta^{+\alpha}, \\ \delta_\lambda h^{++\dot{\alpha}+} & = \mathcal{D}^{++}\lambda^{+\dot{\alpha}} - \lambda^{++}\bar{\theta}^{+\dot{\alpha}}, \\ \delta_\lambda h^{(+4)} & = \mathcal{D}^{++}\lambda^{++}. \end{cases} \tag{5.15}$$

These transformations fully reproduce the linearized gauge freedom of $\mathcal{N}=2$ Weyl multiplet [57]. Note that eqs. (3.8) specifying rigid symmetries of the hypermultiplet, are just the conditions of vanishing of the variations (5.15), $\delta_\lambda h^{++M} = 0$. So one can interpret $\mathcal{N}=2$ rigid superconformal group as the transformations preserving the flat $\mathcal{N}=2$ conformal supergravity background $h^{++M}=0$. This is a consequence of the fact that the multiplet of $\mathcal{N}=2$ conformal supergravity can be obtained through the analytic gauging of rigid $\mathcal{N}=2$ superconformal transformations.

Since we did not impose any conditions on the parameters $\lambda^M(\zeta)$ in section 5.1, the action $S_{free} + S^{(s=2)}_{gauge}$ is exactly invariant under the transformations $\delta_\lambda + \kappa_2\delta_{diff}$ with arbitrary analytic parameters $\lambda^M(\zeta)$:

$$\delta^\lambda_{nonl} \hat{\mathcal{H}}^{++}_{(s=2)} := (\delta_\lambda + \kappa_2 \delta_{diff})\, \hat{\mathcal{H}}^{++}_{(s=2)} = [\mathcal{D}^{++} + \kappa_2 \hat{\mathcal{H}}^{++}_{(s=2)}, \hat{\Lambda}] - \lambda^{++}\mathcal{D}^0. \tag{5.16}$$

In this way we recover the *non-linear gauge freedom of $\mathcal{N}=2$ Weyl multiplet* elaborated in [57]. In the full nonlinear case, superconformal transformations become a subgroup of the full gauge supergroup of conformal supergravity. The latter is realized on the analytic gauge potentials h^{++M} by the same formulas (5.15), however with the replacement

$$\mathcal{D}^{++} \;\Rightarrow\; \mathfrak{D}^{++} := \mathcal{D}^{++} + \kappa_2 \hat{\mathcal{H}}^{++}_{(s=2)}\,. \tag{5.17}$$

In the scalar sector, the action

$$S^{sg}_{hyper} = -\frac{1}{2}\int d\zeta^{(-4)} q^{+a}\mathfrak{D}^{++}q^{+}_{a} \tag{5.18}$$

is reduced to the conformally coupled scalars, $S^{sg}_{hyper} \sim \bar{f}^i(\nabla^2 - \frac{1}{6}R)f^i$.

The superconformal coupling (4.1) of the hypermultiplet to the spin **1** superfield V^{++} is also invariant under the full conformal supergravity gauge supergroup; the $U(1)$ gauge transformations (4.6) are modified just through the replacement (5.17) in the gauge transformation of V^{++}. The sum of the q^{+a} action (5.10) covariantized by $\mathcal{N}=2$ Weyl multiplet and the $(\mathbf{1}, \mathbf{\frac{1}{2}}, \mathbf{\frac{1}{2}})$ coupling (4.1),

$$S = -\frac{1}{2}\int d\zeta^{(-4)} q^{+a}\left(\mathfrak{D}^{++} + \kappa_1 V^{++}J\right) q^{+}_{a}\,, \tag{5.19}$$

is invariant under both the full $\mathcal{N}=2$ conformal supergravity gauge supergroup and the modified gauge $U(1)$ transformations:

$$\delta_\lambda V^{++} = [\mathfrak{D}^{++}, \lambda]. \tag{5.20}$$

So we have constructed the vertex $(\mathbf{1}, \mathbf{\frac{1}{2}}, \mathbf{\frac{1}{2}})$ in $\mathcal{N}=2$ conformal supergravity background. Note that the superfield V^{++} in the action (5.19) does not directly interact with h^{++M}. At the component level, such an interaction is induced as a result of elimination of the auxiliary fields of the hypermultiplet.

5.3 Wess-Zumino gauge: $\mathcal{N}=2$ Weyl supermultiplet

To specify the physical contents of Weyl multiplet, one needs to gauge away the pure gauge degrees of freedom, thus fixing the Wess-Zumino gauge for the set of spin **2** analytic potentials:

$$\begin{cases} h^{++\alpha\dot\alpha} = -4i\theta^{+\rho}\bar\theta^{+\dot\rho}\Phi^{\alpha\dot\alpha}_{\rho\dot\rho} - (\bar\theta^+)^2\theta^+_\rho\psi^{(\alpha\rho)\dot\alpha i}u^-_i + (\theta^+)^2\bar\theta^+_{\dot\rho}\bar\psi^{\alpha(\dot\alpha\dot\rho)i}u^-_i \\ \qquad\qquad\qquad + (\theta^+)^2(\bar\theta^+)^2V^{\alpha\dot\alpha(ij)}u^-_iu^-_j\,, \\ h^{++\mu+} = (\theta^+)^2\bar\theta^+_{\dot\mu}P^{\mu\dot\mu} + \left(\bar\theta^+\right)^2\theta^+_\nu T^{(\nu\mu)} + (\theta^+)^2(\bar\theta^+)^2\chi^{\mu i}u^-_i\,, \\ h^{++\dot\mu+} = \widetilde{h^{++\mu+}}\,, \\ h^{(+4)} = (\theta^+)^2(\bar\theta^+)^2D\,. \end{cases} \tag{5.21}$$

Here we find out the physical content of $\mathcal{N}=2$ Weyl multiplet [57, 64, 65] involving graviton, a doublet of conformal gravitinos, gauge fields for $SU(2)_R$ and γ_5 transformations; all other fields are auxiliary (after some redefinition):

$$\Phi^{\alpha\dot\alpha}_{\rho\dot\rho}, \quad \psi^{(\alpha\beta)\dot\alpha i}, \quad V^{(ij)}_{\alpha\dot\alpha}, \quad P^{\mu\dot\mu}; \qquad T^{(\mu\nu)}, \quad \chi^{\mu i}, \quad D. \tag{5.22}$$

At the linearized level, the residual gauge freedom of the theory is spanned by the parameters:

$$
\begin{cases}
\lambda^{\alpha\dot{\alpha}} \Rightarrow a^{\alpha\dot{\alpha}}(x) - 4i\epsilon^{\alpha i}(x)u^-_i\bar{\theta}^{+\dot{\alpha}} - 4i\theta^{+\alpha}\bar{\epsilon}^{\dot{\alpha}i}(x)u^-_i - 4i\theta^{+\alpha}\bar{\theta}^{+\dot{\alpha}}\lambda^{(ij)}(x)u^-_i u^-_j\,, \\
\lambda^{\mu+} \Rightarrow \epsilon^{\mu i}(x)u^+_i + \theta^{+\nu}\big[\{\tfrac{1}{2}\big[a(x)+ib(x)\big] + \lambda^{(ij)}(x)u^+_i u^-_j\}\delta^\mu_\nu + l_{(\nu}{}^{\mu)}(x)\big] - i(\theta^+)^2\partial^\mu_{\dot{\rho}}\bar{\epsilon}^{\dot{\rho}i}(x)u^-_i\,, \\
\bar{\lambda}^{\dot{\mu}+} \Rightarrow \bar{\epsilon}^{\dot{\mu}i}(x)u^+_i + \bar{\theta}^{+\dot{\nu}}\big[\{\tfrac{1}{2}\big[a(x)-ib(x)\big] + \lambda^{(ij)}(x)u^+_i u^-_j\}\delta^{\dot{\mu}}_{\dot{\nu}} + l_{(\dot{\nu}}{}^{\dot{\mu})}(x)\big] + i(\bar{\theta}^+)^2\partial^{\dot{\mu}}_{\rho}\epsilon^{\rho i}(x)u^-_i\,, \\
\lambda^{++} \Rightarrow \lambda^{ij}(x)u^+_i u^+_j + 4i\theta^{+\alpha}\bar{\theta}^{+\dot{\alpha}}\partial_{\alpha\dot{\alpha}}a(x) + 2i\left(\theta^{+\alpha}\partial_{\alpha\dot{\rho}}\bar{\epsilon}^{\dot{\rho}i}(x) - \partial_{\dot{\alpha}\rho}\epsilon^{\rho i}(x)\bar{\theta}^{+\dot{\alpha}}\right)u^+_i\,.
\end{cases}
\tag{5.23}
$$

These parameters can be identified as follows:

- $a^{\alpha\dot{\alpha}}(x)$ are the remnants of the diffeomorphism parameters which now form the basic gauge freedom of the free spin 2 field;
- $\epsilon^{\hat{\mu}i}(x)$ originate from the parameters of local supersymmetry and provide $\mathcal{N}=2$ counterparts of the local $a^{\alpha\dot{\alpha}}$ transformations;
- $l^{(\mu\nu)}(x)$ and $l^{(\dot{\mu}\dot{\nu})}(x)$ are the former parameters of local Lorentz transformations which can be used to gauge away the antisymmetric part of $\Phi^{\alpha\dot{\alpha}}_{\rho\dot{\rho}}$ and so to leave in the latter only the symmetric part (traceless "conformal graviton" and the trace itself);
- $a(x)$ is a parameter of Weyl transformation;
- $b(x)$ is a parameter of local $U_R(1)$ transformations;
- $\lambda^{(ij)}(x)$ are parameters of local $SU(2)_R$ transformations.

Rigid parameters of special conformal transformations $k_{\alpha\dot{\alpha}}$ and conformal supersymmetry $\eta^{\hat{\mu}i}$ are contained in derivatives of gauge parameters:

$$
k_{\alpha\dot{\alpha}} = \partial_{\alpha\dot{\alpha}}a(x)|, \quad \eta^{\alpha i} = \frac{1}{2}\partial^{\alpha}_{\dot{\rho}}\bar{\epsilon}^{\dot{\rho}i}(x)|, \quad \bar{\eta}^{\dot{\alpha}i} = -\frac{1}{2}\partial^{\dot{\alpha}}_{\rho}\epsilon^{\rho i}(x)|. \tag{5.24}
$$

To find the residual gauge transformations and their action on the component fields, one needs to require the preservation of the Wess-Zumino gauge, that is in $\delta_\lambda h^{++M}$ there should be no terms which could not be compensated by the appropriate transformations of fields in h^{++M}_{WZ}. From this condition one can determine the parameters λ^M_{comp} and the action of these transformations on the fields of $\mathcal{N}=2$ Weyl multiplet. As a result, the linearized transformation law for graviton is:

$$
\delta_\lambda \Phi^{\alpha\dot{\alpha}\rho\dot{\rho}} = \partial^{\rho\dot{\rho}}a^{\alpha\dot{\alpha}} - 2l^{(\alpha\rho)}\epsilon^{\dot{\alpha}\dot{\rho}} - 2l^{(\dot{\alpha}\dot{\rho})}\epsilon^{\alpha\rho} - a\epsilon^{\alpha\rho}\epsilon^{\dot{\alpha}\dot{\rho}}. \tag{5.25}
$$

The decomposition of the field $\Phi^{\alpha\dot{\alpha}\rho\dot{\rho}}$ into the irreducible parts is as follows[10]:

$$
\Phi^{\alpha\dot{\alpha}\rho\dot{\rho}} = \Phi^{(\alpha\rho)(\dot{\alpha}\dot{\rho})} + \epsilon^{\dot{\alpha}\dot{\rho}}\Phi^{(\alpha\rho)} + \epsilon^{\alpha\beta}\Phi^{(\dot{\alpha}\dot{\beta})} + \epsilon^{\alpha\beta}\epsilon^{\dot{\alpha}\dot{\rho}}\Phi. \tag{5.26}
$$

[10] The linearized relation with the metric tensor $g_{ab} = \eta_{ab} + h_{ab}$ is given by:

$$
h^{ab} = \sigma^a_{\alpha\dot{\alpha}}\sigma^b_{\beta\dot{\beta}}\Phi^{(\alpha\beta)(\dot{\alpha}\dot{\beta})} + \frac{1}{2}\eta^{ab}\Phi.
$$

The parameters $l^{(\alpha\rho)}$, $l^{(\dot\alpha\dot\rho)}$ and a can be used to gauge away all the components except for the symmetric part:

$$\delta\Phi^{(\alpha\rho)(\dot\alpha\dot\rho)} = \partial^{(\dot\rho(\rho}a^{\alpha)\dot\alpha)}. \tag{5.27}$$

In this gauge we have $a = \frac{1}{4}\partial_{\rho\dot\rho}a^{\rho\dot\rho}$, $l^{(\alpha\rho)} = \frac{1}{4}\partial^{(\rho\dot\rho}a^{\alpha)}_{\dot\rho}$, $l^{(\dot\alpha\dot\rho)} = \frac{1}{4}\partial^{(\dot\rho\rho}a^{\dot\alpha)}_{\rho}$.

Other gauge fields can be worked out in a similar way. Their irreducible form and gauge transformation laws are given by:

$$\delta\psi^{(\alpha\rho)\dot\alpha i} = \partial^{\dot\alpha(\alpha}\epsilon^{\rho)i}, \qquad \delta V^{(ij)}_{\alpha\dot\alpha} = \partial_{\alpha\dot\alpha}\lambda^{(ij)}, \qquad \delta P_{\alpha\dot\beta} = \frac{1}{2}\partial_{\alpha\dot\alpha}b. \tag{5.28}$$

The fields $T^{(\mu\nu)}, \chi^{\mu i}, D$ are auxiliary and carry no any gauge freedom (after appropriate redefinition), so $\mathcal{N}=2$ Weyl multiplet collects $\mathbf{24}_B + \mathbf{24}_F$ off-shell degrees of freedom.

Note that the same form of WZ gauge for the analytic gauge potentials can be fixed by starting with the full nonlinear $\mathcal{N}=2$ conformal supergravity group from the very beginning.

5.4 $s=2$ superconformal current superfields

According to the superfield version of Noether's theorem, the conserved superfield currents are associated with rigid symmetry transformations. The parameters λ^M that satisfy the relation $[\mathcal{D}^{++}, \hat\Lambda] = 0$ form rigid symmetry of the free hypermultiplet. Using the expression (5.13) for the variation of the action one can easily generalize, to $\mathcal{N}=2$ superconformal case, the current superfields found in [45] for the non-conformal case. Actually, since the variation of the action on shell is vanishing, we obtain a set of conservation laws for each of the unconstrained parameters λ^M. Equivalently, one can obtain these current superfields by varying cubic coupling (5.1) with respect to $\mathcal{N}=2$ Weyl potentials h^{++M}. As a result, we obtain:

$$\begin{aligned}
M=\alpha\dot\alpha \quad &\Rightarrow \quad J^{++}_{\alpha\dot\alpha} = -\frac{1}{2}q^{+a}\partial_{\alpha\dot\alpha}q^+_a, \qquad \mathcal{D}^{++}J^{++}_{\alpha\dot\alpha} = 0;\\
M=\alpha \quad &\Rightarrow \quad J^{+}_{\alpha} = -\frac{1}{2}q^{+a}\partial^-_{\alpha}q^+_a, \qquad \mathcal{D}^{++}J^{+}_{\alpha} = 4i\theta^{+\dot\rho}J_{\alpha\dot\rho};\\
M=\dot\alpha \quad &\Rightarrow \quad J^{+}_{\dot\alpha} = -\frac{1}{2}q^{+a}\partial^-_{\dot\alpha}q^+_a, \qquad \mathcal{D}^{++}J^{+}_{\dot\alpha} = -4i\theta^{+\rho}J_{\rho\dot\alpha};\\
M=++ \quad &\Rightarrow \quad J = -\frac{1}{2}q^{+a}\partial^{--}q^+_a, \qquad \mathcal{D}^{++}J = -\theta^{+\hat\rho}J^+_{\hat\rho}.
\end{aligned} \tag{5.29}$$

As was shown in [45] for the component expansion of the non-conformal currents superfields, the conservation laws of the superfield currents (5.29) lead to the standard x-space conservation of the component currents. All the above current superfields are analytic, but $J^+_{\dot\alpha}$ and J are not invariant under $\mathcal{N}=2$ supersymmetry. The $\mathcal{N}=2$ supersymmetry-invariant supercurrent is defined by the non-analytic superfield:

$$\mathcal{J} := -\frac{1}{2}q^{+a}\mathcal{D}^{--}q^+_a = J + \theta^{-\hat\rho}J_{\hat\rho} - 4i\theta^{-\rho}\bar\theta^{-\dot\rho}J^{++}_{\rho\dot\rho}. \tag{5.30}$$

It embodies all the analytic currents and satisfies the conservation law

$$\mathcal{D}^{++}\mathcal{J} = 0\,, \tag{5.31}$$

which immediately reproduces the conservation laws (5.29).

Note that the supercurrent $\mathcal{J}$ could be obtained from the representation of unconstrained analytic parameters λ^M through an unconstrained non-analytic superfield parameter $l^{--}(\zeta, \theta^-)$:

$$\hat{\Lambda} = \left(D^+\right)^4 \left(l^{--}\mathcal{D}^{--}\right). \tag{5.32}$$

The variation (5.13) of the free hypermultiplet action takes the form:

$$\delta_l^{(s=2)} S_{free} = \int d^4x d^8\theta du \, \left(\mathcal{D}^{++} l^{--}\right) q^{+a} \mathcal{D}^{--} q_a^+ \,, \tag{5.33}$$

which immediately leads to (5.31).

One can also obtain $\mathcal{J}$ by varying the coupling (5.1) with respect to an unconstrained non-analytic prepotential $\Upsilon(\zeta, \theta^-)$ of $\mathcal{N}=2$ conformal supergravity defined as:

$$\hat{\mathcal{H}}^{++}_{(s=2)} := (D^+)^4 \left(\Upsilon \mathcal{D}^{--}\right).$$

This way of representing analytic gauge potentials through the non-analytic Mezincescu-type prepotential was used in refs. [58, 59]. It allows one to relate harmonic gauge potentials to the prepotentials of non-geometric type used for the superfield description of supergravity beyond the HSS approach [38, 60–62].

So we conclude that the transformations (5.11) correspond to the non-analytic current superfield $\mathcal{J}$ defined in (5.30) and obeying the appropriate conservation law (5.31) on shell. This is the "master" current superfield discussed recently in [45] and originally introduced in [59] (see also a recent work [63]). As compared with the non-conformal spin **2** supercurrent [45], we observe the appearance of a new analytic current $J = -\frac{1}{2} q^{+a} \partial^{--} q^+_a$ associated with the rigid conformal parameter λ^{++} in (3.1).

Under the $\mathcal{N}=2$ superconformal transformations of the hypermultiplet, $\mathcal{J}$ transforms as:

$$\delta_{sc} \mathcal{J} = -\hat{\Lambda}\mathcal{J} - \Omega \mathcal{J} + \frac{1}{2} q^{+a} [\mathcal{D}^{--}, \hat{\Lambda}] q_a^+ . \tag{5.34}$$

The last term implies the presence of inhomogeneities in the current transformation laws. E.g., for dilatations (parameter a in (3.9)) we obtain

$$[\mathcal{D}^{--}, \hat{\Lambda}] = -4ia\theta^{-\alpha}\bar{\theta}^{-\dot{\alpha}}\partial_{\alpha\dot{\alpha}} + \frac{1}{2} a \theta^{-\hat{\alpha}} \partial^-_{\hat{\alpha}},$$

so $\mathcal{J}$ transforms under dilatations as:

$$\delta_{(a)} \mathcal{J} = -\hat{\Lambda}_{(a)} \mathcal{J} - \Omega_{(a)} \mathcal{J} + 4ia\theta^{-\alpha}\bar{\theta}^{-\dot{\alpha}} J_{\alpha\dot{\alpha}} - \frac{1}{2} a \theta^{-\hat{\alpha}} J_{\hat{\alpha}}. \tag{5.35}$$

Using the relation (5.30), one can equivalently rewrite this as

$$\delta_{(a)} \mathcal{J} = -\hat{\Lambda}_{(a)} \mathcal{J} - \Omega_{(a)} \mathcal{J} - \frac{1}{2} a \theta^{-\hat{\alpha}} \partial^+_{\hat{\alpha}} \mathcal{J}. \tag{5.36}$$

The last term appeared due to the dilatation rescaling of non-analytic $\theta^{-\hat{\alpha}}$. So, defining $\hat{\Lambda}_{na} := \hat{\Lambda} + \lambda^{-\hat{\alpha}} \partial^+_{\hat{\alpha}}$, the variation (5.36) can be cast in the more suggestive form:

$$\delta_{(a)} \mathcal{J} = -\hat{\Lambda}_{(a)} \mathcal{J} - \Omega_{(a)} \mathcal{J}. \tag{5.37}$$

6 $\mathcal{N}=2$ spin 3 superconformal multiplet and superconformal $(\mathbf{3},\frac{1}{2},\frac{1}{2})$ coupling

The spin **3** superconformal interaction with hypermultiplet is the first non-trivial case which was never discussed before in the HSS approach. The most general form of the two-derivative analytic vertex is:

$$S^{(s=3)}_{int} = -\frac{\kappa_3}{2}\int d\zeta^{(-4)}\, q^{+a}\, h^{++MN}\partial_N\partial_M J\, q^+_a. \tag{6.1}$$

Here we introduced unconstrained analytic gauge potentials $h^{++MN}(\zeta)$, which satisfy the conditions:

$$h^{++MN} = (-1)^{P(M)P(N)} h^{++NM}, \tag{6.2}$$

with

$$P(M) := \begin{bmatrix} 0 & \text{for} \quad M = \alpha\dot{\alpha}, ++ & \text{(bosonic indices)}; \\ 1 & \text{for} \quad M = \alpha+,\ \dot{\alpha}+ & \text{(fermionic indices)}. \end{bmatrix} \tag{6.3}$$

The conditions (6.2) are necessary in order to avoid "double counting" of terms of the same type, for example,

$$h^{++\alpha+\dot{\beta}+}\partial^-_{\dot{\beta}}\partial^-_{\alpha} = h^{++\dot{\beta}+\alpha+}\partial^-_{\alpha}\partial^-_{\dot{\beta}}. \tag{6.4}$$

Taking this into account, the complete expansion of the operator with two derivatives has the form:

$$\begin{aligned} h^{++MN}\partial_N\partial_M =\, & h^{++\alpha\dot{\alpha}\beta\dot{\beta}}\partial_{\beta\dot{\beta}}\partial_{\alpha\dot{\alpha}} \\ & + h^{++[\beta+\gamma]+}\partial^-_{\gamma}\partial^-_{\beta} + h^{++[\dot{\beta}+\dot{\gamma}]+}\partial^-_{\dot{\gamma}}\partial^-_{\dot{\beta}} + h^{(+6)}\partial^{--}\partial^{--} \\ & + 2h^{++\beta+\alpha\dot{\alpha}}\partial_{\alpha\dot{\alpha}}\partial^-_{\beta} + 2h^{++\dot{\beta}+\alpha\dot{\alpha}}\partial_{\alpha\dot{\alpha}}\partial^-_{\dot{\beta}} + 2h^{++\alpha\dot{\alpha}++}\partial^{--}\partial_{\alpha\dot{\alpha}} \\ & + 2h^{++\beta+\dot{\gamma}+}\partial^-_{\dot{\gamma}}\partial^-_{\beta} + 2h^{++++\beta+}\partial^-_{\beta}\partial^{--} + 2h^{++++\dot{\beta}+}\partial^-_{\dot{\beta}}\partial^{--}. \end{aligned} \tag{6.5}$$

We require reality of the action (6.1), so the analytic gauge potentials satisfy the following tilde-conjugation rules:

$$\widetilde{h^{++MN}\partial_N\partial_M} = h^{++MN}\partial_N\partial_M. \tag{6.6}$$

It then follows that the analytic potentials h^{++MN} obey the reality conditions:

$$\widetilde{h^{++\alpha\dot{\alpha}\beta\dot{\beta}}} = h^{++\alpha\dot{\alpha}\beta\dot{\beta}}, \qquad \widetilde{h^{(+6)}} = h^{(+6)}, \tag{6.7a}$$

$$\widetilde{h^{++[\beta+\gamma]+}} = -h^{++[\dot{\beta}+\dot{\gamma}]+}, \qquad \widetilde{h^{++[\dot{\beta}+\dot{\gamma}]+}} = -h^{++[\beta+\gamma]+}, \tag{6.7b}$$

$$\widetilde{h^{++\beta+\alpha\dot{\alpha}}} = -h^{++\dot{\beta}+\alpha\dot{\alpha}}, \qquad \widetilde{h^{++\dot{\beta}+\alpha\dot{\alpha}}} = h^{++\beta+\alpha\dot{\alpha}}, \tag{6.7c}$$

$$\widetilde{h^{++\alpha\dot{\alpha}++}} = h^{++\alpha\dot{\alpha}++}, \qquad \widetilde{h^{++\alpha+\dot{\alpha}+}} = -h^{++\alpha+\dot{\alpha}+}, \tag{6.7d}$$

$$\widetilde{h^{++++\beta+}} = -h^{++++\dot{\beta}+}, \qquad \widetilde{h^{++++\dot{\beta}+}} = h^{++++\beta+}. \tag{6.7e}$$

At this step we deal with the most general form of the analytical gauge potentials h^{++MN}, without assuming in advance any symmetry between the Lorentz spinorial indices hidden in the multi-indices M and N [11].

Next we require $\mathcal{N}=2$ superconformal invariance of vertex (6.1) and determine the minimal set of potentials $h^{++MN}(\zeta)$ needed to secure this invariance. After that we will analyze gauge freedom of the coupling obtained, as well as the irreducible physical field contents of the corresponding superconformal spin **3** supermultiplet.

