$f(T)$ gravity after DESI Baryon Acoustic Oscillation and DES Supernovae 2024 data

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In this *letter* we investigate new constraints on $f(T)$ gravity using the recent Baryon Acoustic Oscillation (BAO) data released by the Dark Energy Spectroscopic Instrument (DESI) and the Type Ia supernovae (SNIa) catalog from the full 5-years of the Dark Energy Survey Supernova Program (DES-SN5YR). The $f(T)$ cosmological models considered are characterised by power law late-time accelerated expansion. Our results show that the combination DESI BAO $+r_d$ CMB Planck suggests a Bayesian preference for late-time $f(T)$ cosmological models over ΛCDM, obtaining a value of $H_0 = 72.4 \pm 2.9$[km/s/Mpc] in agreement with SH0ES collaboration.

The *Hubble tension* has reached a statistical significance reaching a 5σ, strongly proven a mismatch between the cosmic late-time expansion rate H_0 measured through the local distance ladder method using Type Ia supernovae measurements [1, 2] $H_0 = 73\pm1$ [km/s/Mpc], and the inferred H_0 value from observations of the Cosmic Microwave Background (CMB) radiation, $H_0 = 67.4 \pm 0.5$ [km/s/Mpc] [3]. While cautious studies on a possible systematic origin of this mismatch have been performed by the SH0ES collaboration [1], there is no signal that this could be the reason for this H_0 tension issue. This result has brought interesting opportunities to change the view on how the standard cosmological models are designed, allowing us to introduce a path beyond the Λ-Cold Dark Matter(CDM) model.

Current BAO measurements released by the *Dark Energy Spectroscopic Instrument* (DESI) [4, 5] seem to point towards new physics in the dark energy cosmological scheme [6]. Additionally, for the *Dark Energy Survey 5-year SNIa* (DES-SN5YR) release [7], it was found that a time-varying dark energy is preferred with an equation-of-state $w \to 0$ at $\sim 2\sigma$. More discussions on this aspect have been presented in [8]. As part of the efforts to find well-constrained proposals with these releases and also, that can address the cosmological tensions, some studies have been developing in these short period, e.g. new constraints on axion-early dark energy model [9] which are not tighten even after the inclusion of DESI BAO data, interacting dark energy models [10] which shows a value of H_0 in good agreement with SH0ES collaboration, for quintessence scalar field model [11, 12] showing a preference within 2-4% for a kinetic scalar field energy, for dark energy models inspired in thermodynamics and parametrised equation-of-state in Taylor expansions [13, 14], the first one finding that log-corrected parameterisation could alleviate the H_0 tension, and finally Gaussian reconstructions on quintom modified cosmology [15]. All these studies aim to increase the value of H_0 inferred. On one hand, the main challenge in the early CMB measurements seems to be settled in computing the angular scale of the CMB acoustic peaks [3]. Therefore, increasing the value of H_0 without modifying the acoustic scale requires a different post-recombination epoch [16]. On the other hand, late-time cosmic proposals require new physics that can change cosmic distances to compensate for the higher value of H_0, taking into account the preservation of the CMB history.

Within these efforts, *extended theories of gravity* have been proposed as a good description of a fundamental theory of gravity that allows addressing theoretical and observational issues with viable solutions in the observed mismatch [17–25]. To formulate an extension to General Relativity, we consider a construction through the metric-affine gravity [26], where *teleparallel gravity* (TG) has a curvature-free connection [17, 27] with a scenario that include a *teleparallel equivalent of general relativity* (TEGR). This theory has described a set of field equations which are dynamically equivalent to the GR ones. Within this scheme, $f(T)$ gravity emerges as a generalisation of the TEGR Lagrangian with a function of the torsion T as $f(T) = -T + \mathcal{F}(T)$.

In this *letter*, we show that the new DESI BAO plus r_d CMB Planck data release gives a preference for extended $f(T)$ cosmologies within 2σ confidence level (C.L) and that high/low-z observations could be better explained in these models in comparison to ΛCDM and with first principle reasons. In such a scheme, we also consider baseline with DES-SN5YR, which gives a lower value of H_0 for this kind of supernovae catalog.

