

A COUPLED FLUID-DYNAMICS-HEAT TRANSFER MODEL FOR 3D SIMULATIONS OF THE AQUEOUS HUMOR FLOW IN THE HUMAN EYE

Thomas Saigre¹, Christophe Prud'homme¹, Marcela Szopos², and Vincent Chabannes¹

¹Institut de Recherche Mathématique Avancée, UMR 7501 Université de Strasbourg et CNRS

²Université Paris Cité, CNRS, MAP5, F-75006 Paris, France

SUMMARY

Understanding human eye behavior involves intricate interactions between physical phenomena such as heat transfer and fluid dynamics. Accurate computational models are vital for comprehending ocular diseases and therapeutic interventions. This work focuses on modeling and simulating aqueous humor flow in the anterior and posterior chambers of the eye, coupled with overall heat transfer. Aqueous humor dynamics regulates intraocular pressure, crucial for understanding conditions like glaucoma. Convective effects from temperature disparities also influence this flow. Extending prior research, this work develops a comprehensive three-dimensional computational model to simulate coupled fluid-dynamic-heat transfer model, thus contributing to the understanding of ocular physiology.

Key words: *mathematical and computational ophthalmology, finite element method, thermo-fluid dynamics*

1 INTRODUCTION

Understanding the behavior of the human eye is a challenging task, as it involves the study of the interaction between different physical phenomena, such as heat transfer, fluid dynamics, and tissue deformation. It is therefore crucial to develop accurate and efficient computational models to simulate complex multi-physics physiology of the ocular system, in order to gain a better understanding of the mechanisms of ocular diseases and the associated therapeutical interventions. In the present work, we focus on the simulation of the flow of the aqueous humor (AH) in the anterior and posterior chambers of the human eye, and its coupling with the heat transfer inside the eyeball.

The AH is a transparent fluid produced by the ciliary body (see Fig. 1(a)), that flows from the posterior chamber (PC) to the anterior chamber (AC), where it is drained through two pathways, the trabecular meshwork, and the uveoscleral pathway. AH flow plays a fundamental role in the maintenance of the intraocular pressure (IOP) level. IOP is a key parameter since when abnormally elevated, it is a major risk factor for degenerative ocular diseases such as glaucoma [1]. In addition to the hydraulic pressure difference created by production and drainage, AH dynamic is influenced by posture and thermal factors. Specifically, there are convective effects produced by the temperature difference between the external environment at the corneal surface and the internal surface, which is at the body temperature.

Numerical investigations of this complex dynamics have been proposed by several authors, such as [2, 3] where the model describes flow coupled with heat transfer solely in the AC and PC; [4] where the impact of pressure on the AH flow and drainage is studied; or [5, 6] where a coupled model for the thermo-fluid-dynamics of the AH flow in the AC with boundary conditions on the velocity is investigated. The present work develops a three-dimensional computational model allowing to simulate the heat transfer inside the whole human eyeball, taking into account the flow of AH in the AC and PC.

This contribution represents an extension of the work presented in [7], where heat transfer inside the human eyeball was investigated with a special focus on the parametric effects.

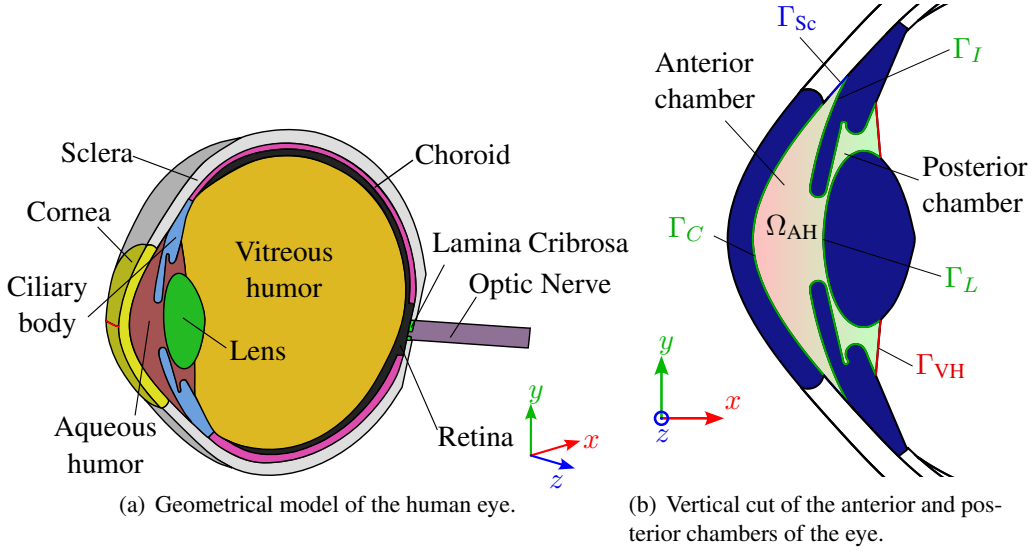


Figure 1: Description of the human eyeball, and the anterior and posterior chambers of the eye.