6.1 $\mathcal{N}=2$ superconformal symmetry

The hypermultiplet transformations (3.1) with arbitrary analytic parameters take the superfield Lagrangian in (6.1), up to total derivative, into:

$$\begin{aligned}\delta_{diff}\left(q^{+a}h^{++MN}\partial_N\partial_M J q^+_a\right) =& q^{+a}(\hat{\Lambda}h^{++MN})\partial_N\partial_M J q^+_a \\ &- 2q^{+a}h^{++MN}(\partial_N\lambda^K)\partial_K\partial_M J q^+_a \\ &+ \frac{1}{2}(-1)^{P(K)}\partial_K\left(h^{++MN}\partial_N\partial_M\lambda^K\right)q^{+a}Jq^+_a \\ &+ \frac{1}{2}(-1)^{P(M)}(\partial_M h^{++MN})(\partial_N\Omega)q^{+a}Jq^+_a.\end{aligned} \tag{6.8}$$

Note that in the process of calculation of this variation we made use of the property

$$J_{ab}=J_{ba}\ \Rightarrow\ q^{+a}J_{ab}(\partial_Z q^{+b}) = \frac{1}{2}\partial_Z(q^a J_{ab} q^b), \tag{6.9}$$

which ensures reducing all terms with one derivative to those without derivatives by integration by parts.

We observe the presence of two types of terms: those with two derivatives acting on the hypermultiplet and terms without derivatives at all. To cancel all these terms one is led to slightly modify the vertex (6.1) by introducing the spin **1** superfield h^{++} and adding the relevant $(\mathbf{1},\frac{\mathbf{1}}{\mathbf{2}},\frac{\mathbf{1}}{\mathbf{2}})$-type vertex:

$$(6.1)\ \Rightarrow\ S^{(s=3)}_{int} = -\frac{\kappa_3}{2}\int d\zeta^{(-4)}\,q^{+a}h^{++MN}\partial_N\partial_M J q^+_a - \frac{\kappa_3}{2}\int d\zeta^{(-4)}q^{+a}h^{++}Jq^+_a. \tag{6.10}$$

Superfield h^{++} satisfies the reality condition $\widetilde{h^{++}} = h^{++}$.

We start our analysis with the two-derivative terms. Requiring local superconformal transformation laws for the analytic potentials,

$$\delta_{diff}h^{++MN} = -\hat{\Lambda}h^{++MN} + h^{++MK}(\partial_K\lambda^N) + (-1)^{P(N)[P(M)+P(K)]}h^{++KN}(\partial_K\lambda^M), \tag{6.11}$$

one can cancel the analogous terms with two derivatives in (6.8). The first term is the transport term, while the second and third ones mix up the potentials carrying different indices. If one chooses as the parameters just the rigid superconformal parameters (3.9)

[11] This is an essential difference from the non-conformal case [44], where all indices of the same chirality were assumed to be symmetrized.

then it is not difficult to make sure that it is necessary to include into the game the whole set of potentials h^{++MN}. For example, under the conformal supersymmetry (parameter η^i_ρ in (3.9)) we have:

$$\delta_{\eta^i_\rho} h^{++\alpha+\dot\beta+} = -\hat\Lambda h^{++\alpha+\dot\beta+} + h^{++(\alpha\beta)\dot\beta+}\left(\eta^i_\beta u^+_i\right) + \dots . \tag{6.12}$$

We observe that the potential $h^{++\alpha+\dot\beta+}$ is mixed with $h^{++(\alpha\beta)\dot\beta+}$.

This peculiarity leads to an important difference of the superconformal vertices from the non-conformal ones constructed in [46]. Indeed, to respect the standard Poincaré supersymmetry (parameters $a^{\alpha\dot\alpha}$ and $\epsilon^{\hat\alpha i}$ in (3.9)) it would be enough to deal only with the restricted set of potentials $h^{++M\alpha\dot\alpha}$.

The superdiffeomorphism transformation of the $(\mathbf{1}, \mathbf{\frac{1}{2}}, \mathbf{\frac{1}{2}})$ part of the vertex reads

$$\delta_{diff} \int d\zeta^{(-4)} q^{+a} h^{++} J q^+_a = \int d\zeta^{(-4)} q^{+a} \left(\delta_{diff} h^{++} + \hat\Lambda h^{++}\right) J q^+_a \,, \tag{6.13}$$

and it is required to cancel the terms without derivatives in (6.8). This is achieved with

$$\begin{aligned} \delta_{diff} h^{++} = &- \hat\Lambda h^{++} - \frac{1}{2}(-1)^{P(K)}\partial_K \left(h^{++MN}\partial_N\partial_M \lambda^K\right) \\ &- \frac{1}{2}(-1)^{P(M)}(\partial_M h^{++MN})(\partial_N \Omega)\,. \end{aligned} \tag{6.14}$$

The first term coincides with the similar term in the superconformal transformation of spin $\mathbf{1}$ multiplet (4.4) and so it automatically leaves the action invariant. Then the appropriate parts of the two-derivative transformations in (6.8) are canceled by the remaining terms in (6.14).

Thus we arrive at the cubic vertices which are *invariant under* $\mathcal{N}=2$ *superdiffeomorphism transformations* with the general analytic parameters λ^M (i.e. invariant under the complete gauge group of $\mathcal{N}=2$ conformal supergravity). The spin $\mathbf{2}$ gauge transformations act on the spin $\mathbf{3}$ potentials according to (6.11) and (6.14), so that the vertex (6.10) is invariant under the sum of these transformations and the hypermultiplet transformations (3.1).

Substituting the superconformal parameters (3.9) into (6.11) and (6.14) yields rigid $\mathcal{N}=2$ superconformal transformation laws of the spin $\mathbf{3}$ analytic potentials. The above reasoning indicates that we need to introduce from the very beginning the most general set of analytic gauge potentials h^{++MN} and h^{++} in order to realize $\mathcal{N}=2$ superconformal symmetry. It is useful to combine the total set of gauge potentials into the spin $\mathbf{3}$ second-order analytic operator as:

$$\hat{\mathcal{H}}^{++}_{(s=3)} := h^{++MN}\partial_N\partial_M + h^{++}. \tag{6.15}$$

The precise realization of rigid $\mathcal{N}=2$ superconformal transformations on the analytic gauge potentials in (6.15) is given in Appendix C. It is shown there that all h^{++MN} with antisymmetric combinations of the Lorentz indices $\alpha, \dot\beta$ form a set closed under $\mathcal{N}=2$ superconformal group, while the remaining "essential" ones (with symmetric combinations of indices) transform through this set and themselves. In other words, h^{++MN} constitute not fully reducible representation. The auxiliary spin **1** gauge potential h^{++} properly transforms through h^{++MN}. The linearized gauge transformations to be discussed in the next subsection are compatible with this not fully reducible superconformal structure: the conformally invariant subset just mentioned is transformed by gauge parameters which do not appear in the gauge transformations of the "essential" potentials. Just this notable group-theoretical property allows one to gauge away the irreducible subset of gauge potentials without breaking of superconformal symmetry and to end up with the essential potentials as carriers of the irreducible $\mathbf{s} = 3$ $\mathcal{N} = 2$ gauge multiplet (in the proper Wess-Zumino gauges).

6.2 Gauge freedom

As the following step we analyze the gauge freedom of the action:

$$S_{free} + S_{int}^{(s=3)} = -\frac{1}{2}\int d\zeta^{(-4)} q^{+a}\left(\mathcal{D}^{++} + \kappa_3 h^{++MN}\partial_N\partial_M J + \kappa_3 h^{++}J\right) q^+_a. \tag{6.16}$$

A generalization of the $\mathbf{s} = 2$ gauge transformations (5.11) is obtained by adding one more derivative, ∂^{--},

$$\partial_M = \{\partial_{\alpha\dot\alpha}, \partial^-_{\hat\alpha}, \partial^{--}\}. \tag{6.17}$$

Then the most general $\mathbf{s} = 3$ generalization of $\mathbf{s} = 2$ gauge freedom (5.11) is given by [12]

$$\delta_\lambda^{(s=3)} q^{+a} = -\frac{\kappa_3}{2}\{\hat\Lambda^M, \partial_M\}_{AGB} J q^{+a} - \frac{\kappa_3}{4}\{\Omega^M, \partial_M\}_{AGB} J q^{+a}. \tag{6.18}$$

Here we have introduced the first-order analytic operators

$$\hat\Lambda^M := \sum_{N\leq M} \lambda^{MN}\partial_N, \tag{6.19}$$

with the analytic parameters satisfying the condition $\lambda^{MN} = (-1)^{P(M)P(N)}\lambda^{NM}$, as well as the analytic weight factor

$$\Omega^M := \sum_{N<M} (-1)^{P(N)}\partial_N\lambda^{NM}. \tag{6.20}$$

The transformation law (6.18) is of the second order in the superspace derivatives.

Gauge parameters satisfy reality conditions, which follows from the requirement of reality of variation (6.18). These conditions have the same form as those for the analytic potentials (6.7):

$$\widetilde{\lambda^{\alpha\dot\alpha\beta\dot\beta}} = \lambda^{\alpha\dot\alpha\beta\dot\beta}, \qquad \widetilde{\lambda^{(+4)}} = \lambda^{(+4)}, \tag{6.21a}$$

$$\widetilde{\lambda^{[\beta+\gamma]+}} = \lambda^{[\dot\beta+\dot\gamma]+}, \qquad \widetilde{\lambda^{[\dot\beta+\dot\gamma]+}} = \lambda^{[\beta+\gamma]+}, \tag{6.21b}$$

$$\widetilde{\lambda^{\beta+\alpha\dot\alpha}} = -\lambda^{\dot\beta+\alpha\dot\alpha}, \qquad \widetilde{\lambda^{\dot\beta+\alpha\dot\alpha}} = \lambda^{\beta+\alpha\dot\alpha}, \tag{6.21c}$$

[12] Anti-graded bracket is defined as $\{F_1, F_2\}_{AGB} := [F_1, F_2]$ for fermionic objects and $\{B_1, B_2\}_{AGB} := \{B_1, B_2\}$ for bosonic ones. Also, $\{F, B\}_{AGB} := [F, B]$.

$$\widetilde{\lambda^{\alpha\dot{\alpha}++}} = \lambda^{\alpha\dot{\alpha}++}, \qquad \widetilde{\lambda^{\alpha+\dot{\alpha}+}} = -\lambda^{\alpha+\dot{\alpha}+}, \tag{6.21d}$$

$$\widetilde{\lambda^{++\beta+}} = -\lambda^{++\dot{\beta}+}, \qquad \widetilde{\lambda^{++\dot{\beta}+}} = \lambda^{++\beta+}. \tag{6.21e}$$

The variation of the free hypermultiplet Lagrangian under the general transformations (6.18) with arbitrary analytic parameters $\lambda^{MN}(\zeta)$ has the form (up to a total derivative)[13]:

$$\begin{aligned}
\delta_{\lambda}^{(s=3)} S_{free} &= \frac{\kappa_3}{4} \int d\zeta^{(-4)}\, q^{+a} \left[\mathcal{D}^{++}, \{\hat{\Lambda}^M, \partial_M\}_{AGB}\right] J q^+_a \\
&= \frac{\kappa_3}{2} \int d\zeta^{(-4)}\, q^{+a} [\mathcal{D}^{++}, \hat{\Lambda}^M] \partial_M J q^+_a \\
&\quad + \frac{\kappa_3}{4} \int d\zeta^{(-4)}\, q^{+a} \left\{\hat{\Lambda}^M, [\mathcal{D}^{++}, \partial_M]\right\}_{AGB} J q^+_a .
\end{aligned} \tag{6.22}$$

The first line involves terms with two derivatives. The second line, modulo integration by parts, collects terms with two derivatives and those without derivatives.

Requiring gauge invariance

$$\delta_{\lambda}^{(s=3)} S_{hyper} + \delta_{\lambda}^{(s=3)} S_{int} = 0 \tag{6.23}$$

to the leading order gives the linearized gauge transformation law for the analytic potentials. It can be formally represented as[14]:

$$\begin{aligned}
\delta_{\lambda}^{(s=3)} \hat{\mathcal{H}}^{++}_{(s=3)} &= \frac{1}{2} \left[\mathcal{D}^{++}, \{\hat{\Lambda}^M, \partial_M\}_{AGB}\right] \\
&= [\mathcal{D}^{++}, \hat{\Lambda}^M] \partial_M + \frac{1}{2} \left\{\hat{\Lambda}^M, [\mathcal{D}^{++}, \partial_M]\right\}_{AGB} .
\end{aligned} \tag{6.24}$$

The action $S_{hyper} + S^{(s=3)}_{int}$ also respects an additional $U(1)$ gauge freedom:

$$\delta_\lambda q^{+a} = -\kappa_3 \lambda J q^{+a}, \qquad \delta_\lambda h^{++} = \mathcal{D}^{++} \lambda. \tag{6.25}$$

6.3 Wess-Zumino gauge: $\mathcal{N}=2$ superconformal spin 3 multiplet

The linearized gauge transformations of independent analytic potentials can be deduced from (6.24):

$$\begin{cases}
\delta_\lambda h^{++\alpha\dot{\alpha}\beta\dot{\beta}} &= \mathcal{D}^{++}\lambda^{\alpha\beta\dot{\alpha}\dot{\beta}} + 2i\left(\lambda^{\alpha\dot{\alpha}\beta+}\bar{\theta}^{+\dot{\beta}} + \lambda^{\beta\dot{\beta}\alpha+}\bar{\theta}^{+\dot{\alpha}}\right) \\
&\qquad - 2i\left(\bar{\lambda}^{\alpha\dot{\alpha}\dot{\beta}+}\theta^{+\beta} + \bar{\lambda}^{\beta\dot{\beta}\dot{\alpha}+}\theta^{+\alpha}\right), \\
2\delta_\lambda h^{++\alpha\dot{\alpha}\beta+} &= \mathcal{D}^{++}\lambda^{\alpha\dot{\alpha}\beta+} - 8i\lambda^{[\alpha+\beta]+}\bar{\theta}^{+\dot{\alpha}} - 8i\lambda^{\beta+\dot{\alpha}+}\theta^{+\alpha} - \lambda^{\alpha\dot{\alpha}++}\theta^{+\beta}, \\
2\delta_\lambda h^{++\alpha\dot{\alpha}++} &= \mathcal{D}^{++}\lambda^{++\alpha\dot{\alpha}} + 4i\lambda^{++\alpha+}\bar{\theta}^{+\dot{\alpha}} - 4i\lambda^{++\dot{\alpha}+}\theta^{+\alpha},
\end{cases} \tag{6.26a}$$

[13] Using this result one can obtain rigid symmetries ("higher-spin" superconformal symmetries) of the free massless hypermultiplet and the corresponding current superfields. We hope to address this issue elsewhere.

[14] One needs to integrate by parts the terms with one derivative and to reduce them to terms without derivatives in order to be able to cancel them by a gauge transformation of the h^{++} term in $\hat{\mathcal{H}}^{++}_{(s=3)}$. In formula (6.24) we assume that such manipulations have been done.

$$
\begin{cases}
\delta_\lambda h^{++[\alpha+\beta]+} & = \mathcal{D}^{++}\lambda^{[\alpha+\beta]+} - \lambda^{++[\alpha+}\theta^{+\beta]}, \\
2\delta_\lambda h^{++\alpha+\dot{\alpha}+} & = 2\mathcal{D}^{++}\lambda^{\alpha+\dot{\alpha}+} - \lambda^{++\alpha+}\bar{\theta}^{+\dot{\alpha}} + \lambda^{++\dot{\alpha}+}\theta^{+\alpha}, \\
2\delta_\lambda h^{++\hat{\alpha}+++} & = \mathcal{D}^{++}\lambda^{\hat{\alpha}+++} - 2\theta^{+\hat{\alpha}}\lambda^{(+4)}, \\
\delta_\lambda h^{(+6)} & = \mathcal{D}^{++}\lambda^{(+4)},
\end{cases}
\tag{6.26b}
$$

$$
\begin{aligned}
\delta_\lambda h^{++} = \mathcal{D}^{++}\lambda &+ 2i\bar{\theta}^{+\dot{\rho}}\left(\partial_{\alpha\dot{\rho}}\partial^-_\beta\lambda^{[\alpha+\beta]+}\right) + 2i\theta^{+\rho}\left(\partial_{\rho\dot{\alpha}}\partial^-_{\dot{\beta}}\lambda^{[\dot{\alpha}+\dot{\beta}]+}\right) \\
&+ 2i\bar{\theta}^{+\dot{\rho}}\left(\partial_{\alpha\dot{\rho}}\partial^-_{\dot{\beta}}\lambda^{\alpha+\dot{\beta}+}\right) - 2i\theta^{+\rho}\left(\partial_{\rho\dot{\alpha}}\partial^-_\beta\lambda^{\dot{\alpha}+\beta+}\right) \\
&+ 2i\bar{\theta}^{+\dot{\rho}}\left(\partial_{\alpha\dot{\rho}}\partial^{--}\lambda^{\alpha+++}\right) - 2i\theta^{+\rho}\left(\partial_{\rho\dot{\beta}}\partial^{--}\lambda^{\dot{\beta}+++}\right) \\
&- 8i\partial_{\alpha\dot{\alpha}}\lambda^{\alpha+\dot{\alpha}+} + \frac{1}{2}\theta^{+\hat{\rho}}\left(\partial^-_{\hat{\rho}}\partial^{--}\lambda^{(+4)}\right) - 2(\partial^{--}\lambda^{(+4)}).
\end{aligned}
\tag{6.26c}
$$

Using these transformations, one can impose Wess-Zumino type gauge. Potentials of the form $h^{++\alpha\dot{\alpha}M}$ span $\mathcal{N}=2$ spin $\mathbf{3}$ superconformal multiplet ($\mathbf{s}=\mathbf{3}$ Weyl multiplet):

$$
\begin{cases}
h^{++(\alpha\beta)(\dot{\alpha}\dot{\beta})} = -4i\theta^+_\rho\bar{\theta}^+_{\dot{\rho}}\Phi^{(\alpha\beta\rho)(\dot{\alpha}\dot{\beta}\dot{\rho})} - (\bar{\theta}^+)^2\theta^+_\rho\psi^{(\alpha\beta\rho)(\dot{\alpha}\dot{\beta})i}u^-_i \\
\qquad\qquad - (\theta^+)^2\bar{\theta}^+_{\dot{\rho}}\bar{\psi}^{(\alpha\beta)(\dot{\alpha}\dot{\beta}\dot{\rho})i}u^-_i + (\theta^+)^2(\bar{\theta}^+)^2 V^{(\alpha\beta)(\dot{\alpha}\dot{\beta})ij}u^-_i u^-_j\,, \\
h^{++(\alpha\beta)\dot{\alpha}+} = (\theta^+)^2\bar{\theta}^+_{\dot{\nu}}\mathcal{P}^{(\alpha\beta)(\dot{\alpha}\dot{\nu})} + (\bar{\theta}^+)^2\theta^+_\nu T^{(\alpha\beta\nu)\dot{\alpha}} + (\theta^+)^4\chi^{(\alpha\beta)\dot{\alpha}i}u^-_i\,, \\
h^{++\alpha(\dot{\alpha}\dot{\beta})+} = \widetilde{h^{++(\alpha\beta)\dot{\alpha}+}}\,, \\
h^{++\alpha\dot{\alpha}++} = (\theta^+)^2(\bar{\theta}^+)^2 D^{\alpha\dot{\alpha}}\,.
\end{cases}
\tag{6.27}
$$

It is essential that the field $\mathcal{P}^{(\alpha\beta)(\dot{\alpha}\dot{\nu})}$ is *real*,

$$
\widetilde{\mathcal{P}^{(\alpha\beta)(\dot{\alpha}\dot{\nu})}} = \mathcal{P}^{(\alpha\nu)(\dot{\alpha}\dot{\beta})}.
$$

The originally present imaginary part of such a field proves to be pure gauge.

All other potentials (including those parts of the original potentials which are antisymmetric in the spinorial indices) can be fully gauged away[15] using the gauge freedom (6.26b) and (6.26c) (see also discussion in appendix C). The technical details of this procedure are collected in appendix A. In the physical sector we are left with the following fields[16]:

[15]Similar pure gauge field parameters were also used in ref. [32] (see sect. 3.4 there). These fields can also be gauged away. After eliminating these redundant fields, gauge transformations cease to be linear in fields. The purpose of introducing extra fields in the work [32] was the desire to close the algebra of gauge transformations. In our case, their introduction is dictated by $\mathcal{N}=2$ superconformal invariance and, since gauge transformations are chosen to have a general form, we expect that the algebra of gauge transformations will be automatically closed.

[16]Some fields require redefinitions, here we assume that such a procedure has been performed. Explicitly, these redefinitions are given in Appendix A. For simplicity and clarity of notation we also use the properly rescaled gauge parameters here. The precise relation between the gauge parameters $a^{(\alpha\beta)(\dot{\alpha}\dot{\beta})}$, $v^{\alpha\dot{\alpha}(ij)}$, $p^{\beta\dot{\beta}}$, $t^{(\alpha\beta)}$, c used below and the components of the analytic superfield parameters λ^{MN} used in Appendix A can be established by comparing with eqs. (A.65), (A.75), (A.69), (A.55), and (A.74).

Bosonic sector :

- *Conformal spin 3 field* with gauge freedom (7 off-shell d.o.f.):

$$\delta\Phi^{(\alpha\beta\rho)(\dot\alpha\dot\beta\dot\rho)} = \partial^{(\dot\rho(\rho}a^{\alpha\beta)\dot\alpha\dot\beta)}. \tag{6.28}$$

- *Triplet of conformal gravitons (spin 2 fields)* (15 off-shell d.o.f.):

$$\delta V^{(\alpha\beta)(\dot\alpha\dot\beta)(ij)} = \partial^{(\dot\alpha(\alpha}v^{\beta)\dot\beta)(ij)}. \tag{6.29}$$

- *Conformal graviton* (5 off-shell d.o.f.):

$$\delta\mathcal{P}^{(\alpha\beta)(\dot\alpha\dot\beta)} = \partial^{(\dot\alpha(\alpha}p^{\beta)\dot\beta)}. \tag{6.30}$$

- *Gauge field for self-dual two-form symmetry* (10 off-shell d.o.f.):

$$\delta T^{(\alpha\beta\rho)\dot\alpha} = \partial^{\dot\alpha(\rho}t^{\alpha\beta)}. \tag{6.31}$$

Fields $T^{(\alpha\beta\gamma)\dot\alpha}$ and complex conjugated $\bar T^{(\dot\alpha\dot\beta\dot\gamma)\alpha}$ are in one-to-one correspondence with a real tensor field $T^{[ab]d}$:

$$T^{[ab]c} = \sigma^{[ab]}_{(\alpha\beta)}\sigma^{c}_{\gamma\dot\gamma}T^{(\alpha\beta\gamma)\dot\gamma} + \tilde\sigma^{[ab]}_{(\dot\alpha\dot\beta)}\sigma^{c}_{\dot\gamma\gamma}\bar T^{(\dot\alpha\dot\beta\dot\gamma)\gamma}. \tag{6.32}$$

Due to the σ-matrices properties, the following identity holds:

$$T^{[abc]} = 0 \qquad \Leftrightarrow \qquad T^{[ab]c} + T^{[bc]a} + T^{[ca]b} = 0. \tag{6.33}$$

These symmetry properties correspond to the simple hook Young diagram $\begin{array}{|c|c|}\hline \; & \; \\ \hline \; & \multicolumn{1}{c}{} \\ \cline{1-1}\end{array}$. Additionally, properties of σ-matrices imply the traceless condition $T^{[ab]}{}_b = 0$. Gauge freedom are given by:

$$\delta T^{[ab]c} = 2\partial^{[a}t^{b]c} - 2\partial^{c}t^{[ab]}, \qquad\qquad t^{[ab]} = \sigma^{[ab]}_{(\alpha\beta)}t^{(\alpha\beta)} + \sigma^{[ab]}_{(\dot\alpha\dot\beta)}\bar t^{(\dot\alpha\dot\beta)}. \tag{6.34}$$

This field is called "hook field" (or conformal pseudo-graviton field). Hook field was firstly studied in [66, 67] as a generalized gauge field[17]. The basic motivation for their consideration was the construction of dual formulations of gauge fields with spin $s \neq 1$. These fields can be viewed as a natural generalization of the notoph field of Ogievetsky and Polubarinov [68][18] (for review see [70]).

- *Spin 1 gauge field* (3 off-shell d.o.f.):

$$\delta D^{\alpha\dot\alpha} = \partial^{\alpha\dot\alpha}c. \tag{6.35}$$

Fermionic sector[19]:

- *Doublet of conformal spin $\frac{5}{2}$ fields* (24 off-shell d.o.f.):

$$\delta\psi^{(\alpha\beta\rho)(\dot\alpha\dot\beta)i} = \partial^{(\dot\alpha(\rho}b^{\alpha\beta)\dot\beta)i}. \tag{6.36}$$

[17] It is not difficult to construct a conformal and gauge-invariant action for the hook field, see for example Appendix C of ref. [26].

[18] This kind of gauge theories was later re-discovered by Kalb and Ramond [69].

[19] The relations between the gauge parameters $b^{(\alpha\beta)\dot\beta i}$, $c^{\beta i}$ and components of the superfield parameters λ^{MN} can be found by comparing with eqs. (A.86) and (A.91).

- *Gauge spin $\frac{3}{2}$ fermion* $\chi^{(\alpha\beta)\dot{\alpha}i}$ (16 off-shell d.o.f.):

$$\delta\chi^{(\alpha\beta)\dot{\alpha}i} = \partial^{\dot{\alpha}(\alpha}c^{\beta)i}. \tag{6.37}$$

So $\mathcal{N}=2$ off-shell superconformal spin **3** multiplet contains $\mathbf{40}_B+\mathbf{40}_F$ off-shell degrees of freedom. Note that the spin 3 and spin 1 fields appear in the same $\mathcal{N}=2$ gauge supermultiplet. This may simplify the implementation of Grigoriev and Tseytlin assumption [21, 22] about the gauge invariance of a system of conformal spins 1 and 3 on an arbitrary curved background.