To derive our extended cosmology, we start with the the action [28–30]:

$$\mathcal{S}_{\mathcal{F}(T)} = \frac{1}{2\kappa^2}\int \mathrm{d}^4x\, e\left[-T + \mathcal{F}(T)\right] + \int \mathrm{d}^4x\, e\mathcal{L}_{\mathrm{m}}\,, \quad (1)$$

where $\kappa^2 = 8\pi G$ and the tetrad determinant is calculated as $e = \det\left(e^a{}_\mu\right) = \sqrt{-g}$. When $\mathcal{F}(T) \to 0$, we recover the concordance ΛCDM model. As it is oftentimes, we consider a flat homogeneous and isotropic geometry as $e^A{}_\mu = \mathrm{diag}\left(1,\, a(t),\, a(t),\, a(t)\right)$ [31, 32], where $a(t)$ is the scale factor. Using the relationship between the metric

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Parameter	DESI+BBN	DESI+CMB	DESI+CC	DESI+CMB +CC+PN$^+$	DESI+CC +PN$^+$	DESI+CC +SNYR5
H_0 [km/s/Mpc]	$68.8^{+1.3}_{-1.2}$	$68.44^{+0.87}_{-0.83}$	$70.6^{+6.8}_{-6.7}$	$68.80^{+0.81}_{-0.84}$	$72.8^{+2.1}_{-2.0}$	$66.3^{+6.2}_{-6.1}$
$\Omega_{\rm cdm}$	$0.239^{+0.027}_{-0.025}$	$0.2382^{+0.0095}_{-0.0097}$	$0.238^{+0.031}_{-0.030}$	$0.2349^{+0.0085}_{-0.0089}$	$0.254^{+0.023}_{-0.022}$	$0.309^{+0.023}_{-0.022}$
w_b	0.02218 ± 0.00077	$0.02259^{+0.00034}_{-0.00033}$	$0.0250^{+0.011}_{-0.0099}$	$0.02271^{+0.00035}_{-0.00034}$	$0.0282^{+0.0038}_{-0.0036}$	$0.0199^{+0.0081}_{-0.0079}$
$\Omega_{\rm m}$	$0.286^{+0.027}_{-0.025}$	$0.286^{+0.011}_{-0.011}$	$0.288^{+0.027}_{-0.025}$	$0.2829^{+0.0095}_{-0.0099}$	$0.308^{+0.022}_{-0.021}$	$0.353^{+0.020}_{-0.019}$
$r_{\rm d}$ [Mpc]	$149.1^{+3.5}_{-3.4}$	$149.06^{+0.73}_{-0.71}$	146.0 ± 14.0	149.03 ± 0.73	$148.6^{+4.6}_{-4.7}$	147^{+15}_{-14}
M	–	–	–	$-19.391^{+0.027}_{-0.033}$	$-19.265^{+0.056}_{-0.059}$	$-19.53^{+0.19}_{-0.21}$
$\chi^2_{\rm min}$	8.92	12.47	23.99	1646.54	1615.32	5933.33

Table I. Constraints at 1-2σ C.L for the ΛCDM model. For all baselines, we provide results with and without BBN constraints. Also, we include the constraints for two SNIa baselines.

Parameter	DESI+BBN	DESI+CMB	DESI+CC	DESI+CMB +CC+PN$^+$	DESI+CC +PN$^+$	DESI+CC +SNYR5
H_0 [km/s/Mpc]	68.5 ± 6.0	69.8 ± 2.5	$70.1^{+7.2}_{-6.9}$	$69.1^{+1.5}_{-1.3}$	72.4 ± 2.2	$66.5^{+6.1}_{-6.4}$
$\Omega_{\rm cdm}$	$0.249^{+0.031}_{-0.034}$	0.246 ± 0.017	$0.243^{+0.042}_{-0.046}$	$0.248^{+0.013}_{-0.012}$	$0.229^{+0.059}_{-0.079}$	$0.251^{+0.053}_{-0.057}$
p_1	$0.04^{+0.45}_{-0.48}$	$-0.12^{+0.21}_{-0.23}$	$0.08^{+0.43}_{-0.47}$	$-0.03^{+0.11}_{-0.12}$	0.32 ± 0.22	0.29 ± 0.20
w_b	$0.02218^{+0.00077}_{-0.00076}$	$0.02260^{+0.00033}_{-0.00033}$	$0.027^{+0.013}_{-0.012}$	$0.02271^{+0.00039}_{-0.00041}$	$0.0377^{+0.011}_{-0.0099}$	$0.026^{+0.013}_{-0.012}$
$\Omega_{\rm m}$	$0.298^{+0.029}_{-0.028}$	$0.292^{+0.021}_{-0.020}$	$0.297^{+0.029}_{-0.029}$	$0.295^{+0.015}_{-0.014}$	$0.301^{+0.049}_{-0.068}$	0.308 ± 0.041
$r_{\rm d}$ [Mpc]	$150.6^{+9.9}_{-8.0}$	$147.12^{+0.79}_{-0.76}$	144^{+14}_{-13}	$147.24^{+0.88}_{-1.0}$	$145.1^{+5.1}_{-4.6}$	147^{+15}_{-14}
M	–	–	–	-19.387 ± 0.040	$-19.261^{+0.065}_{-0.067}$	$-19.51^{+0.20}_{-0.22}$
$\chi^2_{\rm min}$	8.83	11.57	23.86	1646.15	1607.846	5926.77
$\ln \mathcal{B}_{ij}$	0.102	−0.221	−1.68	−2.46	−2.51	2.66

Table II. Constraints at 1-2σ C.L for the f_1 model. For all baselines, we provide results with and without BBN constraints. Also, we include the constraints for two SNIa baselines.