2 METHODOLOGY

We denote by Ω the domain of the human eye presented in Fig. 1(a). This domain is divided in ten subdomains, representing the different parts of the eye, such as the cornea, the lens, the vitreous body, the retina, *etc.*, with various physical properties. Precisely, we denote by Ω_{AH} the anterior and posterior chambers of the eye (brown part in Fig. 1(a)), which is filled with the AH, as shown in Fig. 1(b).

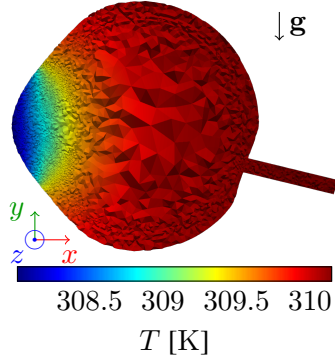


Figure 2: Distribution of the computed temperature over the eyeball in the standing position, on a vertical cut. Mesh discretization is also presented.

In the spirit of [6, 2], we make the following assumptions: the fluid is incompressible, its density is constant, and we neglect the effect of the cornea and the lens on the flow. We hence consider that the steady flow of the aqueous humor is governed by the Navier-Stokes equations, coupled with the heat transfer:

$$\rho(\mathbf{u} \cdot \nabla \mathbf{u}) - \mu \nabla^2 \mathbf{u} + \nabla p = -\rho\beta(T - T_{\text{ref}})\mathbf{g} \quad \text{in } \Omega_{AH}, \quad (1a)$$

$$\nabla \cdot \mathbf{u} = 0 \quad \text{in } \Omega_{AH}, \quad (1b)$$

$$\rho C_p \mathbf{u} \cdot \nabla T - k \nabla^2 T = 0 \quad \text{in } \Omega, \quad (1c)$$

where μ [N s/m²] is the dynamic viscosity of the fluid, ρ [kg/m³] its density (both at reference temperature T_{ref} [K]), C_p [J kg⁻¹ K⁻¹] its specific heat, k [W m⁻¹ K⁻¹] its thermal conductivity. The quantity T [K] is the temperature of the eye, while p [Pa] is the pressure of the aqueous humor fluid (also expressed in mmHg in biologic contexts), and \mathbf{u} [m s⁻¹] is its velocity. The right-hand side term in Eq. (1a) represents the gravitational force per unit volume, along with the Boussinesq approximation, utilized to account for the buoyancy effects. The coefficient β [K⁻¹] is the fluid volume expansion coefficient, and \mathbf{g} [m s⁻²] the gravitational acceleration vector. Depending on the position of the patient (standing, laying supine or prone respectively), \mathbf{g} can be either vertical ($\mathbf{g} = [0, -g, 0]^T$) or horizontal ($\mathbf{g} = [g, 0, 0]^T$, $\mathbf{g} = [-g, 0, 0]^T$ respectively), where g is the gravitational acceleration.

We impose the following boundary conditions for the fluid velocity:

$$\mathbf{u} = \mathbf{0} \quad \text{on } \Gamma_C \cup \Gamma_I \cup \Gamma_L \cup \Gamma_{VH} \cup \Gamma_{Sc}, \quad (2)$$

that, together with Eq. (1a), only incorporates thermal and postural mechanisms driving the AH flow. The hydraulic pressure difference due to production and drainage of AH is not currently accounted for in the model, since previous works [5, 8] already pointed out that buoyancy is observed to be the dominant source of the convective motion, regardless of the postural orientation.

Moreover, the heat transfer inside the eye is governed by the heat equation (1c), with the following boundary conditions, taking into account the convective heat transfer with the surrounding tissues, over a domain denoted as Γ_{body} ; and the heat production due to the metabolism of the eye, as well as the heat transfer with the ambient air, over a domain called Γ_{amb} :

$$-k \frac{\partial T}{\partial \mathbf{n}} = h_{\text{bl}}(T - T_{\text{bl}}) \quad \text{on } \Gamma_{\text{body}}, \quad (3a)$$

$$-k \frac{\partial T}{\partial \mathbf{n}} = (h_{\text{amb}} + h_r)(T - T_{\text{bl}}) + E \quad \text{on } \Gamma_{\text{amb}}. \quad (3b)$$

Thanks to the open source library Feel++ [9], we develop a computational framework allowing to simulate the model (1–2–3), using the finite element method. We solve the monolithic systems with a Newton method, and use a preconditioner based on the PDE system proposed by [10, Ch. 11]. The overall preconditioner is block diagonal with respect to the fluid and heat unknowns. The fluid part preconditioner is block upper triangular with the pressure block being the Schur complement [10], the other ones corresponding to the full velocity preconditioned by an additive Schwarz method (ASM) and the velocity pressure coupling respectively. The heat block is preconditioned also with an ASM.