The residual gauge transformations and their action on these fields can be analyzed in full analogy with the spin **2** case considered in Section 5.3. We do not give the explicit formulas, because the detailed component considerations are beyond the scope of our study.

All the potentials except $h^{++\alpha\dot{\alpha}M}$ can be put equal to zero using the original large gauge freedom. One can choose such a gauge from the very beginning to bring the vertex to the simpler form:

$$S^{(s=3)}_{int|fixed} = -\frac{\kappa_3}{2}\int d\zeta^{(-4)}\, q^{+a}h^{++\alpha\dot{\alpha}M}\partial_M\partial_{\alpha\dot{\alpha}}Jq^+_a\,, \tag{6.38}$$

where, like in the non-conformal case, the spinorial indices of the same chirality in $h^{++\alpha\dot{\alpha}M}$ are assumed to be symmetrized. In such a form the vertex, up to terms involving harmonic derivative ∂^{--}, fully matches the non-conformal $(\mathbf{3},\frac{\mathbf{1}}{\mathbf{2}},\frac{\mathbf{1}}{\mathbf{2}})$ vertex. However in such a gauge one is led to accompany the superconformal transformations (6.11) by the proper compensating gauge transformations in order to preserve the gauge:

$$\delta h^{++MN}_{WZ} = \delta_{diff}h^{++MN}_{WZ} + \delta_\lambda h^{++MN} \sim h^{++MN}_{WZ}. \tag{6.39}$$

From this condition one can determine the parameters λ^{MN} involving the explicit dependence on gauge potentials h^{++MN}. So the vertex (6.38) is invariant under the modified superconformal transformations

$$\delta_{sc|mod}q^{+a} = \delta_{sc}q^{+a} + \delta_{\lambda|WZ}q^{+a}. \tag{6.40}$$

These transformations generically involve the spin **3** potentials and so are essentially nonlinear.

As an example, we quote the explicit form of such a transformation in the sector of conformal supersymmetry (parameter η^i_α). In WZ gage $h^{++\alpha\dot{\beta}++}_{WZ}=0$, which amounts to the condition:

$$\delta h^{++\alpha\dot{\beta}++}_{WZ} = \mathcal{D}^{++}\lambda^{+\alpha\dot{\beta}+} - \lambda^{+\alpha++}\bar{\theta}^{+\dot{\beta}} + \lambda^{+\dot{\beta}++}\theta^{+\alpha} + h^{++(\alpha\beta)\dot{\beta}+}_{WZ}\eta^i_\beta u^+_i = 0. \tag{6.41}$$

Using the explicit form of WZ gauge for $h^{++(\alpha\beta)\dot{\beta}+}_{WZ}$, one has:

$$\lambda^{+\alpha\dot{\beta}+} = -h^{++(\alpha\beta)\dot{\beta}+}_{WZ}\eta^i_\beta u^-_i + \dots. \tag{6.42}$$

Here ellipses stand for possible contributions from $\lambda^{+\hat{\alpha}++}$.

The resulting modified hypermultiplet superconformal transformation with parameter η^i_α (conformal supersymmetry) is found to be:

$$\begin{aligned}\delta^{\eta}_{sc|mod} q^{+a} =& \delta^{\eta}_{sc} q^{+a} + \frac{\kappa_3}{2}\left\{h^{++(\alpha\beta)\dot{\beta}+}_{WZ}\eta^i_\beta u^-_i \partial^-_{\dot{\beta}}, \partial^-_\beta\right\}_{AGB} J q^{+a} \\ &+ \frac{\kappa_3}{4}\left\{\partial^-_{\dot{\beta}} h^{++(\alpha\beta)\dot{\beta}+}_{WZ}\eta^i_\beta u^-_i, \partial^-_\beta\right\}_{AGB} J q^{+a} + \dots\end{aligned} \tag{6.43}$$

To summarize, it was necessary to start with the most general form of the vertex (6.10) in order to realize (local) $\mathcal{N} = 2$ superconformal symmetry linearly on the hypermultiplet. The elimination of the auxiliary analytic potentials leads to the minimal set of the gauge potentials on which rigid $\mathcal{N} = 2$ superconformal group generically acts by nonlinear transformations explicitly involving the spin **3** potentials.

6.4 s = 3 superconformal current superfields

Putting $\delta^{(s=3)}_\lambda S_{free} = 0$ in (6.22), we recover rigid symmetries of the free hypermultiplet action. There exist two ways to derive the corresponding $\mathbf{s} = 3$ Noether current superfields. One can either study the variation (6.22) of the action, or, equivalently, vary the cubic coupling (6.1) with respect to the analytic potentials h^{++MN}. The relevant current superfields are given by the expressions:

$$J^{++}_{MN} = -\frac{1}{2} q^{+a}\partial_N\partial_M J q^+_a, \qquad \mathcal{D}^{++} J^{++}_{MN} = -\frac{1}{2} q^{+a}[\mathcal{D}^{++}, \partial_N\partial_M] J q^+_a. \tag{6.44}$$

When deducing the current conservation condition in (6.44), we made use of the free hypermultiplet equations of motion (2.11). The current superfields obtained in this way are sources of the equations of motion for the spin **3** gauge potentials. In this article we do not discuss the issue of constructing an $\mathcal{N} = 2$ spin **3** superconformal action and the corresponding equations of motion.

In the more detailed notation, we are left with nine independent current superfields:

$$J^{++}_{\alpha\beta\dot{\alpha}\dot{\beta}}, \quad J^+_{\alpha\beta\dot{\alpha}}, \quad J^+_{\dot{\alpha}\dot{\beta}\alpha}, \quad J_{\alpha\beta}, \quad J_{\dot{\alpha}\dot{\beta}}, \quad J_{\alpha\dot{\alpha}}, \quad J^-_\alpha, \quad J^-_{\dot{\alpha}}, \quad J^{--}. \tag{6.45}$$

The current superfields (6.44) (or (6.45)) are analytic but they are not invariant under $\mathcal{N} = 2$ supersymmetry. Like in the $\mathbf{s} = 2$ case, one can introduce non-analytic current superfields which are invariants of $\mathcal{N} = 2$ supersymmetry. In contrast to the $\mathbf{s} = 2$ case, here we deal with few different "master" currents.

The simplest option corresponds to the choice $M = \alpha\dot{\alpha}$. The non-analytic current superfield has the following form:

$$\mathcal{J}_{\alpha\dot{\alpha}} = -\frac{1}{2} q^{+a}\mathcal{D}^{--}\partial_{\alpha\dot{\alpha}} J q^+_a, \qquad \mathcal{D}^{++}\mathcal{J}_{\alpha\dot{\alpha}} = -\frac{1}{2} q^{+a}\partial_{\alpha\dot{\alpha}} J q^+_a. \tag{6.46}$$

This expression satisfies various conservation laws, for example:

$$\mathcal{D}^{++}\left(D^+_{\dot{\beta}}\mathcal{J}_{\alpha\dot{\alpha}}\right) = 0, \qquad \mathcal{D}^{++}\left(D^+_\beta D^+_{\dot{\beta}}\mathcal{J}_{\alpha\dot{\alpha}}\right) = 0. \tag{6.47}$$

For the choice $M = \hat{\alpha}$ we analogously obtain:

$$\mathcal{J}^{-}_{\hat{\alpha}} = -\frac{1}{2} q^{+a} \mathcal{D}^{--} \mathcal{D}^{-}_{\hat{\alpha}} J q^{+}_{a}, \qquad \mathcal{D}^{++} \mathcal{J}^{-}_{\hat{\alpha}} = 0. \tag{6.48}$$

At last, choosing $M = ++$ yields:

$$\mathcal{J}^{--} = -\frac{1}{2} q^{+a} \mathcal{D}^{--} \mathcal{D}^{--} J q^{+}_{a}, \qquad \mathcal{D}^{++} \mathcal{J}^{--} = 0. \tag{6.49}$$

The set of the $\mathbf{s} = 3$ superconformal current superfields $\{\mathcal{J}_{\alpha\dot{\alpha}}, \mathcal{J}^{-}_{\hat{\alpha}}, \mathcal{J}^{--}\}$ incorporate all the analytic supercurrents (6.44), (6.45) in their θ^{-} expansions. For example,

$$\mathcal{J}_{\alpha\dot{\alpha}} = -4i\theta^{-\beta}\bar{\theta}^{-\dot{\beta}} J^{++}_{\alpha\beta\dot{\alpha}\dot{\beta}} + \theta^{-\beta} J^{+}_{\alpha\beta\dot{\alpha}} + J_{\alpha\dot{\alpha}}. \tag{6.50}$$

An alternative way to derive these superconformal currents is through varying cubic couplings with respect to the unconstrained non-analytic prepotentials $\{\Upsilon^{\alpha\dot{\alpha}}, \Upsilon^{+\hat{\alpha}}, \Upsilon^{++}, \Upsilon^{--}\}$ defined as:

$$\hat{\mathcal{H}}^{++}_{(s=3)} := (D^{+})^{4} \left(\Upsilon^{\alpha\dot{\alpha}} \partial_{\alpha\dot{\alpha}} \mathcal{D}^{--} + \Upsilon^{+\hat{\alpha}} \mathcal{D}^{-}_{\hat{\alpha}} \mathcal{D}^{--} + \Upsilon^{++} \mathcal{D}^{--} \mathcal{D}^{--} + \Upsilon^{--} \right). \tag{6.51}$$

From this definition one can deduce the transformation laws of non-analytic prepotentials. In the next section we will show that it is possible to select a gauge $\Upsilon^{+\hat{\alpha}} = 0$, $\Upsilon^{++} = 0$ and $\Upsilon^{--} = 0$. In this gauge we can describe the spin **3** supermultiplet in terms of unconstrained non-analytic prepotential $\Upsilon^{\alpha\dot{\alpha}}$. Such a prepotential (in the gauge where it does not depend on harmonics) can presumably be identified with the one introduced in ref. [41]. Thus the relation (6.51) gives a hint of how the prepotentials of ref. [41] could appear within the harmonic superspace approach. It should be pointed out that all these prepotentials and their gauge freedom are of non-geometric character, like the original Mezincescu potential for $\mathcal{N} = 2$ Maxwell theory. In contrast, the analytic gauge potentials have the clear geometric meaning as the objects covariantizing the analyticity-preserving harmonic derivative $\mathcal{D}^{++}$.

6.5 Summary and superconformal $(\mathbf{3}, \frac{\mathbf{1}}{\mathbf{2}}, \frac{\mathbf{1}}{\mathbf{2}})$ vertex on conformal supergravity background

Let us summarize the results collected in section 6. We started from the action containing the spin **2** and the spin **3** couplings to hypermultiplet ,

$$S = -\frac{1}{2} \int d\zeta^{(-4)}\, q^{+a} \left(\mathcal{D}^{++} + \kappa_2 \hat{\mathcal{H}}^{++}_{(s=2)} + \kappa_3 \hat{\mathcal{H}}^{++}_{(s=3)} J \right) q^{+}_{a}. \tag{6.52}$$

Here the operators $\hat{\mathcal{H}}^{++}_{(s=2)}$ and $\hat{\mathcal{H}}^{++}_{(s=3)}$ were defined in (5.3) and (6.15). This action is exactly invariant under the nonlinear spin **2** gauge transformations (3.1), (5.16), (6.11) (and so is also invariant under rigid $\mathcal{N} = 2$ superconformal transformations), as well as under the linearized spin **3** gauge transformations (6.18), (6.24) (*i.e.*, to the leading order in κ_2, κ_3).

Under the spin **3** gauge transformations (6.18) of the hypermultiplet the vertex $(\mathbf{2}, \frac{\mathbf{1}}{\mathbf{2}}, \frac{\mathbf{1}}{\mathbf{2}})$ transforms as:

$$\begin{aligned} \delta^{(s=3)}_{\lambda} \left(q^{+a} \hat{\mathcal{H}}^{++}_{(s=2)} q^{+}_{a} \right) = &- \frac{\kappa_3}{2} q^{+a} \left[\hat{\mathcal{H}}^{++}_{(s=2)}, \{\hat{\Lambda}^{M}, \partial_{M}\}_{AGB} \right] J q^{+}_{a} \\ &- \frac{\kappa_3}{4} q^{+a} \left[\hat{\mathcal{H}}^{++}_{(s=2)}, \{\Omega^{M}, \partial_{M}\}_{AGB} \right] J q^{+}_{a}. \end{aligned} \tag{6.53}$$

One can cancel these terms (using integrations by parts) by introducing the additional spin **2**-dependent terms in the gauge transformations (6.24) of the spin **3** multiplet:

$$\delta^{ad}_{\lambda}\hat{\mathcal{H}}^{++}_{(s=3)} = \frac{\kappa_2}{2}\left[\hat{\mathcal{H}}^{++}_{(s=2)}, \{\hat{\Lambda}^M, \partial_M\}_{AGB}\right] + \frac{\kappa_2}{4}\left[\hat{\mathcal{H}}^{++}_{(s=2)}, \{\Omega^M, \partial_M\}_{AGB}\right]. \tag{6.54}$$

These terms deform the transformations law (6.24) by the general $\mathcal{N}=2$ conformal supergravity background:

$$\delta^{(s=3)}_{\lambda|full}\hat{\mathcal{H}}^{++}_{(s=3)} = \frac{1}{2}\left[\mathfrak{D}^{++}, \{\hat{\Lambda}^M, \partial_M\}_{AGB}\right] + \frac{\kappa_2}{4}\left[\hat{\mathcal{H}}^{++}_{(s=2)}, \{\Omega^M, \partial_M\}_{AGB}\right]. \tag{6.55}$$

The last term acts only on the h^{++} part of $\hat{\mathcal{H}}^{++}_{(s=3)}$.

As a result, we have found that the action (6.52) is invariant with respect to the spin **3** transformations to the leading order in κ_3 and to any order in κ_2. This means that we have constructed a cubic vertex $(\mathbf{3}, \frac{\mathbf{1}}{\mathbf{2}}, \frac{\mathbf{1}}{\mathbf{2}})$ which is invariant under the gauge transformations of conformal $\mathcal{N}=2$ supergravity. In the component approach this amounts to the property that, after elimination of the auxiliary fields, one will recover the superconformal action of the spin **3** supermultiplet coupling $(\mathbf{3}, \frac{\mathbf{1}}{\mathbf{2}}, \frac{\mathbf{1}}{\mathbf{2}})$ on *generic* $\mathcal{N}=2$ Weyl supergravity background. Note that the spin **3** multiplet fields in the action (6.52) do not directly interact with the supergravity fields; the interaction is mediated by the auxiliary fields of hypermultiplet.

7 Generalization to arbitrary spin s

In this section we generalize the results for the superconformal spin **3** hypermultiplet coupling to the general spin **s** case. We follow the general strategy of section 3.1.

The relevant cubic superconformal $(\mathbf{s}, \frac{\mathbf{1}}{\mathbf{2}}, \frac{\mathbf{1}}{\mathbf{2}})$ vertex has the form:

$$S^{(s)}_{int} = -\frac{\kappa_s}{2}\int d\zeta^{(-4)}\, q^{+a}\hat{\mathcal{H}}^{++}_{(s)}\,(J)^{P(s)}\, q^+_a. \tag{7.1}$$

Here $\hat{\mathcal{H}}^{++}_{(s)}$ is analytic differential operator including general terms with $s-1$, $s-3$, $\dots$ 1/0 (for even s/odd s) derivatives:

$$\hat{\mathcal{H}}^{++}_{(s)} := h^{++M_1\dots M_{s-1}}\partial_{M_{s-1}}\dots\partial_{M_1} + h^{++M_1\dots M_{s-3}}\partial_{M_{s-3}}\dots\partial_{M_1} + \dots + \begin{bmatrix} h^{++M}\partial_M & (\textit{even } s) \\ h^{++} & (\textit{odd } s). \end{bmatrix} \tag{7.2}$$

Like in the $\mathbf{s}=2$ and $\mathbf{s}=3$ cases, the necessity to include the derivatives of general type follows from the requirement of $\mathcal{N}=2$ superconformal invariance. The analytic superfields $h^{++\dots}(\zeta)$ for any pair of adjacent indices satisfy the symmetry conditions:

$$h^{++M_1\dots M_n M_k\dots M_{s-1}} = (-1)^{P(M_k)P(M_n)}h^{++M_1\dots M_k M_n\dots M_{s-1}}. \tag{7.3}$$

From here one can deduce how to permute any 2 indices. Also, the operator $\hat{\mathcal{H}}^{++}_{(s)}$ satisfies the reality condition:

$$\widetilde{\hat{\mathcal{H}}^{++}_{(s)}} = \hat{\mathcal{H}}^{++}_{(s)}. \tag{7.4}$$

7.1 $\mathcal{N}=2$ superconformal symmetry

The analytic superdiffeomorphism transformation (3.17) of the hypermultiplet generates the following transformation of the vertex:

$$\begin{aligned}\delta_{diff} S^{(s)}_{int} =& \frac{\kappa_s}{2} \int d\zeta^{(-4)}\, q^{+a} [\hat{\mathcal{H}}^{++}_{(s)}, \hat{\Lambda}]\, (J)^{P(s)}\, q^+_a + \frac{\kappa_s}{4} \int d\zeta^{(-4)}\, q^{+a} [\hat{\mathcal{H}}^{++}_{(s)}, \Omega]\, (J)^{P(s)}\, q^+_a \\ &- \frac{\kappa_s}{2} \int d\zeta^{(-4)}\, q^{+a} \delta_{diff} \hat{\mathcal{H}}^{++}_{(s)}\, (J)^{P(s)}\, q^+_a .\end{aligned} \tag{7.5}$$

Calculating the commutators in the first line, we get terms with various numbers of derivatives acting on the hypermultiplet, analogously to the spin **3** case (recall eq. (6.8)).

• For even spin **s** one can always reduce the terms with even number of derivatives to those with odd number. In this case, they are entirely compensated by the corresponding transformations of the gauge potentials in (7.2).

For example, the contribution of the two-derivative term in the spin **4** case is:

$$q^{+a} h^{++MNKL} \left(\partial_L \partial_K \partial_N \lambda^R\right) \partial_R \partial_M q^+_a . \tag{7.6}$$

The expression $T^{++MR} := h^{++MNKL} \left(\partial_L \partial_K \partial_N \lambda^R\right)$ has the proper symmetry under permuting the indices R and M:

$$T^{++MR} = (-1)^{P(R)P(M)} T^{++RM} . \tag{7.7}$$

because it is a coefficient of $\partial_R \partial_M$. After integration by parts and omitting total derivatives we obtain:

$$T^{++MR} q^{+a} \partial_R \partial_M q^+_a \Rightarrow -(-1)^{(P(M)+P(R))P(R)} \left(\partial_R T^{++MR}\right) q^{+a} \partial_M q^+_a - T^{++MR} \partial_R q^{+a} \partial_M q^+_a . \tag{7.8}$$

Due to the symmetry (7.7) the second term vanishes. So we have reduced the term with two derivatives acting on q^+_a to a term with one derivative.

In the general case, one should integrate by parts and bring all the terms either to an odd number of derivatives acting on q^+_a (and those without derivatives), which can be compensated by the proper transformation of gauge potentials, or to a term with equal number of derivatives acting on q^{+a} and q^+_a, and then use the identities of the type:

$$h^{++\dots N_1 \dots N_n M_1 \dots M_n \dots} \partial_{N_1} \dots \partial_{N_n} q^{+a} \partial_{M_1} \dots \partial_{M_n} q^+_a = 0, \tag{7.9}$$

which are a direct generalization of the identity $q^{+a} q^+_a = 0$.

• For odd **s** one can also transform the terms with an odd number of derivatives to those with the even number, integrating by parts and making use of the relation:

$$\begin{aligned}& h^{++N_1 \dots N_n M_1 \dots M_n K} \partial_{N_1} \dots \partial_{N_n} q^{+a} \partial_K \partial_{M_1} \dots \partial_{M_n} J q^+_a \\ &\qquad = \frac{1}{2} h^{++N_1 \dots N_n M_1 \dots M_n K} \partial_K \left(\partial_{N_1} \dots \partial_{N_n} q^{+a} \partial_{M_1} \dots \partial_{M_n} J q^+_a\right) .\end{aligned} \tag{7.10}$$

As a result, for any **s** we are able to cancel terms coming from $[\hat{\mathcal{H}}^{++}_{(s)}, \hat{\Lambda}]$ and $[\hat{\mathcal{H}}^{++}_{(s)}, \Omega]$ by the proper transformations of the set of gauge potentials (7.2) and thereby to ensure the diffeomorphism (and so superconformal) invariance of the cubic interaction (7.1). Once again, since we have not used the explicit form of $\mathcal{N} = 2$ superconformal parameters anywhere, this vertex is covariant under the complete gauge group of $\mathcal{N} = 2$ conformal supergravity.

Based upon this reasoning, from (7.5) we can figure out the transformation law of the analytic spin **s** operator $\hat{\mathcal{H}}^{++}_{(s)}$, which can be symbolically written as:

$$\delta_{diff} \hat{\mathcal{H}}^{++}_{(s)} = [\hat{\mathcal{H}}^{++}_{(s)}, \hat{\Lambda}] + \frac{1}{2}[\hat{\mathcal{H}}^{++}_{(s)}, \Omega]. \tag{7.11}$$

Here we assumed that the various terms in the right hand side must be re-organized as was explained above. The auxiliary gauge potentials of the lower spins play the same role as in the spin **3** case: they cancel terms with a lesser number of derivatives, which result from the commutators $[\hat{\mathcal{H}}^{++}_{(s)}, \hat{\Lambda}]$ and $[\hat{\mathcal{H}}^{++}_{(s)}, \Omega]$.

7.2 Gauge freedom

The action

$$S_{free} + S^{(s)}_{int} = -\frac{1}{2} \int d\zeta^{(-4)}\, q^{+a} \left(\mathcal{D}^{++} + \kappa_s \hat{\mathcal{H}}^{++}_{(s)} \left(J\right)^{P(s)}\right) q^{+}_{a} \tag{7.12}$$

is invariant under the hypermultiplet gauge transformations of the form ($k = s, s-2, s-4\ldots$):

$$\begin{aligned} \delta^{(k)}_{\lambda} q^{+a} =& - \frac{\kappa_s}{2} \left\{\hat{\Lambda}^{M_1 \ldots M_{k-2}}, \partial_{M_{k-2}} \ldots \partial_{M_1}\right\}_{AGB} (J)^{P(s)}\, q^{+a} \\ & - \frac{\kappa_s}{4} \left\{\Omega^{M_1 \ldots M_{k-2}}, \partial_{M_{k-2}} \ldots \partial_{M_1}\right\}_{AGB} (J)^{P(s)}\, q^{+a} \end{aligned} \tag{7.13}$$

accompanied by the gauge transformations of the gauge potentials:

$$\begin{aligned} \delta_{\lambda} \hat{\mathcal{H}}^{++}_{(s)} =& \left[\mathcal{D}^{++}, \hat{\Lambda}^{M_1 \ldots M_{k-2}}\right] \partial_{M_{k-2}} \ldots \partial_{M_1} \\ & + \frac{1}{2} \left\{\hat{\Lambda}^{M_1 \ldots M_{k-2}}, [\mathcal{D}^{++}, \partial_{M_{k-2}} \ldots \partial_{M_1}]\right\}_{AGB}. \end{aligned} \tag{7.14}$$

These formulas are a direct generalization of the spin **3** transformations. The spin **s** gauge transformation of the hypermultiplet contains $s-1$ superspace derivatives.

The formula (7.14) is symbolic like (7.11). One needs to reorganize the terms in the second line as was explained above for the diffeomorphisms invariance, using the fact that they act on the hypermultiplet and integrating by parts.

Here we used the following notations for the first-order analytic operator:

$$\hat{\Lambda}^{M_1 \ldots M_k} := \sum_{N \leq M_k \cdots \leq M_1} \lambda^{M_1 \ldots M_k N} \partial_N \tag{7.15}$$

and for the analytic weight factor:

$$\Omega^{M_1 \ldots M_k} := \sum_{N \leq M_k \cdots \leq M_1} (-1)^{P(N)} \partial_N \lambda^{N M_1 \ldots M_k}. \tag{7.16}$$

Analytic parameters satisfy the conditions

$$\lambda^{...MN...} = (-1)^{P(M)P(N)}\lambda^{...NM...} \tag{7.17}$$

for any pair of adjacent indices. These conditions have the same form as those for analytic gauge potentials in (7.3). The reality of the variation (7.13) implies the appropriate reality conditions for the transformation parameters.

These transformations constitute the gauge freedom of the spin $\mathbf{s}$, spin $\mathbf{s}-2$, ... parts of the differential operator $\hat{\mathcal{H}}^{++}_{(s)}$ (i.e. those entering with $\mathbf{s}-1$, $\mathbf{s}-3$, ... derivatives).

7.3 Wess-Zumino gauge: $\mathcal{N}=2$ superconformal spin s multiplet

The gauge freedom (7.14) enables to eliminate a large number of fields. The Wess-Zumino gauge can be imposed quite analogously to the spin **3** case (as described in detail in Appendix A). The field contents of this gauge completely repeats the form of the corresponding Wess-Zumino gauge in the case of spin **3**:

$$\begin{cases} h^{++\alpha(s-1)\dot\alpha(s-1)} = -4i\theta^+_\rho\bar\theta^+_{\dot\rho}\Phi^{(\rho\alpha(s-1))(\dot\rho\dot\alpha(s-1))} - (\bar\theta^+)^2\theta^+_\rho\psi^{(\rho\alpha(s-1))\dot\alpha(s-1)i}u^-_i \\ \qquad\qquad - (\theta^+)^2\bar\theta^+_{\dot\rho}\bar\psi^{\alpha(s-1)(\dot\alpha(s-1)\dot\rho)i}u^-_i + (\theta^+)^2(\bar\theta^+)^2V^{\alpha(s-1)\dot\alpha(s-1)ij}u^-_iu^-_j\,, \\ h^{++\alpha(s-1)\dot\alpha(s-2)+} = (\theta^+)^2\bar\theta^+_{\dot\nu}P^{\alpha(s-1)(\dot\alpha(s-2)\dot\nu)} + (\bar\theta^+)^2\theta^+_\nu T^{(\alpha(s-1)\nu)\dot\alpha(s-2)} \\ \qquad\qquad + (\theta^+)^4\chi^{\alpha(s-1)\dot\alpha(s-2)i}u^-_i\,, \\ h^{++\alpha(s-2)\dot\alpha(s-1)+} = \widetilde{h^{++\alpha(s-1)\dot\alpha(s-2)+}}\,, \\ h^{(+4)\alpha(s-2)\dot\alpha(s-2)} = (\theta^+)^2(\bar\theta^+)^2D^{\alpha(s-2)\dot\alpha(s-2)}\,. \end{cases} \tag{7.18}$$

All the remaining analytic potentials in (7.2) are pure gauge and can be entirely gauged away. Also, as in the spin **3** case, one can consider a special gauge in which only the potentials $h^{++\alpha(s-2)\dot\alpha(s-2)M}$ survive [20]. Generically, in such a gauge the superconformal transformations must be accompanied by the proper gauge transformations, with composite parameters involving gauge potentials (recall, e.g., the spin **3** example (6.43)).

So we have obtained $\mathcal{N}=2$ spin $\mathbf{s}$ superconformal off-shell gauge multiplet as the set of surviving fields in W-Z gauge. It consists of the fields with gauge transformations:

<u>Bosonic sector</u>:

- *Conformal spin s gauge field* ($2s+1$ off-shell d.o.f.):

$$\delta\Phi^{\alpha(s)\dot\alpha(s)} = \partial^{(\alpha(\dot\alpha}a^{\alpha(s-1))\dot\alpha(s-1))}. \tag{7.19}$$

Such fields are also known as Fradkin-Tseytlin fields [15].

- *Triplet of the spin $s-1$ conformal gauge fields* [$3(2s-1)$ off-shell d.o.f.]:

$$\delta V^{\alpha(s-1)\dot\alpha(s-1)ij} = \partial^{(\alpha(\dot\alpha}v^{\alpha(s-2))\dot\alpha(s-2))}. \tag{7.20}$$

[20] In such a gauge one can rewrite the analytic differential operator as

$$\mathcal{H}^{++}_s = h^{++\alpha(s-2)\dot\alpha(s-2)M}\partial_M\partial^{(s-2)}_{\alpha(s-2)\dot\alpha(s-2)} = (D^+)^4\left(\Upsilon^{\alpha(s-2)\dot\alpha(s-2)}\mathcal{D}^{--}\right)\partial^{(s-2)}_{\alpha(s-2)\dot\alpha(s-2)}.$$

This gives a direct connection with the Mezincescu-type prepotentials studied in [41].

• *Conformal spin $s-1$ gauge field* [$3(2s-1)$ off-shell d.o.f.]:

$$\delta P^{\alpha(s-1)\dot\alpha(s-1)} = \partial^{(\alpha(\dot\alpha}p^{\alpha(s-2))\dot\alpha(s-2))}. \tag{7.21}$$

• *Generalized conformal "hook-type" gauge field* [$2(2s-1)$ off-shell d.o.f.]:

$$\delta T^{\alpha(s)\dot\alpha(s-2)} = \partial^{(\alpha(\dot\alpha}t^{\alpha(s-1))\dot\alpha(s-3))}. \tag{7.22}$$

Such a complex gauge field was already considered in the context of $\mathcal{N}=1$ superconformal multiplets in [34] (see eq. (3.16) there) and in [39]. Gauge invariant field strengths and conformal actions for such fields were also presented in [34]. In refs. [24–26] the gauge invariant actions for such fields were constructed in conformally flat spaces.

• *Spin $s-2$ conformal gauge field* ($2s-3$ off-shell d.o.f.):

$$\delta D^{\alpha(s-2)\dot\alpha(s-2)} = \partial^{(\alpha(\dot\alpha}\Omega^{\alpha(s-3))\dot\alpha(s-3))}. \tag{7.23}$$

Fermionic sector:

• *Doublet of the fermionic spin $s-\frac{1}{2}$ gauge field* ($8s$ off-shell d.o.f.):

$$\delta\psi^{\alpha(s)\dot\alpha(s-1)i} = \partial^{(\alpha(\dot\alpha}b^{\alpha(s-1))\dot\alpha(s-2))i}. \tag{7.24}$$

• *Doublet of the spin $s-\frac{3}{2}$ fermionic gauge* fields [$8(s-1)$ off-shell d.o.f.]:

$$\delta\chi^{\alpha(s-1)\dot\alpha(s-2)i} = \partial^{(\alpha(\dot\alpha}c^{\alpha(s-2))\dot\alpha(s-3))i}. \tag{7.25}$$

So the general integer-spin $\mathbf{s}$ $\mathcal{N}=2$ superconformal multiplet encompasses $\mathbf{8(2s-1)}_B+$ $\mathbf{8(2s-1)}_F$ off-shell degrees of freedom[21]. Interestingly, all fields in the $\mathcal{N}=2$ superconformal higher-spin multiplets are gauge fields: no non-gauge auxiliary fields are present (the cases of $\mathcal{N}=2$ spin $\mathbf{1}$ theory and $\mathcal{N}=2$ conformal supergravity are an exception). It is the significant difference from the case of non-conformal $\mathcal{N}=2$ higher spins [44]. It is worth noting that there appear no auxiliary fields in the superconformal $\mathcal{N}=1$ higher spin multiplets as well [34] [22]. In this connection we mention that the $\mathcal{N}=2$ higher-spin superconformal multiplets constructed can be decomposed into the sum of three $\mathcal{N}=1$ supermultiplets: higher-spin $\mathbf{s}$ multiplet ($\mathbf{4s}_B+\mathbf{4s}_F$ off-shell d.o.f), higher-spin $\mathbf{s-1}$ multiplet ($\mathbf{4(s-1)}_B+\mathbf{4(s-1)}_F$ off-shell d.o.f) and higher-spin $\mathbf{s}-\frac{\mathbf{1}}{\mathbf{2}}$ multiplet ($\mathbf{4(2s-1)}_B+\mathbf{4(2s-1)}_F$ off-shell d.o.f.).

[21]In the case of non-conformal $\mathcal{N}=2$ spin $\mathbf{s}$ supermultiplet one deals with $\mathbf{8[s^2+(s-1)^2]}_B+$ $\mathbf{8[s^2+(s-1)^2]}_F$ off-shell degrees of freedom. These multiplets were constructed in [44] as a generalization of off-shell multiplet of $\mathcal{N}=2$ Einstein supergravity. The superconformal multiplets described here naturally generalize the Weyl multiplet of conformal $\mathcal{N}=2$ supergravity [53, 57] to arbitrary integer higher spins. It is interesting to note that the number of d.o.f. in $\mathcal{N}=2$ superconformal multiplet can be parametrized as $\mathbf{8[s^2-(s-1)^2]}_B+\mathbf{8[s^2-(s-1)^2]}_F$. This leads to the conjecture on the structure of superconformal compensators for the general spin $\mathbf{s}$: they should be composed of two towers of all integer $\mathcal{N}=2$ superconformal higher spins.

[22]Based on these affinities, it is reasonable to assume that an arbitrary $\mathcal{N}$-extended superconformal multiplet also does not contain auxiliary fields.

Generalizing the spin **3** superconformal current superfields of Section 6.4 to the general spin **s** is straightforward. For example, for the special case of vector indices we find:

$$\begin{aligned}\mathcal{J}_{\alpha(s-2)\dot{\alpha}(s-2)} &= -\frac{1}{2}q^{+a}\mathcal{D}^{--}\partial^{s-2}_{\alpha(s-2)\dot{\alpha}(s-2)}Jq^{+}_{a}\,,\\ \mathcal{D}^{++}\mathcal{J}_{\alpha(s-2)\dot{\alpha}(s-2)} &= -\frac{1}{2}q^{+a}\partial^{s-2}_{\alpha(s-2)\dot{\alpha}(s-2)}Jq^{+}_{a}\,.\end{aligned} \tag{7.26}$$

These expressions satisfy various conservation laws, e.g.,

$$\mathcal{D}^{++}\left(D^{+}_{\hat{\beta}}\mathcal{J}_{\alpha(s-2)\dot{\alpha}(s-2)}\right) = 0, \qquad \mathcal{D}^{++}\left(D^{+}_{\beta}D^{+}_{\dot{\beta}}\mathcal{J}_{\alpha(s-2)\dot{\alpha}(s-2)}\right) = 0. \tag{7.27}$$

Other supercurrents can be constructed in a similar way. We leave the general case for the future work.

7.4 Summary of the superconformal spin s

The action (7.12) admits the natural generalization to an arbitrary $\mathcal{N} = 2$ conformal supergravity background:

$$S = -\frac{1}{2}\int d\zeta^{(-4)}q^{+a}\left(\mathfrak{D}^{++} + \kappa_s\hat{\mathcal{H}}^{++}_{(s)}(J)^{P(s)}\right)q^{+}_{a}\,. \tag{7.28}$$

The generalized action (7.28) is invariant under:

1. *Nonlinear spin* **2** *gauge transformations* (i.e. $\mathcal{N} = 2$ conformal supergravity group). The action of these transformations on the spin **s** analytic potentials is given in (7.11).

2. *Spin* **s** *gauge transformations* to the leading order in κ_s (like in the spin **3** case, one needs to add the proper $\hat{\mathcal{H}}^{++}_{(s=2)}$ terms to the $\hat{\mathcal{H}}^{++}_{(s)}$ gauge transformation law). The full form of such transformations, with the proper spin **2** part added, is given by:

$$\begin{aligned}\delta_\lambda\hat{\mathcal{H}}^{++}_{(s)} =& \frac{1}{2}\left[\mathfrak{D}^{++}, \left\{\hat{\Lambda}^{M_1\ldots M_{k-2}}, \partial_{M_{k-2}}\ldots\partial_{M_1}\right\}_{AGB}\right]\\ &+ \frac{\kappa_2}{4}\left[\hat{\mathcal{H}}^{++}_{(s=2)}, \{\Omega^{M_1\ldots M_{k-2}}, \partial_{M_{k-2}}\ldots\partial_{M_1}\}_{AGB}\right].\end{aligned} \tag{7.29}$$

   The second term acts only on the lower-spin parts.

As in the cases of interaction of the spin **1** and **3** fields with $\mathcal{N} = 2$ conformal supergravity fields, the interaction of $\mathcal{N} = 2$ spin **s** multiplet with $\mathcal{N} = 2$ conformal supergravity multiplet is mediated by auxiliary hypermultiplet fields.

Thus eq. (7.28) provides the covariant superconformal vertex $(\mathbf{s}, \frac{\mathbf{1}}{\mathbf{2}}, \frac{\mathbf{1}}{\mathbf{2}})$ in an arbitrary $\mathcal{N} = 2$ conformal supergravity background.

8 Fully consistent higher-spin hypermultiplet coupling

In the previous sections we have constructed superconformal cubic vertices $(\mathbf{s}, \frac{\mathbf{1}}{\mathbf{2}}, \frac{\mathbf{1}}{\mathbf{2}})$ consistent to the leading order in higher-spin analogs of Einstein constant. In this section, we will consider the possibility of making the resulting cubic vertices invariant with respect to gauge transformations in the next orders in these coupling constants.

For example, consider the simplest case of the spin **3** on curved superspace:

$$S_{(s=3)} = -\frac{1}{2}\int d\zeta^{(-4)}\, q^{+a}\left(\mathfrak{D}^{++} + \kappa_3 \hat{\mathcal{H}}^{++}_{(s=3)} J\right) q^+_a . \tag{8.1}$$

This action is gauge invariant to the leading order in κ_3. In the next order we have the following gauge transformation of cubic vertex under the spin **3** gauge transformations (6.18) of the hypermultiplet :

$$\delta_\lambda^{(s=3)}\left(-\frac{\kappa_3}{2} q^{+a}\hat{\mathcal{H}}^{++}_{(s=3)} J q^+_a\right) = -\frac{\kappa_3^2}{4} q^{+a}\left[\hat{\mathcal{H}}^{++}_{(s=3)}, \left\{\hat{\Lambda}^M + \frac{1}{2}\Omega^M, \partial_M\right\}_{AGB}\right] q^+_a . \tag{8.2}$$

So we arrived at the differential operator of the third order in superspace derivatives. Making use of the spin **4** superconformal multiplet described in the previous section (modulo integrations by parts), one can compensate this term by deforming the spin **4** differential operator $\hat{\mathcal{H}}^{++}_{s=4}$ transformation law as:

$$\kappa_4 \delta_\lambda^{(s=3)} \hat{\mathcal{H}}^{++}_{s=4} = -\frac{\kappa_3^2}{4}\left[\hat{\mathcal{H}}^{++}_{(s=3)}, \left\{\hat{\Lambda}^M + \frac{1}{2}\Omega^M, \partial_M\right\}_{AGB}\right]. \tag{8.3}$$

Here we assumed that the appropriate integration by parts has been performed, like in the previous sections. Such a modified transformation law mixes different $\mathcal{N}=2$ superconformal multiplets, i.e. it is a nonabelian-type gauge symmetry. So the action

$$S_{s=3,4} = -\frac{1}{2}\int d\zeta^{(-4)}\, q^{+a}\left(\mathfrak{D}^{++} + \kappa_3\hat{\mathcal{H}}^{++}_{(s=3)} J + \kappa_4 \hat{\mathcal{H}}^{++}_{(s=4)}\right) q^+_a \tag{8.4}$$

respects the spin $\mathbf{s} = 3$ gauge invariance to κ_3^2 order. However, the action (8.4) is not invariant in the $\kappa_3\kappa_4$ order. Then the procedure just described can be continued step by step.

To summarize this procedure, we introduce an analytical differential operator that includes all integer higher spins:

$$\hat{\mathcal{H}}^{++} := \sum_{s=1}^{\infty} \kappa_s \hat{\mathcal{H}}^{++}_{(s)} (J)^{P(s)}. \tag{8.5}$$

The action of the infinite tower of integer $\mathcal{N}=2$ superconformal higher spins interacting with the hypermultiplet on an arbitrary $\mathcal{N}=2$ conformal supergravity background reads:

$$S_{full} = -\frac{1}{2}\int d\zeta^{(-4)}\, q^{+a}\left(\mathfrak{D}^{++} + \hat{\mathcal{H}}^{++}\right) q^+_a . \tag{8.6}$$

Then, assuming the proper gauge transformation of $\hat{\mathcal{H}}^{++}$, one can achieve gauge invariance to any order in couplings constants. Collecting the hypermultiplet gauge transformations (7.13) for all spins, we obtain

$$\delta_\lambda q^{+a} = -\hat{\mathcal{U}} q^{+a} = -\sum_{s=1}^{\infty} \kappa_s \hat{\mathcal{U}}_s q^{+a} . \tag{8.7}$$

This transformation acts linearly on the hypermultiplet superfield. For the set of gauge fields we obtain the transformation law:

$$\delta_\lambda \hat{\mathcal{H}}^{++} = \left[\mathfrak{D}^{++} + \hat{\mathcal{H}}^{++}, \hat{\mathcal{U}}\right]. \tag{8.8}$$

Here we also assumed the proper integration by parts, as in the previous sections. This transformation law mixes different spins, so this is a nonabelian deformation of the spin $\mathbf{s}$ transformation laws.

The invariance of (8.6) under $\mathcal{N}=2$ conformal supergravity transformations is automatic for the reasons expounded in the previous section. So we have constructed the fully consistent gauge-invariant and conformally invariant interaction of hypermultiplet with an infinite tower of $\mathcal{N}=2$ higher spins in an arbitrary $\mathcal{N}=2$ conformal supergravity background. To spot some possible hidden subtleties of the general construction, it seems necessary to perform a further deeper inspection of this procedure and, in particular, to make a detailed comparison with the known couplings among higher-spin gauge fields and scalar fields.

9 Conclusions and outlook

In this paper we have derived and discussed in details the structure of the off-shell manifestly $\mathcal{N}=2$ superconformal cubic interaction of $\mathcal{N}=2$, $4D$ hypermultiplet theory with an arbitrary superconformal higher spin $\mathbf{s}$ gauge superfield. The basic results can be summarized as:

- We considered the off-shell hypermultiplet model in $\mathcal{N}=2$, $4D$ harmonic superspace and described its rigid superconformal symmetries. For invariance of the cubic higher-spin vertices $(\mathbf{s}, \frac{\mathbf{1}}{\mathbf{2}}, \frac{\mathbf{1}}{\mathbf{2}})$ under these symmetries it proved necessary to properly modify the superconformal transformations of the hypermultiplet by the corresponding superconformal gauge superfields;

- To this end, we introduced the complete set of $\mathcal{N}=2$, $4D$ unconstrained analytic spin $\mathbf{s}$ superconformal higher-spin potentials, defined their superconformal and gauge transformations and revealed the physical field contents of the corresponding higher-spin Weyl supermultiplets in Wess-Zumino gauges. Their most notable features are: (i) all fields in the multiplets starting from $\mathbf{s}=3$ are gauge; (ii) the sets of bosonic fields necessarily contain "hook-type" generalized gauge fields;

- As a result, we have derived the manifestly $\mathcal{N}=2$ superconformal cubic vertex of the hypermultiplet coupled to superconformal higher spin external gauge superfields. Generically, the vertex has the structure: *higher spin superconformal gauge superfields* $\times$ *superconformal hypermultiplet supercurrents*. The corresponding supercurrents have been explicitly constructed in terms of the hypermultiplet superfields;

- As particular cases, we have constructed and discussed in detail the off-shell $(\mathbf{s}, \frac{\mathbf{1}}{\mathbf{2}}, \frac{\mathbf{1}}{\mathbf{2}})$ vertices in the background of $\mathcal{N}=2$ conformal supergravity for $\mathbf{s}=2,3$.

It should be specially pointed out that the geometric basis of the superconformal $\mathcal{N}=2, 4D$ off-shell gauge supermultiplets and their couplings to q^+ hypermultiplets, like in the previously discussed non-conformal case, proved to be the preservation of $\mathcal{N}=2$ Grassmann harmonic analyticity. First of all, the fundamental gauge potentials encompassing superconformal gauge multiplets are unconstrained $\mathcal{N}=2$ analytic harmonic superfields. Secondly, they are naturally recovered from the demand of analyticity of the q^{+a} Lagrangian and requiring them to be closed under the analyticity-preserving coordinate realization of rigid $\mathcal{N}=2$ superconformal symmetry.

Finally, let us list possible directions of the future study:

- *Dynamical actions for higher-spin $\mathcal{N}=2$ superconformal multiplets*

  The natural foremost task is to construct $\mathcal{N}=2$ Fradkin-Tseytlin superconformal actions for the superconformal multiplets presented, at least at the linearized level. In components, these actions should be reducible to higher-spin generalizations of the square of the linearized generalized Weyl tensors which were firstly introduced in [15]. In the HSS approach, such actions were not considered even for the standard Weyl ($\mathbf{s}=2$) multiplet.

- *Superconformal current superfield and rigid higher-spin superconformal symmetries*

  In this paper, we have addressed the important issue of the rigid symmetries of the free hypermultiplet and of the corresponding superfield currents only in passing. In fact, like in [45], one can easily identify the corresponding rigid symmetries by imposing the obvious conditions on the parameters (7.14):

$$\left[\mathcal{D}^{++}, \hat{\Lambda}^{M_1\dots M_{k-2}}\right]\partial_{M_{k-2}}\dots\partial_{M_1} + \frac{1}{2}\left\{\hat{\Lambda}^{M_1\dots M_{k-2}}, [\mathcal{D}^{++}, \partial_{M_{k-2}}\dots\partial_{M_1}]\right\}_{AGB} = 0 \tag{9.1}$$

  The solutions of these equations (modulo possible terms vanishing after integrations by parts) yield rigid higher-order conformal symmetries of the free hypermultiplet. It would be of significant interest to study the algebra of the corresponding group variations.

  Using $\mathcal{N}=2$ superfield Noether theorem or directly varying the cubic interactions with respect to the superfield gauge potentials, one can derive the conserved supercurrents for these symmetries. As one of the instructive examples one could construct the "master" current superfields, discussed in [45] and briefly sketched here. An interesting task is to study the component current expansion of the supercurrents obtained. Another important problem is the study of the superconformal transformation laws of the current superfields.

- *Induced actions*

  Finding out the manifestly $\mathcal{N}=2$ superconformal interaction vertex for the hypermultiplet coupled to external gauge higher spin superfields opens a principal possibility to study the higher spin quantum effects in such a theory. One of the topical problems in this area is the one-loop effective action of a higher-spin gauge field induced by its interaction with a lower-spin quantum field. For the explicit construction of such an effective action, there exists a general procedure going back to Schwinger and DeWitt and based upon the representation of the effective action as an integral over the proper time (see, e.g., [71]). In general, the induced effective action is essentially non-local. However, it can be perturbatively calculated as a series in the background field derivatives, which makes it possible to obtain various local invariants as functionals of the background gauge fields. In the context of the theory of higher spin fields, this opens up the possibility to find out, by direct algorithmic calculations, new invariants depending on the higher spin gauge fields. To the best of our knowledge, the study of the induced effective action in the conformal theory of higher spin fields was initiated in refs [33] in the world-line approach (see also the later paper [31]). In the theory of the lower-spin fields cubically coupled to conformal gauge higher spin fields, some approaches to the problem of calculating the induced effective actions were worked out in refs. [30], [72], [73], [32], [74] including the superfield approaches for $\mathcal{N} = 1$ and $\mathcal{N} = 2$ supersymmetric higher-spin theories formulated in $\mathcal{N} = 1$ superspace [32], [74]. In the present paper, we have constructed the manifestly $\mathcal{N} = 2$ superconformally invariant cubic interaction vertex for a hypermultiplet coupled to $\mathcal{N} = 2$ higher spin gauge superfields. This makes it possible to develop the manifestly $\mathcal{N} = 2$ supersymmetric proper time technique and use it to calculate the induced effective action depending on $\mathcal{N} = 2$ higher-spin gauge superfields treated as classical external superfields. In other words, knowing the explicit expressions for the general superfield coupling of the hypermultiplet to the $\mathcal{N} = 2$ higher-spin superconformal gauge potentials could help to find out the invariant Lagrangians of the latter.

- *Higher-spin conformal compensators*

  $\mathcal{N} = 2$ supersymmetric extension of Fronsdal theory constructed in [44] generalized merely one of the available versions of $\mathcal{N} = 2$ Einstein supergravity. An important question is how to construct the higher-spin generalization of other versions of $\mathcal{N} = 2$ supergravity. It is well known that the most general set of distinct versions of Einstein (super)gravity can be obtained by making use of the method of (super)conformal compensators. It is of primary interest to learn what is a generalization of this compensator mechanism to higher $\mathcal{N} = 2$ spins[23]. To answer this question it is necessary, first of all, to explore the issue of quartic interacting conformal vertices. The severe restrictions imposed by extended supersymmetry and harmonic superspace methods could greatly simplify the problem of constructing such vertices[24]. On the other hand, the generic matter conformal compensator for $\mathcal{N} = 2$ supergravity is just the massless hypermultiplet with the wrong sign of kinetic term (plus vector $\mathcal{N} = 2$ compensator with the analogous "wrong" sign of the kinetic term) [51, 57][25]. So there naturally

[23]Few earlier ideas regarding conformal compensators for higher spins were adduced in [33].
[24]One of the possible sources of such vertices in the HSS approach was addressed in a recent paper [48].
[25]The relevant off-shell version of Einstein $\mathcal{N} = 2$ supergravity was dubbed "principal version" in [51, 57];

emerges the problem of extending this picture to higher-spin $\mathcal{N}=2$ supergravity. It is obvious in advance that, in order to recover the hypermultiplet coupling of ref. [46], one needs to start with a conformal system involving at least two independent hypermultiplet superfields, one being a compensator.

- *AdS background*

  One more actual problem is to develop a similar formalism for $\mathcal{N}=2$ higher spins in the AdS and other conformally flat backgrounds. Since the super AdS group is a subgroup of $\mathcal{N}=2$ superconformal group, we hope that such a problem can be attacked, based largely on the results of the present work.

- *Construction of more general interactions*

  An important task is to generalize supercurrents and cubic vertices constructed here for hypermultiplets to the more general cases of interaction with other matter $\mathcal{N}=2$ multiplets, e.g., with $\mathcal{N}=2$ Maxwell multiplet (massless or massive). We hope to tackle this task (closely related also to the issue of conformal compensators) elsewhere.

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A Wess-Zumino gauge for superconformal spin 3

In this appendix, we expound how to fix the Wess-Zumino gauges for the spin **3** analytic potentials h^{++MN}. The relevant linearized gauge transformations are collected in (6.26). We show that one can fix gauge in such a way that all superfields, except their subset $h^{++M\alpha\dot{\alpha}}$, are gauged away. Then we deduce the Wess-Zumino form of the residual gauge potentials and find out the irreducible off-shell component content of the $\mathbf{s}=3$ $\mathcal{N}=2$ gauge multiplet.

A.1 Fixing "harmonic" freedom

As the first important step, consider the analytic superfield $h^{(+n)K}$ with the following gauge freedom:

$$\delta_\lambda h^{(+n)K} = \mathcal{D}^{++}\lambda^{(+(n-2))K}. \tag{A.1}$$

Here K is an arbitrary multi-index, $\lambda^{(+(n-2))K}$ is an unconstrained analytic superfield parameter. Terms of just this type appear in the transformation laws of all gauge potentials, see (6.26). Once this gauge freedom is fixed, we can inspect contributions of other terms.

it is the only one which admits the most general $\mathcal{N}=2$ matter off-shell couplings.