The geometry presented in Fig. 1(a) is discretized, with a significant effort made on the AC and PC domains, where the coupled fluid-thermal problem is solved. This results in a mesh composed of $5.78 \cdot 10^6$ elements. The refinement of the mesh is presented among results in Fig. 2.

3 RESULTS AND CONCLUSIONS

We present the results of the simulations, for a set of parameters presented in Tab. 1. These parameters are chosen to fit the case simulating a healthy patient, with nominal physiological values.

We show in Fig. 2 the distribution of the computed temperature over the eyeball in the standing position, on a vertical cut. The temperature is higher at the back part of the eye, as this part is inside the human body, while the front part, where heat exchanges with the ambient air are present, is colder. This is consistent with previous findings [7, 5].

Now we focus on the impact of the position of the subject on the flow of the AH in the AC. We present in Fig. 3, the results for standing and laying positions, prone and supine. The main striking result is the difference in the flow patterns: it follows the gravitational force, and is more important in the standing position (Fig. 3(a)). Moreover, we notice the formation of Krukenberg’s spindle and phenomena of recirculation in the AC, as observed in the literature [6, 2, 3].

Parameter	Value
μ	$0.001 \text{ kg m}^{-1} \text{ s}^{-1}$
ρ	1000 kg m^{-3}
C_p	$4178 \text{ J kg}^{-1} \text{ K}^{-1}$
β	$3 \times 10^{-4} \text{ K}^{-1}$
k_{AH}	$0.576 \text{ W m}^{-1} \text{ K}^{-1}$
g	9.81 m s^{-2}
T_{ref}	298 K
h_{bl}	$65 \text{ W m}^{-2} \text{ K}^{-1}$
h_{amb}	$10 \text{ W m}^{-2} \text{ K}^{-1}$
h_r	$6 \text{ W m}^{-2} \text{ K}^{-1}$
E	40 W m^{-3}
T_{bl}	310 K
T_{amb}	307 K

Table 1: Parameters used for the simulations. Sources can be found in [5, 2].

Conclusions and perspectives. This work presented a modeling and computational framework allowing to simulate the heat transfer inside the human eyeball, coupled with the flow of the AH, in the anterior and posterior chambers of a healthy eye. The results on flow patterns and temperature distribution are in line with previous numerical studies. In addition, the impact of the postural orientation on the flow recirculations is accurately captured. One main drawback of the present model is the computational cost, which is too high for real-time simulations. In order to overcome this issue, we are currently working on the development of a model order reduction technique for the present problem, to enable real-time simulations of coupled flow-heat transfer inside the human eyeball. From the clinical perspective, such an approach could be utilized to assess the effect of the topical administration of different drugs (eye drops) that are nowadays the standards therapeutical approach for ocular diseases such as glaucoma.

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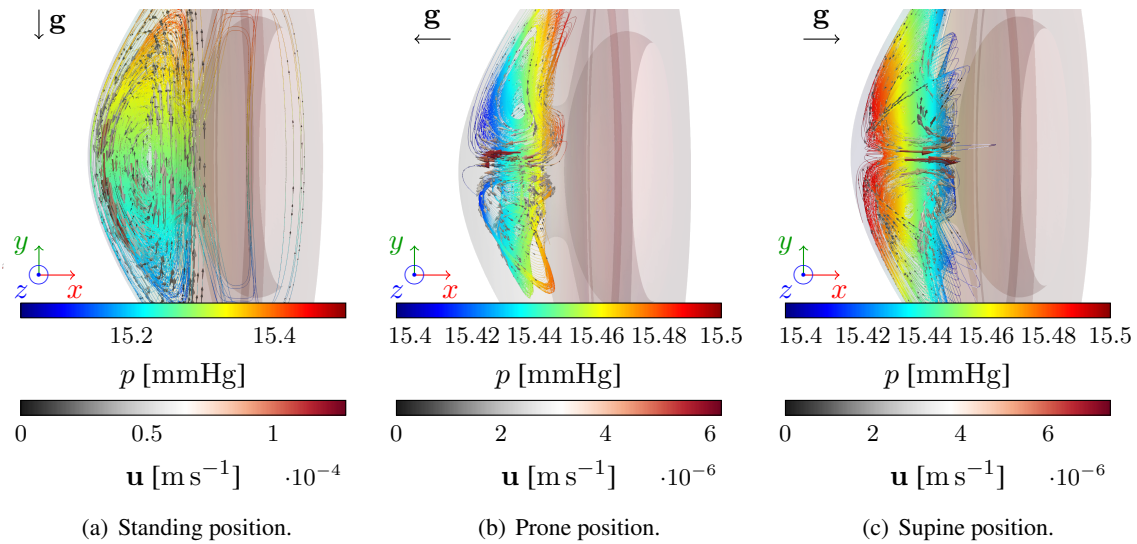


Figure 3: Results of simulation for various orientation of the eye. Streamlines are colored according to the pressure, and the arrows show the fluid velocity.

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