The generic component expansions of $h^{(+n)K}(\zeta)$ and $\lambda h^{(+n)K}$ read, respectively,

$$
\begin{aligned}
h^{(+n)K}(\zeta) =& A^{(+n)K} + \theta^{+\hat{\rho}} B^{(+(n-1))K}_{\hat{\rho}} \\
&+ (\theta^+)^2 C_1^{(+(n-2))K} + (\bar{\theta}^+)^2 C_2^{(+(n-2))K} + \theta^{+\alpha}\bar{\theta}^{+\dot{\alpha}} C^{(+(n-2))K}_{\alpha\dot{\alpha}} \\
&+ (\theta^+)^2 \bar{\theta}^{+\dot{\alpha}} D^{(+(n-3))K}_{\dot{\alpha}} + (\bar{\theta}^+)^2 \theta^{+\alpha} D^{(+(n-3))K}_{\alpha} + (\theta^+)^4 E^{(+(n-4))K},
\end{aligned}
\tag{A.2}
$$

$$
\begin{aligned}
\lambda^{(+(n-2))K}(\zeta) =& a^{(+(n-2))K} + \theta^{+\hat{\rho}} b^{(+(n-3))K}_{\hat{\rho}} \\
&+ (\theta^+)^2 c_1^{(+(n-4))K} + (\bar{\theta}^+)^2 c_2^{(+(n-4))K} + \theta^{+\alpha}\bar{\theta}^{+\dot{\alpha}} c^{(+(n-4))K}_{\alpha\dot{\alpha}} \\
&+ (\theta^+)^2 \bar{\theta}^{+\dot{\alpha}} d^{(+(n-5))K}_{\dot{\alpha}} + (\bar{\theta}^+)^2 \theta^{+\alpha} d^{(+(n-5))K}_{\alpha} + (\theta^+)^4 e^{(+(n-6))K}.
\end{aligned}
\tag{A.3}
$$

The coefficients $A, B\dots$ and $a, b\dots$ are arbitrary x-dependent harmonic functions with the properly fixed harmonic charges.

The result of action of the partial harmonic derivative ∂^{++} on (A.3) is as follows:

$$
\begin{aligned}
\partial^{++}\lambda^{(+(n-2))K}(\zeta) =& \partial^{++} a^{(+(n-2))K} + \theta^{+\hat{\rho}} \partial^{++} b^{(+(n-3))K}_{\hat{\rho}} \\
&+ (\theta^+)^2 \partial^{++} c_1^{(+(n-4))K} + (\bar{\theta}^+)^2 \partial^{++} c_2^{(+(n-4))K} \\
&+ \theta^{+\alpha}\bar{\theta}^{+\dot{\alpha}} \partial^{++} c^{(+(n-4))K}_{\alpha\dot{\alpha}} \\
&+ (\theta^+)^2 \bar{\theta}^{+\dot{\alpha}} \partial^{++} d^{(+(n-5))K}_{\dot{\alpha}} + (\bar{\theta}^+)^2 \theta^{+\alpha} \partial^{++} d^{(+(n-5))K}_{\alpha} \\
&+ (\theta^+)^4 \partial^{++} e^{(+(n-6))K}.
\end{aligned}
\tag{A.4}
$$

In this expression, the harmonic derivative produces general harmonic functions if the charge of the corresponding function ≥ 0. Then, for the harmonic charges with $n \geq 5$, one can gauge away all the components by the gauge transformations (A.1). For $n = 4$, one cannot gauge away by this mechanism the highest component in the harmonic expansion of E^K , for $n = 3$ those in the expansion of $D^K_{\hat{\alpha}}, E^K$, and so forth.

The corresponding residual gauge freedom is specified by the lowest components of the $\lambda^{(+(n-2))K}(\zeta)$ coefficients with the positive harmonic charge. For example, in $n = 3$ case these parameters are a^{iK}, b^K. Due to the presence of the term with x-derivative in $\mathcal{D}^{++}$, these surviving parameters (with derivatives on them) can appear in the transformations of some other non-vanishing components. Also, the appropriate contributions from the terms with explicit θ s in (6.26) can modify the residual gauge transformations and ensure some additional gauge conditions. All these subtleties can be uniquely fixed from the condition of preserving the final Wess-Zumino type gauges.

Now we can proceed to the precise discussion of the gauge-fixing procedure for the superconformal spin **3** potentials. Using merely terms with harmonic derivatives, and based on the reasoning around eqs. (A.3) - (A.4), we can partially fix the gauge as:

$$
\begin{aligned}
h^{++\alpha\dot\alpha\beta\dot\beta} &= i(\theta^+)^2 C^{\alpha\dot\alpha\beta\dot\beta} - i(\bar\theta^+)^2 \bar C^{\alpha\dot\alpha\beta\dot\beta} - 4i\theta^{+\rho}\bar\theta^{+\dot\rho}\Phi^{\alpha\dot\alpha\beta\dot\beta}_{\rho\dot\rho} \\
&\quad + (\bar\theta^+)^2\theta^{+\rho}\psi^{\alpha\dot\alpha\beta\dot\beta i}_{\rho}u^-_i + (\theta^+)^2\bar\theta^{+\dot\rho}\bar\psi^{\alpha\dot\alpha\beta\dot\beta i}_{\dot\rho}u^-_i + (\theta^+)^4 V^{\alpha\dot\alpha\beta\dot\beta ij}u^-_i u^-_j\,, \\
h^{++\alpha\dot\alpha\beta+} &= (\theta^+)^2\bar\theta^{+\dot\rho}P^{\alpha\dot\alpha\beta}_{\dot\rho} + (\bar\theta^+)^2\theta^{+\rho}T^{\alpha\dot\alpha\beta}_{\rho} + (\theta^+)^4\chi^{\alpha\dot\alpha\beta i}u^-_i\,, \\
h^{++\alpha\dot\alpha++} &= (\theta^+)^4 D^{\alpha\dot\alpha}, \\
h^{++[\alpha+\beta]+} &= (\theta^+)^4 K^{[\alpha\beta]}, \\
h^{++\alpha+\dot\alpha+} &= i(\theta^+)^4 K^{\alpha\dot\alpha}, \\
h^{++\hat\alpha+++} &= 0, \\
h^{(+6)} &= 0.
\end{aligned}
\tag{A.5}
$$

The reality conditions for the involved fields can be figured out from the generalized reality conditions for the analytic gauge potentials (6.7). The ultimate effect of the shift transformations (6.26) on the component fields in (A.5) can be determined by considering separately various sectors. Using these transformations, one can find the transformation laws of the remaining fields and learn which fields survive after the WZ gauges have been completely fixed (up to the residual gauge transformations involving only x-derivatives of the relevant parameters).

A.2 Further gauge-fixing

Inspecting the transformations (6.26) more carefully, we found that, besides the primary gauge-fixing (A.5), based on the general properties of the harmonic expansions, the further steps of the gauge fixing can be effected, which become possible due to the presence of the explicit θs in (6.26). Namely, it is self-consistent to put

$$
\hat h^{(+4)} = 0\,, \quad \hat h^{++} = 0\,, \quad \hat h^{+3\hat\alpha} = 0\,, \quad h^{++\alpha+\dot\beta+} = 0 \;\; \text{(and c.c.)}\,, \tag{A.6}
$$

where

$$
\begin{aligned}
&\hat h^{++} := \epsilon_{\alpha\beta}\epsilon_{\dot\alpha\dot\beta}h^{++\alpha\dot\alpha\,\beta\dot\beta}\,,\; \hat h^{+3\dot\alpha} := \epsilon_{\alpha\beta}h^{++\alpha+\beta\dot\alpha}, \\
&\bar{\hat h}^{+3\alpha} = \epsilon_{\dot\alpha\dot\beta}h^{++\dot\alpha+\alpha\dot\beta},\; \hat h^{(+4)} := \frac{1}{2}\epsilon_{\beta\alpha}h^{++[\beta+\alpha]+} \quad \text{(and c.c.)}\,.
\end{aligned}
\tag{A.7}
$$

Then all physical fields are contained in the remaining parts of the original gauge potentials

$$
h^{++(\alpha\,\beta)(\dot\alpha\,\dot\beta)}\,, \quad h^{++(\alpha+\beta)\dot\alpha}\,, \quad h^{++(\dot\alpha+\dot\beta)\alpha}\,, \quad h^{++\alpha\dot\alpha++}\,. \tag{A.8}
$$

Requiring the gauges (A.6) and the last two gauges in (A.5) to be preserved under the general linearized gauge transformations (6.26) impose the following constraints on the relevant residual analytic gauge parameters:

$$h^{(+6)} = 0 \quad \Longrightarrow \quad \mathcal{D}^{++}\lambda^{(+4)} = 0\,, \tag{A.9}$$
$$h^{+5\hat{\beta}} = 0 \quad \Longrightarrow \quad \mathcal{D}^{++}\lambda^{+3\hat{\beta}} - 2\lambda^{(+4)}\theta^{+\hat{\beta}} = 0\,, \tag{A.10}$$
$$\hat{h}^{(+4)} = 0 \quad \Longrightarrow \quad \mathcal{D}^{++}\hat{\lambda}^{+2} - \lambda^{+3\alpha}\theta^+_\alpha = 0\,, \quad \text{and c.c.}\,, \tag{A.11}$$
$$h^{++\beta+\dot{\gamma}+} = 0 \quad \Longrightarrow \quad \mathcal{D}^{++}\lambda^{\beta+\dot{\gamma}+} - \frac{1}{2}\big(\lambda^{+3\beta}\bar{\theta}^{+\dot{\gamma}} - \lambda^{+3\dot{\gamma}}\theta^{+\beta}\big) = 0\,, \tag{A.12}$$
$$\hat{h}^{+3\dot{\alpha}} = 0 \quad \Longrightarrow \quad \mathcal{D}^{++}\hat{\lambda}^{+\dot{\alpha}} + 8i\hat{\lambda}^{+2}\bar{\theta}^{+\dot{\alpha}} + \hat{\lambda}^{++\alpha\dot{\alpha}}\theta^+_\alpha = 0\,, \quad \text{and c.c.}\,, \tag{A.13}$$
$$\hat{h}^{++} = 0 \quad \Longrightarrow \quad \mathcal{D}^{++}\hat{\lambda} - 4i\hat{\lambda}^{+\dot{\beta}}\bar{\theta}^+_{\dot{\beta}} + 4i\hat{\lambda}^{+\beta}\theta^+_\beta = 0\,, \quad \text{and c.c.}\,, \tag{A.14}$$

where

$$\hat{\lambda} := \epsilon_{\alpha\beta}\epsilon_{\dot{\alpha}\dot{\beta}}\lambda^{\alpha\beta\dot{\alpha}\dot{\beta}}, \quad \hat{\lambda}^{+\dot{\beta}} := \epsilon_{\alpha\beta}\lambda^{\alpha+\beta\dot{\beta}}\,\text{and c.c.}\,, \quad \hat{\lambda}^{+2} := \epsilon_{\alpha\beta}\lambda^{[\alpha+\beta]+} \quad \text{and c.c.}\,,$$
$$\hat{\lambda}^{++\alpha\dot{\alpha}} := \lambda^{++\alpha\dot{\alpha}} - 8i\lambda^{\alpha+\dot{\alpha}+}\,. \tag{A.15}$$

The gauge transformations of the "physical" set (A.8) are given by

$$\delta h^{++(\alpha\beta)(\dot{\alpha}\dot{\beta})} = \mathcal{D}^{++}\lambda^{(\alpha\beta)(\dot{\alpha}\dot{\beta})} + 4i\big(\lambda^{(\beta+\alpha)(\dot{\alpha}}\bar{\theta}^{+\dot{\beta})} - \lambda^{(\dot{\alpha}+\dot{\beta})(\beta}\theta^{+\alpha)}\big), \tag{A.16}$$
$$2\delta h^{++(\beta+\alpha)\dot{\alpha}} = \mathcal{D}^{++}\lambda^{(\alpha+\beta)\dot{\alpha}} - \theta^{+(\alpha}\big(\lambda^{\beta)\dot{\alpha}++} + 8i\lambda^{\beta)+\dot{\alpha}+}\big)\,, \quad \text{and c.c.}\,, \tag{A.17}$$
$$2\delta h^{++\alpha\dot{\alpha}++} = \mathcal{D}^{++}\lambda^{++\alpha\dot{\alpha}} + 4i\big(\lambda^{+3\alpha}\bar{\theta}^{+\dot{\alpha}} - \lambda^{+3\dot{\alpha}}\theta^{+\alpha}\big). \tag{A.18}$$

We observe that the eqs. (A.9) - (A.13) involve some gauge parameters which appear also in (A.16) - (A.18). So we need first to fully exhibit the consequences of (A.9) - (A.12).

A.3 Bosonic sector

We will start from the bosonic sector.

For what follows we will need the component structure of the analytic gauge parameters. Firstly we present it for the gauge parameters associated with the pure gauge potentials appearing in (A.9) - (A.14):

$$\lambda^{(+4)} = \ell^{+4} + \ell^{+2}(\theta^+)^2 + \bar{\ell}^{+2}(\bar{\theta}^+)^2 + \ell^{+2\alpha\dot{\alpha}}\theta^+_\alpha\bar{\theta}^+_{\dot{\alpha}} + \ell(\theta^+)^4,$$
$$\lambda^{+3\alpha} = \mu^{+2\alpha}_\beta\theta^{+\beta} + \mu^{+2\alpha}_{\dot{\beta}}\bar{\theta}^{+\dot{\beta}} + \gamma^\alpha_\beta(\bar{\theta}^+)^2\theta^{+\beta} + \gamma^\alpha_{\dot{\beta}}(\theta^+)^2\bar{\theta}^{+\dot{\beta}}\,,$$
$$\hat{\lambda}^{+2} = \sigma^{+2} + \sigma(\theta^+)^2 + \sigma'(\bar{\theta}^+)^2 + \sigma^{\alpha\dot{\alpha}}_1\theta^+_\alpha\bar{\theta}^+_{\dot{\alpha}} + \sigma^{-2}_2(\theta^+)^4,$$
$$\lambda^{\alpha+\dot{\beta}+} = \psi^{++\alpha\dot{\beta}} + \psi^{\alpha\dot{\beta}}_1(\theta^+)^2 - \bar{\psi}^{\alpha\dot{\beta}}_1(\bar{\theta}^+)^2 + \psi^{\alpha\dot{\beta}\gamma\dot{\gamma}}_2\theta^+_\gamma\bar{\theta}^+_{\dot{\gamma}} + \psi^{-2\alpha\dot{\beta}}_3(\theta^+)^4. \tag{A.19}$$
$$\hat{\lambda}^{+\alpha} = \nu^\alpha_\gamma\theta^{+\gamma} + \nu^\alpha_{\dot{\gamma}}\bar{\theta}^{+\dot{\gamma}} + \phi^{-2\alpha}_\gamma(\bar{\theta}^+)^2\theta^{+\gamma} + \phi^{-2\alpha}_{\dot{\gamma}}(\theta^+)^2\bar{\theta}^{+\dot{\gamma}}\,,$$
$$\hat{\lambda} = \beta + \beta^{-2}(\theta^+)^2 + \bar{\beta}^{-2}(\bar{\theta}^+)^2 + \beta^{-2\gamma\dot{\gamma}}\theta^+_\gamma\bar{\theta}^+_{\dot{\gamma}} + \beta^{-4}(\theta^+)^4\,. \tag{A.20}$$

The analogous expansions for the remaining superfield gauge parameters read

$$\lambda^{(\alpha\beta)(\dot{\alpha}\dot{\beta})} = \rho^{(\alpha\beta)(\dot{\alpha}\dot{\beta})} + \big[\rho^{-2(\alpha\beta)(\dot{\alpha}\dot{\beta})}_1(\theta^+)^2 + \text{c.c.}\big] + \rho^{-2(\alpha\beta)(\dot{\alpha}\dot{\beta})\gamma\dot{\gamma}}_2\theta^+_\gamma\bar{\theta}^+_{\dot{\gamma}}$$
$$+\,\rho^{-4(\alpha\beta)(\dot{\alpha}\dot{\beta})}_3(\theta^+)^4, \tag{A.21}$$
$$\lambda^{(\beta+\alpha)\dot{\beta}} = \omega^{(\alpha\beta)\dot{\beta}\gamma}\theta^+_\gamma + \omega^{(\alpha\beta)\dot{\beta}\dot{\gamma}}\bar{\theta}^+_{\dot{\gamma}} + \omega^{-2(\alpha\beta)\dot{\beta}\gamma}_1(\bar{\theta}^+)^2\theta^+_\gamma + \omega^{-2(\alpha\beta)\dot{\beta}\dot{\gamma}}_2(\theta^+)^2\bar{\theta}^+_{\dot{\gamma}}\,, \tag{A.22}$$
$$\lambda^{+2\alpha\dot{\beta}} = \chi^{+2\alpha\dot{\beta}} + \chi^{\alpha\dot{\beta}}_1(\theta^+)^2 + \bar{\chi}^{\alpha\dot{\beta}}_1(\bar{\theta}^+)^2 + \chi^{\alpha\dot{\beta}\gamma\dot{\gamma}}_2\theta^+_\gamma\bar{\theta}^+_{\dot{\gamma}} + \chi^{-2\alpha\dot{\beta}}_3(\theta^+)^4. \tag{A.23}$$

The conjugation rules for the component gauge parameters follow from the superfield ones listed earlier.

Eqs. (A.9) - (A.12) yield the following restrictions on the four sets of the component parameters in (A.19):

$$
\begin{aligned}
&(a)\ \ell^{+4} = \ell^{+4}_{(0)},\ \ell^{+2} = \ell^{+2}_{(0)},\ \ell^{+2\alpha\dot\alpha} = \ell^{+2\alpha\dot\alpha}_{(0)} + 4i\,\partial^{\alpha\dot\alpha}\ell^{+3-}_{(0)},\\
&(b)\ \ell = \ell_{(0)} + i\partial_{\alpha\dot\alpha}\ell^{+-\alpha\dot\alpha}_{(0)} - 2\Box\ell^{+2-2}_{(0)},
\end{aligned} \tag{A.24}
$$

$$
\begin{aligned}
&(a)\ \mu^{+2\beta}_{\alpha} = \mu^{+2\beta}_{\alpha}{}_{(0)} + 2\delta^{\beta}_{\alpha}\ell^{+3-}_{(0)}\,,\ \mu^{+2\beta}_{\dot\alpha} = \mu^{+2\beta}_{\dot\alpha}{}_{(0)}\,,\\
&\gamma^{\alpha}_{\beta} = \gamma^{\alpha}_{\beta\,(0)} - 2i\,\partial^{\dot\alpha}_{\alpha}\,\mu^{+-\beta}_{\dot\alpha}{}_{(0)} + 2\,\delta^{\alpha}_{\beta}\ell^{+-}_{(0)}\,,\\
&(b)\ \gamma^{\beta}_{\dot\alpha} = \gamma^{\beta}_{\dot\alpha\,(0)} - 2i\,\partial^{\alpha}_{\dot\alpha}\,\mu^{+-\beta}_{\alpha}{}_{(0)} - \ell^{+-\beta}_{\dot\alpha}{}_{(0)} - 4i\,\partial^{\beta}_{\dot\alpha}\,\ell^{+2-2}_{(0)}
\end{aligned} \tag{A.25}
$$

$$
\begin{aligned}
&(a)\ \sigma^{+2} = \sigma^{+2}_{(0)}\,,\ \sigma' = \sigma'_{(0)}\,,\ \sigma = \sigma_{(0)} + \frac{1}{2}\mu^{+-\beta}_{\beta(0)} + \ell^{+2-2}_{(0)}\,,\\
&(b)\ \sigma^{\gamma\dot\gamma}_{1} = \sigma^{\gamma\dot\gamma}_{1(0)} + 4i\,\partial^{\gamma\dot\gamma}\,\sigma^{+-}_{(0)} + \mu^{+-\gamma\dot\gamma}_{(0)}\\
&(c)\ \sigma^{-2}_{2} = -2\Box\sigma^{-2}_{(0)} + \bar\ell^{-2}_{(0)} + i\partial_{\alpha\dot\alpha}\mu^{-2\alpha\dot\alpha}_{(0)}\,,\quad (d)\ \gamma^{\alpha}_{\alpha\,(0)} = -2i\partial_{\gamma\dot\gamma}\,\sigma^{\gamma\dot\gamma}_{1(0)}\,,
\end{aligned} \tag{A.26}
$$

$$
\begin{aligned}
&(a)\ \psi^{+2\alpha\dot\alpha} = \psi^{+2\alpha\dot\alpha}_{(0)}\,,\ \psi^{\alpha\dot\alpha}_{1} = \psi^{\alpha\dot\alpha}_{1(0)} + \frac{1}{4}\bar\mu^{+-\alpha\dot\alpha}_{(0)}\,,\\
&(b)\ \psi^{\alpha\dot\beta\gamma\dot\gamma} = \psi^{\alpha\dot\beta\gamma\dot\gamma}_{(0)} + 4i\partial^{\gamma\dot\gamma}\,\psi^{+-\alpha\dot\beta}_{(0)} + \frac{1}{2}\varepsilon^{\rho\gamma}\varepsilon^{\dot\beta\dot\gamma}[\mu^{+-\alpha}_{\rho(0)} + \ell^{+2-2}_{(0)}\delta^{\alpha}_{\rho}]\\
&\qquad + \frac{1}{2}\varepsilon^{\alpha\gamma}\varepsilon^{\dot\alpha\dot\gamma}[\bar\mu^{+-\dot\beta}_{\dot\alpha(0)} + \ell^{+2-2}_{(0)}\delta^{\dot\beta}_{\dot\alpha}],\\
&(c)\ \psi^{-2\alpha\dot\alpha}_{3} = -2\Box\psi^{-2\alpha\dot\alpha}_{(0)} + \frac{i}{2}[\partial^{\rho\dot\alpha}\,\mu^{-2\alpha}_{\rho(0)} + \partial^{\alpha\dot\beta}\bar\mu^{-2\dot\alpha}_{\dot\beta(0)}] + \frac{1}{4}\ell^{-2\alpha\dot\alpha}_{(0)} + i\partial^{\alpha\dot\alpha}\ell^{-3+}_{(0)}\,,\\
&(d)\ \gamma^{\alpha\dot\beta}_{(0)} - \bar\gamma^{\alpha\dot\beta}_{(0)} = 4i\partial_{\gamma\dot\gamma}\psi^{\alpha\dot\beta\gamma\dot\gamma}_{2(0)}\,.
\end{aligned} \tag{A.27}
$$

Analogously, eqs. (A.13), (A.14) yield, for the component parameters in (A.20),

$$
\begin{aligned}
&(a)\ \nu^{\alpha}_{\gamma} = \nu^{\alpha}_{\gamma(0)} + 8i\delta^{\alpha}_{\gamma}\,\bar\sigma^{+-}_{(0)}\,,\ \ \nu^{\alpha}_{\dot\gamma} = \nu^{\alpha}_{\dot\gamma(0)} + \left[\chi^{+-\alpha}_{\dot\gamma(0)} - 8i\,\psi^{+-\alpha}_{\dot\gamma(0)}\right],\\
&(b)\ \phi^{-2\alpha}_{\gamma} = -2i\partial^{\dot\gamma}_{\gamma}\left[\chi^{-2\alpha}_{\dot\gamma(0)} - 8i\,\psi^{-2\alpha}_{\dot\gamma(0)}\right] - 4i\mu^{-2\alpha}_{\gamma(0)} - \frac{8i}{3}\,\delta^{\alpha}_{\gamma}\,\ell^{-3+}_{(0)},\\
&\quad \phi^{-2\alpha\dot\gamma} = 16\,\partial^{\alpha\dot\gamma}\,\bar\sigma^{-2}_{(0)}\,,\\
&(c)\ \chi^{\alpha\dot\gamma}_{1(0)} - 8i\,\psi^{\alpha\dot\gamma}_{1(0)} = 2i\,\partial^{\gamma\dot\gamma}\,\nu^{\alpha}_{\gamma(0)} - 4i\,\bar\sigma^{\alpha\dot\gamma}_{1(0)}\,,
\end{aligned} \tag{A.28}
$$

$$
\begin{aligned}
&(a)\ \beta = \beta_{(0)}\,,\quad \beta^{-2} = 16\bar\sigma^{-2}_{(0)}\,,\quad \nu^{\alpha}_{\alpha(0)} = \bar\nu^{\dot\alpha}_{\dot\alpha(0)} = 0\,,\\
&(b)\ \beta^{-2\gamma\dot\gamma} = -4i\left[\chi^{-2\gamma\dot\gamma}_{(0)} - 8i\,\psi^{-2\gamma\dot\gamma}_{(0)}\right],\quad \nu^{\gamma\dot\gamma}_{(0)} + \bar\nu^{\gamma\dot\gamma}_{(0)} = \partial^{\gamma\dot\gamma}\,\beta_{(0)}\,,\\
&(c)\ \beta^{-4} = -\frac{16}{3}\ell^{-4}_{(0)},\quad \mu^{-2\alpha}_{\alpha(0)} + \bar\mu^{-2\dot\alpha}_{\dot\alpha(0)} = \frac{3}{2}\,\partial_{\gamma\dot\gamma}\left[\chi^{-2\gamma\dot\gamma}_{(0)} - 8i\,\psi^{-2\gamma\dot\gamma}_{(0)}\right].
\end{aligned} \tag{A.29}
$$

Hereafter, the suffix (0) denotes the lowest-order terms in the relevant harmonic expansions.

Thus we have shown that plenty of the superconformal gauge potentials, including $h^{(+6)}$ and $h^{++\hat\alpha+++}$ in (A.5) and those in (A.6), can be completely gauged away and we are left with the restricted set of the analytic superfield potentials which eventually encompass the irreducible $\mathbf{s} = 3$ multiplet. In the bosonic sector, these basic gauge potentials have the following component expansions (before passing to the partly gauge-fixed form (A.5)):

$$\begin{aligned} h^{++(\alpha\beta)(\dot\alpha\dot\beta)} &= h_0^{++(\alpha\beta)(\dot\alpha\dot\beta)} + \left[h_1^{(\alpha\beta)(\dot\alpha\dot\beta)}(\theta^+)^2 + \text{c.c.}\right] + h_2^{(\alpha\beta)(\dot\alpha\dot\beta)\gamma\dot\gamma}\theta^+_\gamma\bar\theta^+_{\dot\gamma} \\ &\quad + h_3^{-2(\alpha\beta)(\dot\alpha\dot\beta)}(\theta^+)^4, \qquad\qquad (A.30) \\ h^{++(\beta+\alpha)\dot\alpha} &= h_{1\gamma}^{+2(\beta\alpha)\dot\alpha}\theta^{+\gamma} + h_{2\dot\gamma}^{+2(\beta\alpha)\dot\alpha}\bar\theta^{+\dot\gamma} + h_{3\gamma}^{(\beta\alpha)\dot\alpha}(\bar\theta^+)^2\theta^{+\gamma} \\ &\quad + h_{4\dot\gamma}^{(\beta\alpha)\dot\alpha}(\theta^+)^2\bar\theta^{+\dot\gamma}, \qquad\qquad (A.31) \\ h^{+4\alpha\dot\alpha} &= h_0^{+4\alpha\dot\alpha} + h_1^{+2\alpha\dot\alpha}(\theta^+)^2 + \bar h_1^{+2\alpha\dot\alpha}(\bar\theta^+)^2 + h_2^{+2\alpha\dot\alpha\gamma\dot\gamma}\theta^+_\gamma\bar\theta^+_{\dot\gamma} + h_3^{\alpha\dot\alpha}(\theta^+)^4. \qquad (A.32) \end{aligned}$$

The component fields of $h^{++(\alpha\beta)(\dot\alpha\dot\beta)}$ have the following gauge transformation laws:

$$\begin{aligned} \delta h_0^{++(\alpha\beta)(\dot\alpha\dot\beta)} &= \partial^{++}\rho^{(\alpha\beta)(\dot\alpha\dot\beta)}, \\ \delta h_1^{(\alpha\beta)(\dot\alpha\dot\beta)} &= \partial^{++}\rho_1^{-2(\alpha\beta)(\dot\alpha\dot\beta)} - 2i\,\bar\omega_{(0)}^{(\alpha\beta)(\dot\alpha\dot\beta)}, \text{ (and c.c.)}, \qquad (A.33) \\ \delta h_2^{(\alpha\beta)(\dot\alpha\dot\beta)\gamma\dot\gamma} &= \partial^{++}\rho_2^{-2(\alpha\beta)(\dot\alpha\dot\beta)\gamma\dot\gamma} - 4i\,\partial^{\gamma\dot\gamma}\rho^{(\alpha\beta)(\dot\alpha\dot\beta)} \\ &\quad + 4i\left[\omega^{(\alpha\beta)\gamma(\dot\alpha}\varepsilon^{\dot\beta)\dot\gamma} + \bar\omega^{(\dot\alpha\dot\beta)\dot\gamma(\alpha}\varepsilon^{\beta)\gamma}\right], \qquad (A.34) \\ \delta h_3^{-2(\alpha\beta)(\dot\alpha\dot\beta)} &= \partial^{++}\rho_3^{-4(\alpha\beta)(\dot\alpha\dot\beta)} - i\,\partial_{\gamma\dot\gamma}\rho_2^{-2(\alpha\beta)(\dot\alpha\dot\beta)\gamma\dot\gamma} \\ &\quad + 2i\left[\omega_2^{-2(\alpha\beta)(\dot\alpha\dot\beta)} - \bar\omega_2^{-2(\alpha\beta)(\dot\alpha\dot\beta)}\right]. \qquad (A.35) \end{aligned}$$

Using eqs.(A.33), one can impose the gauges

$$\begin{aligned} &h_0^{++(\alpha\beta)(\dot\alpha\dot\beta)} = 0 \;\Rightarrow\; \rho^{(\alpha\beta)(\dot\alpha\dot\beta)} = \rho_{(0)}^{(\alpha\beta)(\dot\alpha\dot\beta)}, \\ &h_1^{(\alpha\beta)(\dot\alpha\dot\beta)} = 0\,, \;\Rightarrow\; \rho_1^{-2(\alpha\beta)(\dot\alpha\dot\beta)} = 0\,,\; \omega_{(0)}^{(\alpha\beta)(\dot\alpha\dot\beta)} = 0\,, \;\Rightarrow\; \omega_{(0)\;\dot\beta}^{(\alpha\beta)\dot\alpha} = \frac{1}{2}\,\delta^{\dot\alpha}_{\dot\beta}\,\omega_{(0)}^{(\alpha\beta)}. \qquad (A.36) \end{aligned}$$

The analysis of consequences of eqs. (A.34) and (A.35) requires more effort. First of all, we need the gauge transformations of the component fields in the other two superfield potentials (A.32) and (A.31)

$$\begin{aligned} 2\delta h_0^{+4\alpha\dot\alpha} &= \partial^{++}\chi^{+2\alpha\dot\alpha}, \qquad (A.37) \\ 2\delta h_1^{+2\alpha\dot\alpha} &= \partial^{++}\chi_1^{\alpha\dot\alpha} + 2i\,\bar\mu^{+2\alpha\dot\alpha}, \qquad (A.38) \\ 2\delta\bar h_1^{+2\alpha\dot\alpha} &= \partial^{++}\bar\chi_1^{\alpha\dot\alpha} - 2i\,\mu^{+2\alpha\dot\alpha}, \qquad (A.39) \\ 2\delta h_2^{+2\alpha\dot\alpha\gamma\dot\gamma} &= \partial^{++}\chi_2^{\alpha\dot\alpha\gamma\dot\gamma} - 4i\left[\partial^{\gamma\dot\gamma}\chi^{++\alpha\dot\alpha} - \varepsilon^{\beta\gamma}\varepsilon^{\dot\alpha\dot\gamma}\mu_\beta^{+2\alpha} - \varepsilon^{\alpha\gamma}\varepsilon^{\dot\beta\dot\gamma}\bar\mu_{\dot\beta}^{+2\dot\alpha}\right], \qquad (A.40) \\ 2\delta h_3^{\alpha\dot\alpha} &= \partial^{++}\chi_3^{-2\alpha\dot\alpha} - i\partial_{\gamma\dot\gamma}\chi_2^{\alpha\dot\alpha\gamma\dot\gamma} - 2i\left(\gamma^{\alpha\dot\alpha} - \bar\gamma^{\alpha\dot\alpha}\right), \qquad (A.41) \end{aligned}$$

$$\begin{aligned} 2\delta h_{1\gamma}^{+2(\beta\alpha)\dot\alpha} &= -\left[\partial^{++}\omega_\gamma^{(\alpha\beta)\dot\alpha} + \Sigma^{+2(\alpha\dot\alpha}\delta_\gamma^{\beta)}\right], \qquad (A.42) \\ 2\delta h_{2\dot\gamma}^{+2(\beta\alpha)\dot\alpha} &= -\partial^{++}\omega_{\dot\gamma}^{(\alpha\beta)\dot\alpha}, \qquad (A.43) \\ 2\delta h_3^{(\beta\alpha)\dot\alpha\gamma} &= -\left[\partial^{++}\omega_1^{-2(\alpha\beta)\dot\alpha\gamma} + 2i\partial^{\gamma\dot\gamma}\omega_{\dot\gamma}^{(\alpha\beta)\dot\alpha} - \bar\Sigma_1^{(\alpha\dot\alpha}\varepsilon^{\beta)\gamma}\right], \qquad (A.44) \\ 2\delta h_4^{(\beta\alpha)\dot\alpha\dot\gamma} &= -\left[\partial^{++}\omega_2^{-2(\alpha\beta)\dot\alpha\dot\gamma} + 2i\partial^{\gamma\dot\gamma}\omega_\gamma^{(\beta\alpha)\dot\alpha} - \frac{1}{2}\Sigma_2^{(\alpha\beta)\dot\alpha\dot\gamma}\right], \qquad (A.45) \end{aligned}$$

where

$$\Sigma^{++\alpha\dot\alpha} := \chi^{++\alpha\dot\alpha} + 8i\psi^{++\alpha\dot\alpha}\,,\; \Sigma_1^{\alpha\dot\alpha} := \chi_1^{\alpha\dot\alpha} + 8i\,\psi_1^{\alpha\dot\alpha}\,,\; \bar\Sigma_1^{\alpha\dot\alpha} = \bar\chi_1^{\alpha\dot\alpha} - 8i\,\bar\psi_1^{\alpha\dot\alpha}$$
$$\Sigma_2^{\alpha\dot\alpha\gamma\dot\gamma} := \chi_2^{\alpha\dot\alpha\gamma\dot\gamma} + 8i\,\psi_2^{\alpha\dot\alpha\gamma\dot\gamma} + 4i\partial^{\gamma\dot\gamma}\big[\chi^{+-\alpha\dot\alpha} + 8i\psi^{+-\alpha\dot\alpha}\big]. \tag{A.46}$$

We start with the analysis of (A.37) - (A.41). One observes that the fields $h_0^{+4\alpha\dot\alpha}$, $h_1^{+2\alpha\dot\alpha}$ and $h_2^{+2\alpha\dot\alpha\gamma\dot\gamma}$ can be completely gauged away:

$$h_0^{+4\alpha\dot\alpha} = 0 \;\Rightarrow\; \chi^{+2\alpha\dot\alpha} = \chi_{(0)}^{+2\alpha\dot\alpha}\,,\; h_1^{+2\alpha\dot\alpha} = 0 \;\Rightarrow\; \chi_1^{\alpha\dot\alpha} = \chi_{1(0)}^{\alpha\dot\alpha} - 2i\bar\mu_{(0)}^{+1\alpha\dot\alpha}\,,$$
$$h_2^{+2\alpha\dot\alpha\gamma\dot\gamma} = 0 \;\Rightarrow\; \chi_2^{\alpha\dot\alpha\gamma\dot\gamma} = \chi_{2(0)}^{\alpha\dot\alpha\gamma\dot\gamma} + 4i\big[\partial^{\gamma\dot\gamma}\chi_{(0)}^{+-\alpha\dot\alpha} - \varepsilon^{\beta\gamma}\varepsilon^{\dot\alpha\dot\gamma}\mu_{\beta(0)}^{+-\alpha} - \varepsilon^{\alpha\gamma}\varepsilon^{\dot\beta\dot\gamma}\bar\mu_{\dot\beta(0)}^{+-\dot\alpha}$$
$$-2\varepsilon^{\alpha\gamma}\varepsilon^{\dot\alpha\dot\gamma}\ell_{(0)}^{+2-2}\big]. \tag{A.47}$$

In this gauge we also have

$$\Sigma^{++\alpha\dot\alpha} = \Sigma_{(0)}^{++\alpha\dot\alpha} = \chi_{(0)}^{++\alpha\dot\alpha} + 8i\psi_{(0)}^{++\alpha\dot\alpha}\,,\; \Sigma_1^{\alpha\dot\alpha} = \Sigma_{1(0)}^{\alpha\dot\alpha} = \chi_{1(0)}^{\alpha\dot\alpha} + 8i\,\psi_{1(0)}^{\alpha\dot\alpha}\,,$$
$$\Sigma_2^{\alpha\dot\alpha\gamma\dot\gamma} = \Sigma_{2(0)}^{\alpha\dot\alpha\gamma\dot\gamma} + 4i\,\partial^{\gamma\dot\gamma}\Sigma_{(0)}^{+-\alpha\dot\alpha}\,,\quad \Sigma_{2(0)}^{\alpha\dot\alpha\gamma\dot\gamma} = \chi_{2(0)}^{\alpha\dot\alpha\gamma\dot\gamma} + 8i\,\psi_{2(0)}^{\alpha\dot\alpha\gamma\dot\gamma}, \tag{A.48}$$

where we made use of eq. (A.27b).

Finally, looking at $\delta h_3^{\alpha\dot\alpha}$ we find that it is possible to impose one more gauge,

$$h_3^{\alpha\dot\alpha} = h_{3(0)}^{\alpha\dot\alpha} = D^{\alpha\dot\alpha} \;\Rightarrow\; 2\,\delta D^{\alpha\dot\alpha} = -i\partial_{\gamma\dot\gamma}\chi_{2(0)}^{\alpha\dot\alpha\gamma\dot\gamma} - 2i\big[\gamma_{(0)}^{\alpha\dot\alpha} - \bar\gamma_{(0)}^{\alpha\dot\alpha}\big],$$
$$2\,\delta D^{\alpha\dot\alpha} = -i\partial_{\gamma\dot\gamma}\Omega^{\alpha\dot\alpha\gamma\dot\gamma}\,,\quad \Omega^{\alpha\dot\alpha\gamma\dot\gamma} := \Sigma_{2(0)}^{\alpha\dot\alpha\gamma\dot\gamma}\,, \tag{A.49}$$

where we used eq. (A.27d). Below we show that (A.49) amounts to the standard Maxwell gauge transformation for the properly redefined vector field $\tilde D^{\alpha\dot\alpha}$.

The next steps towards the eventual WZ gauge are based on the transformations (A.42) - (A.45). Eqs. (A.42), (A.43) imply the possibility to choose the gauge

$$h^{+2(\beta\alpha)\dot\alpha\gamma} = 0 \;\Rightarrow\; \omega^{(\beta\alpha)\dot\alpha\gamma} = \omega_{(0)}^{(\beta\alpha)\dot\alpha\gamma} + \Sigma_{(0)}^{+-(\alpha\dot\alpha}\varepsilon^{\beta)\gamma}\,, \tag{A.50}$$
$$h^{+2(\beta\alpha)\dot\alpha\dot\gamma} = 0 \;\Rightarrow\; \omega^{(\beta\alpha)\dot\alpha\dot\gamma} = \omega_{(0)}^{(\beta\alpha)\dot\alpha\dot\gamma} = \frac{1}{2}\varepsilon^{\dot\gamma\dot\alpha}\,\omega_{(0)}^{(\alpha\beta)}\,, \tag{A.51}$$

where we used eq. (A.36). Eq. (A.44) permits the gauge choice

$$h_3^{(\alpha\beta)\dot\alpha\gamma} = h_{3(0)}^{(\alpha\beta)\dot\alpha\gamma} \;\Rightarrow\; \omega_1^{-2(\alpha\beta)\dot\alpha\gamma} = 0\,, \tag{A.52}$$
$$\delta h_{3(0)}^{(\alpha\beta)\dot\alpha\gamma} = -\frac{1}{2}\big(i\,\partial^{\gamma\dot\alpha}\,\omega_{(0)}^{(\alpha\beta)} + \varepsilon^{\gamma(\beta}\,\bar\Sigma_{1(0)}^{\alpha)\dot\alpha}\big). \tag{A.53}$$

It is clear from (A.53) that the further gauge-fixing is possible,

$$h_{3(0)}^{(\alpha\beta)\dot\alpha\gamma} = T^{(\alpha\beta\gamma)\dot\alpha} \;\Rightarrow\; \bar\Sigma_{1(0)}^{\beta\dot\alpha} = -\frac{2i}{3}\,\partial_{\gamma}^{\dot\alpha}\,\omega_{(0)}^{(\beta\gamma)}\,, \tag{A.54}$$
$$\delta T^{(\alpha\beta\gamma)\dot\alpha} = i\partial_{\dot\gamma}^{(\gamma}\,\omega^{\beta\alpha)\dot\alpha\dot\gamma} = -\frac{i}{2}\,\partial^{\dot\alpha(\gamma}\omega_{(0)}^{\beta\alpha)}\,, \tag{A.55}$$

where we used the notations introduced in (A.5). The field $T^{(\alpha\beta\gamma)\dot\alpha}$ is just the "hook" gauge field. It involves $\mathbf{16} - \mathbf{6} = \mathbf{10}$ essential off-shell degrees of freedom.

It remains to reveal the consequences of the gauge freedom (A.45). First, it allows for the gauge choice

$$h_4^{(\beta\alpha)\dot\alpha\dot\gamma} = h_{4(0)}^{(\beta\alpha)\dot\alpha\dot\gamma} \;\Rightarrow\; \omega_2^{-2(\alpha\beta)\dot\alpha\dot\gamma} = 2i\,\partial^{\dot\gamma(\beta}\Sigma_{(0)}^{-2\alpha)\dot\alpha}\,, \tag{A.56}$$

$$\delta h_{4(0)}^{(\beta\alpha)\dot\alpha\dot\gamma} = \frac{1}{4}\Omega^{(\alpha\dot\alpha\beta)\dot\gamma} - i\partial^{\gamma\dot\gamma}\omega_{\gamma(0)}^{(\alpha\beta)\dot\alpha}\,. \tag{A.57}$$

Now, let us decompose

$$\Omega^{\alpha\dot\alpha\beta\dot\beta} = \Omega^{(\alpha\beta)(\dot\alpha\dot\beta)} + \varepsilon^{\alpha\beta}\Omega^{(\dot\alpha\dot\beta)} - \varepsilon^{\dot\alpha\dot\beta}\Omega^{(\alpha\beta)} - i\varepsilon^{\alpha\beta}\varepsilon^{\dot\alpha\dot\beta}\Omega\,, \tag{A.58}$$

$$\overline{(\Omega^{(\alpha\beta)(\dot\alpha\dot\beta)})} = -\Omega^{(\alpha\beta)(\dot\alpha\dot\beta)}\,,\; \overline{(\Omega^{(\alpha\gamma)})} = \Omega^{(\dot\alpha\dot\gamma)}\,,\; \bar\Omega = \Omega\,. \tag{A.59}$$

Then, we can impose the further gauge

$$h_{4(0)}^{(\beta\alpha)\dot\alpha\dot\gamma} = h_{4(0)}^{(\beta\alpha)(\dot\alpha\dot\gamma)} \;\Rightarrow$$

$$\Omega^{(\alpha\beta)} = -2i\partial_{\gamma\dot\alpha}\,\omega_{(0)}^{(\alpha\beta)\dot\alpha\gamma}\,,\quad \Omega^{(\dot\alpha\dot\beta)} = 2i\partial_{\alpha\dot\gamma}\,\bar\omega_{(0)}^{(\dot\alpha\dot\beta)\alpha\dot\gamma}\,, \tag{A.60}$$

and, using the complex conjugation rules (A.59), make $h_{4(0)}^{(\beta\alpha)(\dot\alpha\dot\gamma)}$ real

$$h_{4(0)}^{(\beta\alpha)(\dot\alpha\dot\gamma)} = \bar h_{4(0)}^{(\beta\alpha)(\dot\alpha\dot\gamma)} = P^{(\beta\alpha)(\dot\alpha\dot\gamma)} \;\Rightarrow$$

$$\Omega^{(\alpha\beta)(\dot\alpha\dot\beta)} = -i\big[\partial_\gamma^{\dot\gamma}\,\omega_{(0)}^{(\alpha\beta)\dot\alpha\gamma} + \partial_\gamma^{\dot\alpha}\,\omega_{(0)}^{(\alpha\beta)\dot\gamma\gamma} + \partial_{\dot\rho}^{\alpha}\,\bar\omega_{(0)}^{(\dot\alpha\dot\gamma)\beta\dot\rho} + \partial_{\dot\rho}^{\alpha}\,\bar\omega_{(0)}^{(\dot\alpha\dot\gamma)\beta\dot\rho}\big]\,. \tag{A.61}$$

At this step, we are left with the following gauge transformation of real $P^{(\beta\alpha)(\dot\alpha\dot\gamma)}$

$$\delta P^{(\beta\alpha)(\dot\alpha\dot\gamma)} = \frac{i}{4}\big[\partial_\gamma^{\dot\gamma}\,\omega_{(0)}^{(\alpha\beta)\dot\alpha\gamma} + \partial_\gamma^{\dot\alpha}\,\omega_{(0)}^{(\alpha\beta)\dot\gamma\gamma} - \partial_{\dot\rho}^{\alpha}\,\bar\omega_{(0)}^{(\dot\alpha\dot\gamma)\beta\dot\rho} - \partial_{\dot\rho}^{\beta}\,\bar\omega_{(0)}^{(\dot\alpha\dot\gamma)\alpha\dot\rho}\big]\,. \tag{A.62}$$

Now we should be back to the discussion of the structure of gauge potential $h^{++(\alpha\beta)(\dot\alpha\dot\beta)}$. The gauge transformation (A.34) implies that the whole harmonic-dependent part of $h^{(\alpha\beta)(\dot\alpha\dot\beta)\gamma\dot\gamma}$ can be gauged away, in agreement with the general structure (A.5),

$$h_2^{(\alpha\beta)(\dot\alpha\dot\beta)\gamma\dot\gamma} = h_{2(0)}^{(\alpha\beta)(\dot\alpha\dot\beta)\gamma\dot\gamma} \;\Rightarrow$$

$$\rho^{-2(\alpha\beta)(\dot\alpha\dot\beta)\gamma\dot\gamma} = -2i\big[\varepsilon^{\beta\gamma}\Sigma_{(0)}^{-2\alpha(\dot\alpha}\varepsilon^{\dot\beta)\dot\gamma} + (\alpha\leftrightarrow\beta)\big], \tag{A.63}$$

$$\delta h_{2(0)}^{(\alpha\beta)(\dot\alpha\dot\beta)\gamma\dot\gamma} = -4i\partial^{\gamma\dot\gamma}\rho_{(0)}^{(\alpha\beta)(\dot\alpha\dot\beta)} + 4i\big[\omega_{(0)}^{(\alpha\beta)\gamma(\dot\alpha}\varepsilon^{\dot\beta)\dot\gamma} + \bar\omega_{(0)}^{(\dot\alpha\dot\beta)\dot\gamma(\alpha}\varepsilon^{\beta)\gamma}\big]. \tag{A.64}$$

From (A.64) we also observe that all parts of $h_{2(0)}^{(\alpha\beta)(\dot\alpha\dot\beta)\gamma\dot\gamma}$, excepting the totally symmetric one, can be gauged away, leading to the following gauge transformation for the conformal spin 3 gauge field (in the notation of (A.5)):

$$\delta\Phi^{(\alpha\beta\gamma)(\dot\alpha\dot\beta\dot\gamma)} = \partial^{(\gamma\dot\gamma}\rho_{(0)}^{(\alpha\beta)(\dot\alpha\dot\beta))} \tag{A.65}$$

(where total symmetrizations with respect to dotted and undotted indices are assumed). Thus we are left with $\mathbf{16} - \mathbf{9} = \mathbf{7}$ off-shell degrees of freedom in $\Phi^{(\alpha\beta\gamma)(\dot\alpha\dot\beta\dot\gamma)}$.

Preserving the gauge on $h_{2(0)}^{(\alpha\beta)(\dot\alpha\dot\beta)\gamma\dot\gamma}$ just mentioned, yields the following restrictions on the gauge ω-parameters

$$\omega_{(0)}^{(\alpha\beta\gamma)\dot\beta} = -\frac{2}{3}\,\partial_{\dot\alpha}^{(\gamma}\,\rho_{(0)}^{\alpha\beta)(\dot\alpha\dot\beta)}\,, \quad \bar\omega_{(0)}^{(\dot\alpha\dot\beta\dot\gamma)\beta} = -\frac{2}{3}\,\partial_{\alpha}^{(\dot\gamma}\,\rho_{(0)}^{\dot\alpha\dot\beta)(\alpha\beta)}\,, \tag{A.66}$$

$$\omega_{(0)}^{\beta\dot\beta} + \bar\omega_{(0)}^{\beta\dot\beta} = -\frac{2}{3}\partial_{\alpha\dot\alpha}\rho_{(0)}^{(\alpha\beta)(\dot\alpha\dot\beta)}\,, \quad \omega_{(0)}^{\beta\dot\beta} := \omega_{(0)\ \alpha}^{(\alpha\beta)\dot\beta}\,,\ \bar\omega_{(0)}^{\beta\dot\beta} := \bar\omega_{\dot\alpha\,(0)}^{(\dot\beta\dot\alpha)\beta}\,. \tag{A.67}$$

Defining new independent gauge parameter

$$p^{\beta\dot\beta} := i\big(\omega_{(0)}^{\beta\dot\beta} - \bar\omega_{(0)}^{\beta\dot\beta}\big) \tag{A.68}$$

and substituting (A.66), (A.67) in (A.62), we obtain

$$\delta P^{(\beta\alpha)(\dot\alpha\dot\gamma)} = -\frac{1}{3}\partial^{(\beta(\dot\gamma}p^{\alpha)\dot\alpha)}\,. \tag{A.69}$$

So $P^{(\beta\alpha)(\dot\alpha\dot\gamma)}$ is "conformal graviton": it carries $\mathbf{9}-\mathbf{4}=\mathbf{5}$ off-shell degrees of freedom.

At this step let us come back to the transformation law (A.49). One can check that

$$\delta D^{\alpha\dot\alpha} = -\frac{1}{2}\partial^{\alpha\dot\alpha}\big[\Omega + \frac{2}{9}\,\partial_{\beta\dot\beta}\partial_{\gamma\dot\gamma}\rho_{(0)}^{(\beta\gamma)(\dot\beta\dot\gamma)}\big] - \frac{2}{9}\Box\partial_{\gamma\dot\gamma}\rho_{(0)}^{(\alpha\gamma)(\dot\alpha\dot\gamma)} \tag{A.70}$$

In order to pass to the gauge field with the standard gradient transformation law, let us define

$$Z^{\alpha\dot\alpha} := \partial_{\beta\dot\beta}\partial_{\gamma\dot\gamma}\Phi^{(\alpha\beta\gamma)(\dot\alpha\dot\beta\dot\gamma)}. \tag{A.71}$$

Under the spin $\mathbf{s}=3$ gauge transformations:

$$\delta Z^{\alpha\dot\alpha} = \frac{1}{27}\big[\partial^{\alpha\dot\alpha}(\partial_{\beta\dot\beta}\partial_{\gamma\dot\gamma}\rho_{(0)}^{(\beta\gamma)(\dot\beta\dot\gamma)}) + 5\Box\partial_{\gamma\dot\gamma}\rho_{(0)}^{(\alpha\gamma)(\dot\alpha\dot\gamma)}\big]. \tag{A.72}$$

Then it is easy to check that

$$\tilde D^{\alpha\dot\alpha} = D^{\alpha\dot\alpha} + \frac{6}{5}Z^{\alpha\dot\alpha} \tag{A.73}$$

has the correct spin $\mathbf{s}=1$ gauge transformation with the properly redefined gauge parameter

$$\delta\tilde D^{\alpha\dot\alpha} = -\frac{1}{2}\partial^{\alpha\dot\alpha}\tilde\Omega\,, \quad \tilde\Omega := \Omega + \frac{2}{15}\,(\partial_{\beta\dot\beta}\partial_{\gamma\dot\gamma}\rho_{(0)}^{(\beta\gamma)(\dot\beta\dot\gamma)}). \tag{A.74}$$

The final step is to reveal the role of the gauge transformation (A.35). It admits imposing the gauge

$$\begin{aligned} &h_3^{-2(\alpha\beta)(\dot\alpha\dot\beta)} = h_{3(0)}^{-2(\alpha\beta)(\dot\alpha\dot\beta)} \quad \Rightarrow \quad \rho_3^{-4(\alpha\beta)(\dot\alpha\dot\beta)} = 0\,,\\ &\delta h_{3(0)}^{-2(\alpha\beta)(\dot\alpha\dot\beta)} = -12\partial^{(\dot\alpha(\beta}\Sigma_{(0)}^{-2\alpha)\dot\beta)}, \end{aligned} \tag{A.75}$$

where we used the expressions (A.63) for $\rho^{-2(\alpha\beta)(\dot\alpha\dot\beta)\gamma\dot\gamma}$ and (A.56) for $\omega_2^{-2(\alpha\beta)(\dot\alpha\dot\beta)}$. The triplet gauge field $h_{3(0)}^{-2(\alpha\beta)(\dot\alpha\dot\beta)} := V^{(\alpha\beta)(\dot\alpha\dot\beta)(ij)}u_i^-u_j^-$ carries $\mathbf{27}-\mathbf{12}=\mathbf{15}$ off-shell degrees of freedom.

To summarize, the whole set of bosonic gauge fields carries just total of $\mathbf{15}+\mathbf{3}+\mathbf{7}+\mathbf{5}+\mathbf{10}=\mathbf{40}$ essential off-shell degrees of freedom. All essential bosonic fields are gauge, in contradistinction to the lower spin ($\mathbf{s}=1$ and $\mathbf{s}=2$) multiplets containing also auxiliary fields in the bosonic sector.

A.4 Fermionic sector

The analysis of the component structure of the conformal $\mathcal{N}=2$, $\mathbf{s}=3$ gauge supermultiplet in the fermionic sector basically follows the same pattern as in the bosonic one, so we will concentrate on the final answers rather than on the intermediate computations.

We will need the following fermionic terms in the general analytic gauge parameters:

$$
\begin{aligned}
&\lambda^{+4} \;\Rightarrow\; (\bar\theta^+)^2\theta^{+\alpha}\ell^+_\alpha - (\theta^+)^2\bar\theta^{+\dot\alpha}\bar\ell^+_{\dot\alpha}\,,\\
&\lambda^{\alpha+\dot\beta+} \;\Rightarrow\; \theta^{+\gamma}p^{+\alpha\dot\beta}_\gamma + \bar\theta^{+\dot\gamma}\bar p^{+\alpha\dot\beta}_{\dot\gamma},\\
&\lambda^{++\alpha\dot\beta} \;\Rightarrow\; \theta^{+\gamma}k^{+\alpha\dot\beta}_\gamma - \bar\theta^{+\dot\gamma}\bar k^{+\alpha\dot\beta}_{\dot\gamma},\\
&\lambda^{(\beta+\alpha)\dot\alpha} \;\Rightarrow\; \tau^{+(\beta\alpha)\dot\alpha} + (\theta^+)^2\tau_1^{-(\beta\alpha)\dot\alpha} + (\bar\theta^+)^2\hat\tau_1^{-(\beta\alpha)\dot\alpha} + \theta^{+\rho}\bar\theta^{+\dot\rho}\tau_{1\,\rho\dot\rho}^{-(\beta\alpha)\dot\alpha}\\
&\qquad\qquad + (\theta^+)^4\tau_2^{-3(\beta\alpha)\dot\alpha}\,,\\
&\lambda^{(\beta\alpha)(\dot\alpha\dot\beta)} \;\Rightarrow\; \theta^{+\rho}\lambda_\rho^{-(\alpha\beta)(\dot\alpha\dot\beta)} - \bar\theta^{+\dot\rho}\bar\lambda_{\dot\rho}^{-(\alpha\beta)(\dot\alpha\dot\beta)} + (\bar\theta^+)^2\theta^{+\rho}\lambda_\rho^{-3(\alpha\beta)(\dot\alpha\dot\beta)}\\
&\qquad\qquad - (\theta^+)^2\bar\theta^{+\dot\rho}\bar\lambda_{\dot\rho}^{-3(\alpha\beta)(\dot\alpha\dot\beta)}\,.
\end{aligned}
\tag{A.76}
$$

We can impose the same preliminary gauge conditions as in the bosonic case, in particular, fully gauge away the set of the gauge potentials $h^{(+6)}$, $h^{++\hat\alpha+++}$, $\hat h^{+4}$, $\hat h^{++}$, $\hat h^{+3\hat\alpha}$, $h^{++\alpha+\dot\beta+}$ also in the fermionic sector. The whole fermionic part of $h^{+4\alpha\dot\alpha}$ can also be gauged away. These gaugings imply certain conditions on the residual component gauge parameters. In particular, the conditions

$$
h_F^{++\alpha+\dot\beta+} = 0\,, \qquad h_F^{++\alpha+\dot\beta+} = 0 \tag{A.77}
$$

give rise to

$$
p^{+\alpha\dot\beta\gamma} = p^{+\alpha\dot\beta\gamma}_{(0)} = p^{\alpha\dot\beta\gamma\,i}u^+_i\,, \quad k^{+\alpha\dot\beta\gamma} = k^{+\alpha\dot\beta\gamma}_{(0)} = k^{\alpha\dot\beta\gamma\,i}u^+_i \;\; (\text{and c.c.}). \tag{A.78}
$$

The rest of constraints can also be straightforwardly solved. The corresponding reduced gauge parameters are of no interest for our purposes.

We end up with the symmetrized vielbeins $h_F^{+3(\alpha\beta)\dot\alpha}$ and $h_F^{+2(\alpha\beta)(\dot\alpha\dot\beta)}$:

$$
\begin{aligned}
h_F^{+3(\alpha\beta)\dot\alpha} &= h_1^{+3(\alpha\beta)\dot\alpha} + (\theta^+)^2h_2^{+(\alpha\beta)\dot\alpha} + (\bar\theta^+)^2\hat h_2^{+(\alpha\beta)\dot\alpha} + \theta^+_\rho\bar\theta_{\dot\rho}h_3^{+(\alpha\beta)\dot\alpha\rho\dot\rho}\\
&\quad + (\theta^+)^4h_4^{-(\beta\alpha)\dot\alpha}\,,
\end{aligned}
\tag{A.79}
$$

$$
\begin{aligned}
h_F^{+2(\alpha\beta)(\dot\alpha\dot\beta)} &= \theta^{+\gamma}h_\gamma^{+(\alpha\beta)(\dot\alpha\dot\beta)} - \bar\theta^{+\dot\rho}\bar h_{\dot\gamma}^{+(\dot\alpha\dot\beta)(\alpha\beta)} + (\bar\theta^+)^2\theta^{+\gamma}\hat h_\gamma^{-(\alpha\beta)(\dot\alpha\dot\beta)}\\
&\quad -(\theta^+)^2\bar\theta^{+\dot\gamma}\hat{\bar h}_{\dot\gamma}^{-(\dot\alpha\dot\beta)(\alpha\beta)}\,.
\end{aligned}
\tag{A.80}
$$

Firstly we elaborate on (A.79). From the gauge conditions,

$$
h_1^{+3(\alpha\beta)\dot\alpha} = 0\,,\; h_2^{+(\alpha\beta)\dot\alpha} = 0\,,\; \hat h_2^{+(\alpha\beta)\dot\alpha} = 0\,,\; h_3^{+(\alpha\beta)\dot\alpha\rho\dot\rho} = 0 \tag{A.81}
$$

we find the constraints on the residual gauge parameters

$$
\begin{aligned}
&\tau^{+(\beta\alpha)\dot\alpha} = \tau^{+(\beta\alpha)\dot\alpha}_{(0)} = \tau^{(\beta\alpha)\dot\alpha i}u^+_i\,,\; \hat\tau_1^{-(\beta\alpha)\dot\alpha} = 0\,,\\
&\tau_1^{-(\beta\alpha)\dot\alpha} = \frac{1}{2}\varepsilon^{\rho(\alpha}R_\rho^{-\beta)\dot\alpha}\,, \quad R_\rho^{-\beta\dot\alpha} := k^{-\beta\dot\alpha}_{\rho(0)} + 8ip^{-\beta\dot\alpha}_{\rho(0)} = R_\rho^{\beta\dot\alpha i}u^-_i\,,\\
&\tau_{1\,\rho\dot\rho}^{-(\beta\alpha)\dot\alpha} = 4i\partial_{\rho\dot\rho}\tau^{-(\beta\alpha)\dot\alpha}_{(0)} - \delta_\rho^{(\beta}\,\bar R_{\dot\rho}^{-\alpha)\dot\alpha}.
\end{aligned}
\tag{A.82}
$$

For the surviving part of the gauge potential $h_4^{-(\beta\alpha)\dot\alpha}$, that is $h_{4(0)}^{-(\beta\alpha)\dot\alpha} := \chi^{-(\beta\alpha)\dot\alpha} = \chi^{(\beta\alpha)\dot\alpha i}u_i^-$ (recall "master gauge" (A.5)), at this step we obtain the following gauge transformation

$$\delta\chi^{-(\beta\alpha)\dot\alpha} = i\partial^{(\beta\dot\rho}\bar R_{\dot\rho}^{-\alpha)\dot\alpha} + 2\Box\,\tau_{(0)}^{-(\alpha\beta)\dot\alpha}\,. \tag{A.83}$$

In order to find the final form of this gauge transformation, we need first to work out (A.80). The choice of the gauge

$$h_\gamma^{+(\alpha\beta)(\dot\alpha\dot\beta)} = 0 \;\; \text{(and c.c.)}, \qquad \hat h_\gamma^{-(\alpha\beta)(\dot\alpha\dot\beta)} = \hat h_{\gamma(0)}^{-(\alpha\beta)(\dot\alpha\dot\beta)}, \tag{A.84}$$

implies the relations

$$\lambda_\rho^{-(\alpha\beta)(\dot\alpha\dot\beta)} = -4i\delta_\rho^{(\alpha}\bar\tau^{-(\dot\alpha\dot\beta)\beta)}\,, \quad \bar\lambda_{\dot\rho}^{-(\alpha\beta)(\dot\alpha\dot\beta)} = -4i\delta_{\dot\rho}^{(\dot\alpha}\tau^{-(\alpha\beta)\dot\beta)}\,,$$
$$\lambda_\rho^{-3(\alpha\beta)(\dot\alpha\dot\beta)} = 0 \quad \text{(and c.c.)},$$

and the following gauge transformation of $\hat h_{\gamma(0)}^{-(\alpha\beta)(\dot\alpha\dot\beta)} := \psi_\gamma^{(\alpha\beta)(\dot\alpha\dot\beta)i}u_i^-$

$$\delta\psi_\rho^{-(\alpha\beta)(\dot\alpha\dot\beta)} = -8\partial_\rho^{(\dot\alpha}\,\tau_{(0)}^{-(\beta\alpha)\dot\beta} + 4i\,\delta_\rho^{(\alpha}\,\bar\tau_1^{-(\dot\beta\dot\alpha)\beta} + 2i\,\tau_{1\rho}^{-(\beta\alpha)(\dot\alpha\dot\beta)}. \tag{A.85}$$

Using the expressions (A.82), this variation can be rewritten as

$$\delta\psi_\rho^{-(\alpha\beta)(\dot\alpha\dot\beta)} = -16\,\partial_\rho^{(\dot\alpha}\,\tau_{(0)}^{-(\alpha\beta)\dot\beta)} + 4i\,\delta_\rho^{(\alpha}\,\varepsilon^{\dot\gamma(\dot\beta}\bar R_{\dot\gamma}^{-\beta)\dot\alpha)}\,.$$

This transformation law implies

$$\delta\psi^{-(\alpha\beta\rho)(\dot\alpha\dot\beta)} = -16\,\partial^{(\dot\alpha(\rho}\,\tau_{(0)}^{-\alpha\beta)\dot\beta)}\,. \tag{A.86}$$

while the rest of components in $\psi_\rho^{-(\alpha\beta)(\dot\alpha\dot\beta)}$ can be gauged away

$$\psi_\alpha^{-(\alpha\beta)(\dot\alpha\dot\beta)} = 0 \;\Rightarrow\; \bar R^{-(\dot\alpha\dot\beta)\alpha} = \frac{8i}{3}\,\partial_\rho^{(\dot\alpha}\tau_{(0)}^{-(\beta\rho)\dot\beta)} \tag{A.87}$$

The transformation law (A.86) means that the complex field $\psi^{-(\alpha\beta\rho)(\dot\alpha\dot\beta)} = \psi^{(\alpha\beta\rho)(\dot\alpha\dot\beta)i}u_i^-$ encompasses the $SU(2)$ doublet of the spin $\mathbf{5/2}$ gauge fields with $\mathbf{48} - \mathbf{24} = \mathbf{24}$ essential degrees of freedom off shell.

As the next step, one can define

$$w^{-(\alpha\beta)\dot\beta} := \partial_{\gamma\dot\alpha}\psi^{-(\alpha\beta\gamma)(\dot\beta\dot\alpha)}\,, \tag{A.88}$$

$$\delta w^{-(\alpha\beta)\dot\beta} = -\frac{4}{3}\left[\partial^{(\alpha\dot\beta}\,b^{-\beta)} + 5\,\Box\,\tau_{(0)}^{-(\alpha\beta)\dot\beta}\right], \quad b^{-\beta} := \partial_{\gamma\dot\gamma}\tau_{(0)}^{-(\beta\gamma)\dot\gamma}\,. \tag{A.89}$$

Now, coming back to eq. (A.83) and redefining

$$\hat\chi^{-(\beta\alpha)\dot\alpha} := \chi^{-(\beta\alpha)\dot\alpha} + \frac{1}{10}w^{-(\alpha\beta)\dot\alpha}\,, \tag{A.90}$$

we find that $\hat{\chi}^{-(\beta\alpha)\dot{\alpha}}$ is transformed as

$$\delta\hat{\chi}^{-(\beta\alpha)\dot{\alpha}} = \frac{i}{2}\partial^{(\alpha\dot{\alpha}}c^{-\beta)}\,, \quad c^{-\beta} := \bar{R}^{-\beta\dot{\gamma}}_{\dot{\gamma}} - \frac{12i}{5}b^{-\beta}\,. \tag{A.91}$$

Since $c^{-\beta} = c^{\beta i}u^-_i$ involves just 8 independent real gauge parameters $c^{\beta i}$, the field $\hat{\chi}^{-(\beta\alpha)\dot{\alpha}} = \hat{\chi}^{(\beta\alpha)\dot{\alpha}\,i}u^-_i$ describes $SU(2)$ doublet of the spin $\mathbf{3/2}$ gauge fields with $\mathbf{24} - \mathbf{8} = \mathbf{16}$ essential degrees of freedom off shell.

Thus in the fermionic sector we end up with the spin $\mathbf{5/2}$ and spin $\mathbf{3/2}$ conformal gauge fields with the total of $\mathbf{40}$ off-shell essential degrees of freedom. This number precisely matches the number of essential degrees of freedom in the bosonic sector, and it remains to show that the last gauge potential h^{++} does not contribute any degree of freedom in the full WZ gauge.

A.5 h^{++} gauge potential

Using the $\mathcal{D}^{++}\lambda$ gauge freedom, one can fix the gauge:

$$\begin{aligned} h^{++} =& \theta^{+\alpha}\bar{\theta}^{+\dot{\alpha}}A_{\alpha\dot{\alpha}} + (\theta^+)^2\phi + (\bar{\theta}^+)^2\bar{\phi} \\ &+ 4(\bar{\theta}^+)^2\theta^{+\alpha}\xi^i_\alpha u^-_i + 4(\theta^+)^2\bar{\theta}^{+\dot{\alpha}}\bar{\xi}^i_{\dot{\alpha}}u^-_i + (\theta^+)^4 D^{ij}u^-_i u^-_j\,. \end{aligned} \tag{A.92}$$

This is the standard WZ gauge for the spin $\mathbf{1}$ multiplet. However, the full h^{++} gauge transformation law (6.26c) contain additional terms which can be used to gauge away all fields in (A.92). In the process, only those terms in the θ and u-expansions of (6.26c) are of interest, which have the form (A.92). All other terms can be absorbed into the redefinition of the gauge parameters which were used to ensure (A.92).

After some straightforward algebra, using eqs. (A.24) - (A.27), we obtain (up to some $U(1)$ gauge transformation of $A^{\gamma\dot{\gamma}}$):

$$\delta\phi = -i\partial_{\alpha\dot{\alpha}}\bar{\sigma}^{\alpha\dot{\alpha}}_{1(0)}, \qquad \delta\bar{\phi} = i\partial_{\alpha\dot{\alpha}}\sigma^{\alpha\dot{\alpha}}_{1(0)}\,, \tag{A.93}$$

$$\delta A^{\gamma\dot{\gamma}} = \frac{1}{2}\partial_{\alpha\dot{\alpha}}\Big[\Sigma^{(\alpha\gamma)(\dot{\alpha}\dot{\gamma})}_{-} - \frac{4i}{9}\Box\,\rho^{(\alpha\gamma)(\dot{\alpha}\dot{\gamma})}_{(0)}\Big], \tag{A.94}$$

$$\Sigma^{(\alpha\gamma)(\dot{\alpha}\dot{\gamma})}_{-} := \chi^{(\alpha\gamma)(\dot{\alpha}\dot{\gamma})}_{2(0)} - 8i\psi^{(\alpha\gamma)(\dot{\alpha}\dot{\gamma})}_{2(0)}\,,$$

$$\delta D^{-2} = -4i\partial_{\alpha\dot{\alpha}}\Big[\ell^{-2\alpha\dot{\alpha}}_{(0)} + \frac{15i}{16}\Box\chi^{-2\alpha\dot{\alpha}}_{(0)} - \frac{3}{2}\Box\psi^{-2\alpha\dot{\alpha}}_{(0)}\Big], \tag{A.95}$$

$$\delta\xi^-_\rho = \partial^{--}\ell^+_{\rho(0)} + \dots\,, \tag{A.96}$$

where ellipses stand for some terms with x-derivatives.

Thus we see that all bosonic fields in (A.92) are shifted by divergences of the appropriate vector parameters. Since the parameters in (A.93), (A.94) and (A.95) are unconstrained and independent (they are new compared to those which were used earlier in fixing various WZ gauges), these parameters are capable to gauge away all bosonic fields in (A.92). The fermionic field ξ^i_ρ is shifted by an unconstrained parameter ℓ^i_ρ (defined in (A.76)), so one can also choose $\xi^i_\alpha = 0$. As a result, one can fix the gauge $h^{++} = 0$.

Note that the gauge transformations of the form $\delta h_{\alpha(s)\dot{\alpha}(s)} = \partial^{\beta\dot{\beta}}\lambda_{\beta\alpha(s)\dot{\beta}\dot{\alpha}(s)}$ are frequently encountered in the free theory of massless higher spins [13, 14] (see also [5, 40] for review).

B On residual parameters and reparametrization freedom of free hypermultiplet

Note that among the parameters of the spin **3** transformations (6.26) there are special parameters which do not appear in transformations of the gauge potentials h^{++MN} and h^{++}.

For instance, the transformation with parameter

$$\lambda^{(+4)} = (\theta^+)^4 e(x) \tag{B.1}$$

acts only on the hypermultiplet[26]:

$$\delta_{e(x)} q^{+a} = (\theta^+)^4 e(x) \partial^{--}\partial^{--} J q^{+a}. \tag{B.2}$$

This is the exact off-shell symmetry of the free part of the hypermultiplet action, $\delta_{e(x)} S_{free} = 0$. This transformation and other symmetries of similar kind mix the auxiliary fields of the hypermultiplet,

$$q^{+a} = \dots + K^{(ijk)a} u_i^+ u_j^+ u_k^- + \dots + (\theta^+)^4 F^{(ijk)a} u_i^- u_j^- u_k^- + \dots , \tag{B.3}$$

$$\delta_{e(x)} K^{(ijk)a}(x) = e(x) J F^{(ijk)a}(x), \tag{B.4}$$

and seemingly have no impact on the structure of superconformal vertices.

C Superconformal transformations of $\mathcal{N} = 2$ superspace derivatives and gauge potentials

For checking the transformation properties of various analytic vielbeins under the rigid $\mathcal{N} = 2$ superconformal group, it is useful to be aware of the superconformal transformation laws of the partial derivatives with respect to the co-ordinates of the analytic harmonic superspace. It suffices to know such laws for rigid $\mathcal{N} = 2$ supersymmetry and special conformal transformations, since the whole superconformal group is the closure of these two.

Using the infinitesimal superconformal coordinate shifts (3.9), we obtain

Supersymmetry:

$$\delta_\epsilon \partial^-_\alpha = 4i\bar{\epsilon}^{-\dot\beta}\partial_{\alpha\dot\beta}\,, \; \delta_\epsilon \bar\partial^-_{\dot\alpha} = -4i\epsilon^{-\beta}\partial_{\beta\dot\alpha}\,, \; \delta_\epsilon \partial^{--} = -\epsilon^{-\alpha}\partial^-_\alpha - \bar\epsilon^{-\dot\alpha}\bar\partial-_{\dot\alpha}\,, \; \delta_\epsilon \partial_{\alpha\dot\alpha} = 0\,; \tag{C.1}$$

Special conformal transformations:

$$\begin{aligned}
&\delta_k \partial_{\alpha\dot\alpha} = -(k_{\alpha\dot\beta}x^{\gamma\dot\beta}\partial_{\gamma\dot\alpha} + k_{\gamma\dot\alpha}x^{\gamma\dot\beta}\partial_{\alpha\dot\beta}) - k_{\beta\dot\alpha}\theta^{+\beta}\partial^-_\alpha - k_{\alpha\dot\beta}\bar\theta^{+\dot\beta}\bar\partial^-_{\dot\alpha}\,, \quad \delta_k\partial^{--} = 0\,, \\
&\delta_k \partial^-_\alpha = -k_{\alpha\dot\beta}x^{\beta\dot\beta}\,\partial^-_\beta - 4ik_{\alpha\dot\beta}\bar\theta^{+\dot\beta}\partial^{--}\,, \quad \delta_k\bar\partial^-_{\dot\alpha} = -k_{\gamma\dot\alpha}x^{\gamma\dot\beta}\,\bar\partial^-_{\dot\beta} + 4ik_{\gamma\dot\alpha}\theta^{+\gamma}\partial^{--}\,.
\end{aligned} \tag{C.2}$$

[26]One can define even more general off-shell symmetry transformation of the hypermultiplet:

$$\delta_{c_{(ab)}} q^{+a} = (\theta^+)^4 c^a_b(x)(\partial^{--})^2 q^{+b}$$

with an arbitrary symmetric matrix $c_{(ab)}(x)$.

It is straightforward to calculate the corresponding transformation properties of various products of these derivatives, e.g. of the bilinear products $\partial_N\partial_M$ appearing in (6.1), (6.10). As an example we first present the passive form (without "transport" term) of the transformation rules of the analytic gauge potentials of the spin **2** case:

$$
\begin{aligned}
&\delta^*_\epsilon h^{++\alpha\dot\alpha} = -4i\big(\epsilon^{-\alpha}\bar h^{++\dot\alpha+} - \bar\epsilon^{-\dot\alpha}h^{++\alpha+}\big)\,, \quad \delta^*_\epsilon h^{+4} = 0\,,\\
&\delta^*_\epsilon h^{++\alpha+} = \epsilon^{-\alpha}h^{+4}\,, \quad \delta^*_\epsilon h^{++\dot\alpha+} = \bar\epsilon^{-\dot\alpha}h^{+4}\,, \qquad (C.3)\\
&\delta^*_k h^{++\alpha\dot\alpha} = k_{\gamma\dot\beta}\big(h^{++\gamma\dot\alpha}\,x^{\alpha\dot\beta} + h^{++\alpha\dot\beta}\,x^{\gamma\dot\alpha}\big)\,, \quad \delta^*_k h^{+4} = 4ik_{\gamma\dot\beta}\big(h^{++\gamma+}\bar\theta^{+\dot\beta} - h^{++\dot\beta+}\theta^{+\gamma}\big),\\
&\delta^*_k h^{++\alpha+} = k_{\gamma\dot\beta}\big(h^{++\gamma+}\,x^{\alpha\dot\beta} - h^{++\alpha\dot\beta}\theta^{+\gamma}\big),\\
&\delta^*_k h^{++\dot\alpha+} = k_{\gamma\dot\beta}\big(h^{++\dot\beta+}\,x^{\gamma\dot\alpha} - h^{++\gamma\dot\alpha}\bar\theta^{+\dot\beta}\big). \qquad (C.4)
\end{aligned}
$$

It is also useful to explicitly give how the $\mathbf{s} = 3$ gauge potentials defined in (6.5) are transformed by $\mathcal{N} = 2$ superconformal group (before any gauge-fixing). We skip the passive transformation rules of the products of various partial derivatives and quote at once the transformation laws of the analytic potentials

Supersymmetry:

$$
\begin{aligned}
&\delta^*_\epsilon h^{++\alpha\dot\alpha\,\beta\dot\beta} = 4i\big[\bar\epsilon^{-\dot\beta}h^{++\beta+\alpha\dot\alpha} - \epsilon^{-\beta}h^{++\dot\beta+\alpha\dot\alpha}\big] + (\alpha,\dot\alpha \Leftrightarrow \beta,\dot\beta)\,,\\
&\delta^*_\epsilon h^{++[\beta+\gamma]+} = 2\epsilon^{-[\beta}\,h^{++++\,\gamma]+}\,, \quad \delta^*_\epsilon h^{++[\dot\beta+\dot\gamma]+} = 2\bar\epsilon^{-[\dot\beta}\,h^{++++\,\dot\gamma]+}\,,\\
&\delta^*_\epsilon h^{++\beta+\alpha\dot\alpha} = \epsilon^{-\beta}\,h^{++\alpha\dot\alpha++} + 4i\big(\epsilon^{-\alpha}\,h^{++\beta+\dot\alpha+} - \bar\epsilon^{-\dot\alpha}h^{++[\beta+\alpha]+}\big)\\
&\delta^*_\epsilon h^{++\dot\beta+\alpha\dot\alpha} = \delta^*_\epsilon\widetilde{h^{++\beta+\alpha\dot\alpha}}\,, \quad \delta^*_\epsilon h^{(+6)} = 0\,,\\
&\delta^*_\epsilon h^{++++\beta+} = \epsilon^{-\beta}h^{(+6)}\,, \quad \delta^*_\epsilon h^{++++\dot\beta+} = \delta^*_\epsilon\widetilde{h^{++++\beta+}}\,,\\
&\delta^*_\epsilon h^{++\alpha\dot\alpha++} = -4i\big(\epsilon^{-\alpha}h^{++++\dot\alpha+} - \bar\epsilon^{-\dot\alpha}h^{++++\alpha+}\big)\,,\\
&\delta^*_\epsilon h^{++\alpha+\dot\alpha+} = \epsilon^{-\alpha}h^{++++\dot\alpha+} - \bar\epsilon^{-\dot\alpha}h^{++++\alpha+}\,. \qquad (C.5)
\end{aligned}
$$

Special conformal transformations:

$$
\begin{aligned}
&\delta^*_k h^{++\alpha\dot\alpha\,\beta\dot\beta} = k_{\lambda\dot\rho}x^{\alpha\dot\rho}h^{++\lambda\dot\alpha\,\beta\dot\beta} + k_{\rho\dot\lambda}x^{\rho\dot\alpha}h^{++\alpha\dot\lambda\,\beta\dot\beta} + (\alpha,\dot\alpha \Leftrightarrow \beta,\dot\beta)\,,\\
&\delta^*_k h^{++[\beta+\gamma]+} = (k\cdot x)h^{++[\beta+\gamma]+} + 2k^{\rho}_{\dot\alpha}\theta^+_\rho h^{++[\beta+\gamma]\dot\alpha}\,, \quad \delta^*_k h^{++[\dot\beta+\dot\gamma]+} = -(\delta^*_k\widetilde{h^{++[\dot\beta+\dot\gamma]+}})\\
&\delta^*_k h^{(+6)} = 8i\big(k_{\gamma\dot\beta}\theta^{+\gamma}\,h^{++++\dot\beta+} - k_{\beta\dot\beta}\bar\theta^{+\dot\beta}\,h^{++++\beta+}\big),\\
&\delta^*_k h^{++\beta+\alpha\dot\alpha} = k_{\lambda\dot\rho}x^{\beta\dot\rho}h^{++\lambda+\alpha\dot\alpha} + k_{\lambda\dot\rho}x^{\alpha\dot\rho}h^{++\beta+\lambda\dot\alpha} + k_{\rho\dot\lambda}x^{\rho\dot\alpha}h^{++\beta+\alpha\dot\lambda} + k_{\rho\dot\beta}\theta^{+\rho}\,h^{++\alpha\dot\alpha\,\beta\dot\beta}\,,\\
&\delta^*_k h^{++\dot\beta+\alpha\dot\alpha} = \delta^*_k\widetilde{h^{++\beta+\alpha\dot\alpha}}\,,\\
&\delta^*_k h^{++++\beta+} = k_{\lambda\dot\rho}x^{\beta\dot\rho}\,h^{++++\lambda+} + 4ik_{\gamma\dot\rho}\bar\theta^{+\dot\rho}\,h^{++[\beta+\gamma]+}\,, \ \delta^*_k h^{++++\dot\beta+} = (\delta^*_k\widetilde{h^{++++\beta+}})\,,\\
&\delta^*_k h^{++\alpha\dot\alpha++} = k_{\lambda\dot\rho}x^{\beta\dot\rho}\,h^{++\lambda\dot\alpha++} + k_{\rho\dot\lambda}x^{\rho\dot\alpha}\,h^{++\alpha\dot\lambda++} + 4i\big(k_{\lambda\dot\rho}\theta^{+\lambda}\,h^{++\dot\rho+\alpha\dot\alpha}\epsilon^{-\alpha} - k_{\lambda\dot\rho}\bar\theta^{+\dot\rho}\,h^{++\lambda+\alpha\dot\alpha}\big),\\
&\delta^*_k h^{++\alpha+\dot\alpha+} = k_{\lambda\dot\rho}x^{\beta\dot\rho}\,h^{++\lambda+\dot\alpha+} + k_{\rho\dot\lambda}x^{\rho\dot\alpha}\,h^{++\alpha+\dot\lambda+} + k_{\lambda\dot\rho}\theta^{+\lambda}\,h^{++\dot\rho+\alpha\dot\alpha}\epsilon^{-\alpha} - k_{\lambda\dot\rho}\bar\theta^{+\dot\rho}\,h^{++\lambda+\alpha\dot\alpha}.
\end{aligned}
$$
(C.6)

A curious feature of the realization (C.5) and (C.6) is that with respect to it the set of analytic gauge superfields is divided into an invariant subset and a quotient over this subset. The invariant subspace is spanned by the potentials

$$\begin{aligned}&\hat{h}^{++} := \epsilon_{\alpha\beta}\epsilon_{\dot{\alpha}\dot{\beta}}h^{++\alpha\dot{\alpha}\,\beta\dot{\beta}}\,,\ \hat{h}^{+3\dot{\alpha}} := \epsilon_{\alpha\beta}h^{++\alpha+\beta\dot{\alpha}},\ \bar{h}^{+3\dot{\alpha}},\ \hat{h}^{+4} := \frac{1}{2}\epsilon_{\alpha\beta}h^{++[\beta+\alpha]+}\,,\ \bar{\hat{h}}^{+4}\,,\\ &h^{(+6)}\,,\ h^{++++\alpha+},\ h^{++++\dot{\alpha}+} = \widetilde{h^{++++\alpha+}},\ \hat{g}^{+4\alpha\dot{\alpha}} := h^{++\alpha\dot{\alpha}++} - 4ih^{++\alpha+\dot{\alpha}+},\end{aligned} \tag{C.7}$$

while the quotient by

$$h^{++(\alpha\beta)\,(\dot{\alpha}\dot{\beta})},\ h^{+++(\alpha\beta)\dot{\alpha}},\ \widetilde{h^{+++(\alpha\beta)\dot{\alpha}}},\ g^{+4\alpha\dot{\alpha}} := h^{++\alpha\dot{\alpha}++} + 4ih^{++\alpha+\dot{\alpha}+}. \tag{C.8}$$

The closedness of (C.7) under both (C.5) and (C.6) can be readily checked. The remaining set (C.8) transforms through (C.7) and itself.

Inspecting the linearized gauge transformations (6.26a) and (6.26b), we observe that the gauge potentials from the set (C.5) are transformed through the restricted set of gauge parameters

$$\begin{aligned}&\epsilon_{\alpha\beta}\epsilon_{\dot{\alpha}\dot{\beta}}\lambda^{\alpha\beta\dot{\alpha}\dot{\beta}},\quad \epsilon_{\alpha\beta}\lambda^{\alpha\dot{\beta}\beta}\,(\text{and } c.c.),\quad \lambda^{(+4)}\,,\quad \epsilon_{\alpha\beta}\lambda^{[\alpha+\beta]+}\,(\text{and } c.c.),\\ &\lambda^{++\alpha+}\,(\text{and } c.c.),\quad \lambda^{++\alpha\dot{\alpha}} - 8i\lambda^{\alpha+\dot{\alpha}+}\,.\end{aligned} \tag{C.9}$$

Based on this observation, we can choose the gauge in which all potentials from the set (C.7) are equal to zero and end up with (C.8) as encoding the irreducible gauge $\mathbf{s} = 3$ supermultiplet. Such a gauge does not break rigid superconformal symmetry at all. Note that, instead of choosing the gauge $h^{++\alpha\dot{\alpha}++} - 4ih^{++\alpha+\dot{\alpha}+} = 0$, in section 6.3 (and Appendix A) we imposed the equivalent gauge $h^{++\alpha+\dot{\alpha}+} = 0$, which is technically more convenient. Looking at the ϵ and k-transformations of $h^{++\alpha+\dot{\alpha}}$ in (C.5) and (C.6) we observe that in the latter case the r.h.s. of the k-transformation contains the "physical" non-zero gauge potentials $h^{+3(\dot{\rho}\dot{\alpha})\alpha}$ and $h^{+3(\lambda\alpha)\dot{\alpha}}$. So this gauge seemingly breaks superconformal covariance. However, it is easy to check that in the WZ gauge (6.27) for these gauge potentials the sum of the problematic terms in $\delta^*_k h^{++\alpha+\dot{\alpha}}$ vanishes. So the breaking just mentioned is in fact fictitious.

As the last topic of this Appendix, we discuss the modification (before imposing any gauge) of the superconformal properties of h^{++} compared to the standard superconfomal law (4.4) of the spin $\mathbf{1}$ analytic gauge potential. As before, we will deal with the passive form of the conformal transformations. The modification appears only in the realization of special conformal transformations due to the property that such transformations of the bilinear products of partial derivatives in $\hat{\mathcal{H}}_{s=3}$ contain terms with one derivative. After integrating by parts, with taking into account that Ω_{sc} defined in (3.10) is reduced to $2(x\cdot k)$ for k-transformations, we obtain the following addition to the conformal transformation of h^{++}:

$$\delta^*_k h^{++} = 2k_{\alpha\dot{\alpha}}\big(\partial^-_{\beta}h^{+++(\beta\alpha)\dot{\alpha}} + \partial^-_{\dot{\beta}}h^{+++(\dot{\beta}\dot{\alpha})\alpha} - \partial_{\beta\dot{\beta}}h^{++(\alpha\beta)(\dot{\alpha}\dot{\beta})} - \frac{1}{2}\partial^{--}h^{++\alpha\dot{\alpha}++}\big).$$

It is easy to find the compensating gauge transformation of h^{++} of the type (6.26c), which ensures the k-invariance of the gauge $h^{++} = 0$ (with WZ gauge (6.27) for all other potentials).

References

[1] E. S. Fradkin, M. Ya. Palchik, *Conformal Quantum Field Theory in D-dimensions*, Springer, 1996, 466 p.

[2] P. Di Francesco, P. Mathieu, D. Senechal, *Conformal field theory*, Springer, 1997, 911 p.

[3] M. R. Gaberdiel, *An introduction to conformal field theory*, Rept. Prog. Phys. **63** (2000) 607-667, [arXiv: hep-th/9910156].

[4] S. Rychkov, *EPFL Lectures on Conformal Field Theory in $D \geq 3$ Dimensions*, [arXiv:1601.05000 [hep-th]].

[5] I. L. Buchbinder, S. M. Kuzenko, *Ideas and Methods of Supersymmetry and Supergravity or a Walk Through Superspace*, IOP Publishing, 1996, 656 p.

[6] D. Z. Freedman and A. Van Proeyen, *Supergravity*, Cambridge Univ. Press, 2012, 626 p.

[7] M. A. Vasiliev, *Higher spin gauge theories in various dimensions*, PoS **JHW2003** (2003) 003, [arXiv:hep-th/0401177 [hep-th]].

[8] X. Bekaert, S. Cnockaert, C. Iazeolla and M. A. Vasiliev, *Nonlinear higher spin theories in various dimensions*, [arXiv:hep-th/0503128 [hep-th]].

[9] X. Bekaert, N. Boulanger, A. Campoleoni, M. Chiodaroli, D. Francia, M. Grigoriev, E. Sezgin and E. Skvortsov, *Snowmass White Paper: Higher Spin Gravity and Higher Spin Symmetry*, [arXiv:2205.01567 [hep-th]].

[10] A. A. Tseytlin, *On limits of superstring in $AdS(5) \times S^5$*, Theor. Math. Phys. **133** (2002) 1376-1389, [arXiv:hep-th/0201112 [hep-th]].

[11] M. R. Gaberdiel and R. Gopakumar, *String Theory as a Higher Spin Theory*, JHEP **09** (2016) 085, [arXiv:1512.07237 [hep-th]].

[12] D. Ponomarev, *Basic introduction to higher-spin theories*, [arXiv:2206.15385 [hep-th]].

[13] C. Fronsdal, *Massless Fields with Integer Spin*, Phys. Rev. D **18** (1978) 3624.

[14] J. Fang and C. Fronsdal, *Massless Fields with Half Integral Spin*, Phys. Rev. D **18** (1978) 3630.

[15] E. S. Fradkin and A. A. Tseytlin, *Conformal supergravity*, Phys. Rept. **119** (1985) 233-362.

[16] R. R. Metsaev, *Ordinary-derivative formulation of conformal totally symmetric arbitrary spin bosonic fields*, JHEP **06** (2012) 062, [arXiv:0709.4392 [hep-th]].

[17] M. A. Vasiliev, *Bosonic conformal higher-spin fields of any symmetry*, Nucl. Phys. B **829** (2010) 176-224, [arXiv:0909.5226 [hep-th]].

[18] E. Joung and K. Mkrtchyan, *A note on higher-derivative actions for free higher-spin fields*, JHEP **11** (2012) 153, [arXiv:1209.4864 [hep-th]].

[19] T. Nutma and M. Taronna, *On conformal higher spin wave operators*, JHEP **06** (2014) 066, [arXiv:1404.7452 [hep-th]].

[20] T. Basile, X. Bekaert and E. Joung, *Conformal Higher-Spin Gravity: Linearized Spectrum = Symmetry Algebra*, JHEP **11** (2018) 167, [arXiv:1808.07728 [hep-th]].

[21] M. Grigoriev and A. A. Tseytlin, *On conformal higher spins in curved background*, J. Phys. A **50** (2017) 125401, [arXiv:1609.09381 [hep-th]].

[22] M. Beccaria and A. A. Tseytlin, *On induced action for conformal higher spins in curved background*, Nucl. Phys. B **919** (2017) 359-383, [arXiv:1702.00222 [hep-th]].

[23] R. Manvelyan and G. Poghosyan, *Geometrical structure of Weyl invariants for spin three gauge field in general gravitational background in* $d = 4$, Nucl. Phys. B **937** (2018) 1-27, [arXiv:1804.10779 [hep-th]].

[24] S. M. Kuzenko and M. Ponds, *Conformal geometry and (super)conformal higher-spin gauge theories*, JHEP **05** (2019) 113, [arXiv:1902.08010 [hep-th]].

[25] S. M. Kuzenko and M. Ponds, *Generalised conformal higher-spin fields in curved backgrounds*, JHEP **04** (2020) 021, [arXiv:1912.00652 [hep-th]].

[26] S. M. Kuzenko, M. Ponds and E. S. N. Raptakis, *New locally (super)conformal gauge models in Bach-flat backgrounds*, JHEP **08** (2020) 068, [arXiv:2005.08657 [hep-th]].

[27] T. Basile, M. Grigoriev and E. Skvortsov, *Covariant action for conformal higher spin gravity*, [arXiv:2212.10336 [hep-th]].

[28] E. S. Fradkin and V. Y. Linetsky, *Cubic Interaction in Conformal Theory of Integer Higher Spin Fields in Four-dimensional Space-time*, Phys. Lett. B **231** (1989) 97-106.

[29] E. S. Fradkin and V. Y. Linetsky, *Superconformal Higher Spin Theory in the Cubic Approximation*, Nucl. Phys. B **350** (1991) 274-324.

[30] X. Bekaert, E. Joung and J. Mourad, *Effective action in a higher-spin background*, JHEP **02** (2011) 048, [arXiv:1012.2103 [hep-th]].

[31] R. Bonezzi, *Induced Action for Conformal Higher Spins from Worldline Path Integrals*, Universe **3** (2017) 64, [arXiv:1709.00850 [hep-th]].

[32] S. M. Kuzenko, M. Ponds and E. S. N. Raptakis, *Conformal Interactions Between Matter and Higher-Spin (Super)Fields*, Fortsch. Phys. **71** (2023) 1, [arXiv:2208.07783 [hep-th]].

[33] A. Y. Segal, *Conformal higher spin theory*, Nucl. Phys. B **664** (2003) 59-130, [arXiv:hep-th/0207212 [hep-th]].

[34] S. M. Kuzenko, R. Manvelyan and S. Theisen, *Off-shell superconformal higher spin multiplets in four dimensions*, JHEP **07** (2017) 034, [arXiv:1701.00682 [hep-th]].

[35] S. M. Kuzenko, A. G. Sibiryakov and V. V. Postnikov, *Massless gauge superfields of higher half integer superspins*, JETP Lett. **57** (1993) 534-538.

[36] S. M. Kuzenko and A. G. Sibiryakov, *Massless gauge superfields of higher integer superspins*, JETP Lett. **57** (1993) 539-542.

[37] S. M. Kuzenko and A. G. Sibiryakov, *Free massless higher superspin superfields on the anti-de Sitter superspace*, Phys. Atom. Nucl. **57** (1994) 1257-1267, [arXiv:1112.4612 [hep-th]].

[38] P. S. Howe, K. S. Stelle and P. K. Townsend, *Suprecurrents*, Nucl. Phys. B **192** (1981) 332-352.

[39] S. M. Kuzenko, M. Ponds and E. S. N. Raptakis, *Generalised superconformal higher-spin multiplets*, JHEP **03** (2021) 183, [arXiv:2011.11300 [hep-th]].

[40] J. Hutomo and S. M. Kuzenko, *The massless integer superspin multiplets revisited*, JHEP **02** (2018) 137, [arXiv:1711.11364 [hep-th]].

[41] S. M. Kuzenko and E. S. N. Raptakis, *Extended superconformal higher-spin gauge theories in four dimensions*, JHEP **12** (2021) 210, [arXiv:2104.10416 [hep-th]].

[42] D. Hutchings, S. M. Kuzenko and E. S. N. Raptakis, *The $\mathcal{N} = 2$ superconformal gravitino multiplet*, Phys. Lett. B **845** (2023), 138132 [arXiv:2305.16029 [hep-th]].

[43] D. Butter, *N=2 Conformal Superspace in Four Dimensions*, JHEP **10** (2011) 030, [arXiv:1103.5914 [hep-th]].

[44] I. Buchbinder, E. Ivanov and N. Zaigraev, *Unconstrained off-shell superfield formulation of 4D, $\mathcal{N} = 2$ supersymmetric higher spins*, JHEP **12** (2021) 016, [arXiv:2109.07639 [hep-th]].

[45] I. Buchbinder, E. Ivanov and N. Zaigraev, *$\mathcal{N} = 2$ higher spins: superfield equations of motion, the hypermultiplet supercurrents, and the component structure*, JHEP **03** (2023) 036, [arXiv:2212.14114 [hep-th]].

[46] I. Buchbinder, E. Ivanov and N. Zaigraev, *Off-shell cubic hypermultiplet couplings to $\mathcal{N} = 2$ higher spin gauge superfields*, JHEP **05** (2022) 104, [arXiv:2202.08196 [hep-th]].

[47] I. Buchbinder, E. Ivanov and N. Zaigraev, *Unconstrained $\mathcal{N} = 2$ higher-spin gauge superfields and their hypermultiplet couplings*, JHEP **03** (2023) 036, [arXiv:2211.09501 [hep-th]].

[48] E. Ivanov, *Higher Spins in Harmonic Superspace*, Teor. Mat. Fiz. **217** (2023) 3 515-532 [Theor. Math. Phys. **217** (2023) 1855-1869], [arXiv:2306.10401 [hep-th]].

[49] A. Galperin, E. Ivanov, V. Ogievetsky, E. Sokatchev, *Harmonic superspace: key to $N = 2$ supersymmetric theories*, Pis'ma ZhETF **40** (1984) 155 [JETP Lett. **40** (1984) 912].

[50] A. S. Galperin, E. A. Ivanov, S. Kalitzin, V. I. Ogievetsky, E. S. Sokatchev, *Unconstrained $\mathcal{N} = 2$ Matter, Yang-Mills and Supergravity Theories in Harmonic Superspace*, Class. Quant. Grav. **1** (1984) 469-498 [Erratum: Class. Quant. Grav. **2** (1985) 127].

[51] A. S. Galperin, E. A. Ivanov, V. I. Ogievetsky, E. S. Sokatchev, *Harmonic superspace*, Cambridge Monographs on Mathematical Physics, Cambridge University Press, 2001, 306 p.

[52] A. Galperin, E. Ivanov, V. Ogievetsky and E. Sokatchev, *Hyperkahler Metrics and Harmonic Superspace*, Commun. Math. Phys. **103** (1986) 515.

[53] E. Ivanov, *$\mathcal{N} = 2$ Supergravities in Harmonic Superspace*, [arXiv:2212.07925 [hep-th]].

[54] A. Galperin, E. Ivanov, V. Ogievetsky and E. Sokatchev, *Conformal invariance in harmonic superspace*, JINR-E2-85-363.

[55] A. Fotopoulos, N. Irges, A. C. Petkou and M. Tsulaia, *Higher-Spin Gauge Fields Interacting with Scalars: The Lagrangian Cubic Vertex*, JHEP **10** (2007) 021, [arXiv:0708.1399 [hep-th]].

[56] A. S. Galperin, N. A. Ky and E. Sokatchev, *$\mathcal{N}$=2 Supergravity in Superspace: Solution to the Constraints*, Class. Quant. Grav. **4** (1987) 1235.

[57] A. S. Galperin, E. A. Ivanov, V. I. Ogievetsky and E. Sokatchev, *$\mathcal{N}$=2 Supergravity in Superspace: Different Versions and Matter Couplings*, Class. Quant. Grav. **4** (1987) 1255.

[58] B. M. Zupnik, *Background harmonic superfields in $\mathcal{N}$=2 supergravity*, Theor. Math. Phys. **116** (1998) 964-977, [arXiv:hep-th/9803202 [hep-th]].

[59] S. M. Kuzenko and S. Theisen, *Correlation functions of conserved currents in $\mathcal{N}$=2 superconformal theory*, Class. Quant. Grav. **17** (2000) 665-696, [arXiv:hep-th/9907107 [hep-th]].

[60] V. O. Rivelles and J. G. Taylor, *Linearized $\mathcal{N}$=2 superfield supergravity*, J. Phys. A **15** (1982) 163.

[61] S. J. Gates, Jr. and W. Siegel, *Linearized $\mathcal{N}$=2 superfield supergravity*, Nucl. Phys. B **195** (1982) 39-60.

[62] D. Butter and S. M. Kuzenko, *$\mathcal{N}$=2 supergravity and supercurrents*, JHEP **12** (2010) 080, [arXiv:1011.0339 [hep-th]].

[63] S. M. Kuzenko and E. S. N. Raptakis, *On higher-spin $\mathcal{N}$ = 2 supercurrent multiplets*, JHEP **05** (2023) 056, [arXiv:2301.09386 [hep-th]].

[64] E. Bergshoeff, M. de Roo and B. de Wit, *Extended Conformal Supergravity*, Nucl. Phys. B **182** (1981) 173-204.

[65] M. Muller, *Minimal $\mathcal{N} = 2$ Supergravity in Superspace*, Nucl. Phys. B **282** (1987) 329-348.

[66] T. L. Curtright and P. G. O. Freund, *Masssive dual fields*, Nucl. Phys. B **172** (1980) 413-424.

[67] T. Curtright, *Generalized gauge fields*, Phys. Lett. B **165** (1985) 304-308.

[68] V. I. Ogievetsky and I. V. Polubarinov, *The notoph and its possible interactions*, Yad. Fiz. **4** (1966) 216-223.

[69] M. Kalb and P. Ramond, *Classical direct interstring action*, Phys. Rev. D **9** (1974) 2273-2294.

[70] E. A. Ivanov, *Gauge Fields, Nonlinear Realizations, Supersymmetry*, Phys. Part. Nucl. **47** (2016) no.4 508-539 [arXiv:1604.01379 [hep-th]].

[71] I. L. Buchbinder, I. L. Shapiro, *Introduction to Quantum Field Theory with Applicaions to Quantum Gravity*, Oxford Graduate Texts, Oxford Univ. Press, 2021, 525 p.

[72] L. Bonora, M. Cvitan, P. Dominis Prester, S. Giaccari, B. Lima de Souza, T. Stemberga, *One-loop effective actions and higher spins*, JHEP **12** (2016) 084, [arXiv:1609.02088 [hep-th]].

[73] L. Bonora, M. Cvitan, P. Dominis Prester, S. Giaccari, T. Stemberga, *One-loop effective actions and higher spins. Part II*, JHEP **01** (2018) 080, [arXiv:1709.01738 [hep-th]].

[74] S.M. Kuzenko, J. La Fontaine and M. Ponds, *Induced action for superconformal higher-spin multiplets using SCFT techniques*, Phys. Lett. B **839** (2023) 137818, [arXiv:2212.00468 [hep-th]].