Parameter	DESI+BBN	DESI+CMB	DESI+CC	DESI+CMB +CC+PN$^+$	DESI+CC +PN$^+$	DESI+CC +SNYR5
H_0 [km/s/Mpc]	$68.7^{+2.9}_{-6.2}$	$67.9^{+1.6}_{-2.1}$	$69.1^{+7.2}_{-7.1}$	68.5 ± 1.1	72.3 ± 2.0	$66.5^{+6.1}_{-6.3}$
$\Omega_{\rm cdm}$	$0.258^{+0.030}_{-0.027}$	$0.257^{+0.018}_{-0.015}$	$0.254^{+0.033}_{-0.033}$	0.251 ± 0.011	$0.245^{+0.041}_{-0.039}$	$0.309^{+0.037}_{-0.043}$
$1/p_2$	$0.28^{+0.38}_{-0.34}$	$0.15^{+0.18}_{-0.17}$	$0.29^{+0.37}_{-0.34}$	$0.16^{+0.14}_{-0.19}$	$0.38^{+0.20}_{-0.22}$	$0.21^{+0.21}_{-0.23}$
w_b	$0.02219^{+0.00076}_{-0.00076}$	$0.02260^{+0.00034}_{-0.00034}$	$0.026^{+0.011}_{-0.011}$	$0.02277^{+0.00044}_{-0.00057}$	$0.0336^{+0.0073}_{-0.0076}$	$0.0215^{+0.0096}_{-0.0090}$
$\Omega_{\rm m}$	$0.308^{+0.032}_{-0.028}$	$0.306^{+0.021}_{-0.017}$	$0.308^{+0.030}_{-0.028}$	0.300 ± 0.014	$0.309^{+0.034}_{-0.032}$	$0.357^{+0.031}_{-0.036}$
$r_{\rm d}$ [Mpc]	$150.7^{+6.2}_{-5.4}$	$147.30^{+0.74}_{-0.73}$	144^{+14}_{-13}	$147.32^{+0.98}_{-0.94}$	$145.8^{+4.7}_{-4.3}$	145^{+15}_{-14}
M	–	–	–	$-19.396^{+0.033}_{-0.044}$	$-19.258^{+0.062}_{-0.066}$	$-19.52^{+0.20}_{-0.21}$
$\chi^2_{\rm min}$	8.64	12.47	23.749	1644.85	1606.53	5932.84
$\ln \mathcal{B}_{ij}$	0.839	1.03	−0.321	1.37	2.43	4.73

Table III. Constraints at 1-2σ C.L for the f_2 model. For all baselines, we provide results with and without BBN constraints. Also, we include the constraints for two SNIa baselines.

and the tetrad $g_{\mu\nu} = e^{A}{}_{\mu} e^{B}{}_{\nu} \eta_{AB}$, we can write the flat Friedmann–Lemaître–Robertson–Walker (FLRW) metric as

$$\mathrm{d}s^2 = \mathrm{d}t^2 - a^2(t)\left(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2\right), \qquad (2)$$

with $H = \dot{a}/a$. Subsequently, we can derive the Friedmann equations:

$$H^2 + \frac{T}{3}\mathcal{F}_T - \frac{\mathcal{F}}{6} = \frac{\kappa^2}{3}\rho \qquad (3)$$

$$\dot{H}\left(1 - \mathcal{F}_T - 2T\mathcal{F}_{TT}\right) = -\frac{\kappa^2}{2}\left(\rho + p\right), \qquad (4)$$

where ρ and p, are the energy density and pressure, respectively. We selected $f(T)$ cases where it is possible to reproduce naturally a late-time cosmic acceleration:

- *Power Law Model (f_1)*[33]. – This model is of the form: $f_1(T) = (-T)^{p_1}$, where p_1 is a constant. We can recover ΛCDM model when $p_1 = 0$. Otherwise, if $p_1 = 1$, the extra term gives a re-scaled gravitational constant related to the GR limit. Furthermore, when $p_1 < 1$ gives an accelerating universe. To compare the new constraints for this model using DESI 2024, in [34] was considered BAO measurements from Two-Degree Field Galaxy Redshift Survey (2dFGRS) and SDSS DR7, where it was found that $H_0 = 69.4 \pm 0.8$[km/s/Mpc], $\Omega_m = 0.298 \pm 0.07$ and $p_1 = -0.10^{+0.09}_{-0.07}$, where it is clear that exists a hint of a deviation from the ΛCDM model.
- *Linder Model (f_2)*[29]. – This model is described as: $f_2(T) = T_0(1 - e^{[-p_2\sqrt{T/T_0}]})$, where p_2 is a constant and $T_0 = T|_{t=t_0} = -6H_0^2$. Notice that this model recovers ΛCDM in the limit $p_2 \to +\infty$. As in the latter case, this model was tested using BAO from 2dFGRS and SDSS obtaining $H_0 = 69.6 \pm 0.9$[km/s/Mpc], $\Omega_m = 0.296 \pm 0.07$ and $1/p_2 = 0.13^{+0.09}_{-0.11}$ [34], again, denoting an interesting deviation.

We implement each of $f(T)$ cosmological model described and test them using the constraining parameters method through MCMC analysis using [1] for the cosmology and the baselines with the extract of constraints using `GetDist`[2]. Additionally, we assume flat priors on the set of $\{\Omega_b h^2, \Omega_c, H_0\}$. The baselines considered in our analysis are: *(a)* **DESI(BAO)** release obtained from observations of galaxies and quasars [4], and Lyman-α [5] measurements. These trasers are described through the transverse comoving distance $D_\mathrm{M}/r_\mathrm{d}$, the angle-averaged distance $D_\mathrm{V}/r_\mathrm{d}$, where r_d is the comoving sound horizon at the drag epoch, and the Hubble horizon $D_\mathrm{H}/r_\mathrm{d}$. *(b)* **CMB** Planck-2018 distant priors, which provide information on three parameters: the shift parameter R that measures the peak spacing of the temperature in the CMB spectrum, the acoustic scale l_A from we can measure the temperature in the transverse direction, and finally the combination $\Omega_b h^2$ [35]. *(c)* Cosmic Chronometers **CC**, which are measurements of $H(z)$ from the relative ages of passively-evolving galaxies [36]. We conservatively use the galactic spectra to obtain $\mathrm{d}t/\mathrm{d}z$ [37]. The final sample contains 31 data points up to $z \sim 2$ with the covariance matrix generated given in [38]. *(d)* **PN$^+$** Pantheon-plus catalog [39], with SH0ES Cepheid host distances calibrators [1], and *(e)* **DES-SN5YR** Type Ia supernovae measured during the full 5-years of DES Supernova Program, which includes 1635 SNIa in the redshift range $0.10 < z < 1.13$ [7].

[1] emcee.readthedocs.io
[2] getdist.readthedocs.io

We divide our analysis into these baselines since H_0 and r_d are degenerate in the DESI BAO release. Due to this degeneracy, we will test the set in different schemes: (i) Using the combination of parameters Ω_m and $r_d h$ in Mpc to avoid the degeneracy between $h = H_0/100$[km/s/Mpc] and r_d. This yields the results with the 95% confidence intervals for the ΛCDM model:

$$\begin{cases} \Omega_m = 0.286^{+0.028}_{-0.026}, \\ r_d h = 102.6 \pm 2.5 \text{ [Mpc]}, \end{cases} \text{DESI(BAO)}$$

which is in 2σ interval from the results reported by the DESI collaboration [6]. Meanwhile, for the $f_1(T)$ model:

$$\begin{cases} \Omega_m = 0.282^{+0.031}_{-0.033}, \\ r_d h = 102.0 \pm 4.2 \text{ [Mpc], DESI(BAO)} \\ p_1 = 0.06^{+0.44}_{-0.48}. \end{cases}$$

we can notice that the data prefers a slightly lower fractional matter density with a similar product $r_d h$ and that the free parameter for the power-law model p_1 is within 2σ region. This recovers ΛCDM with a minor positive deviation. In this case, the $f_1(T)$ model contains a Bayes factor of $\ln \mathcal{B}_{ij} = +1.34$, which indicates a preference for the ΛCDM model. For the $f_2(T)$ model:

$$\begin{cases} \Omega_m = 0.307^{+0.041}_{-0.039}, \\ r_d h = 100.6^{+4.5}_{-5.1} \text{ [Mpc], DESI(BAO)} \\ 1/p_2 = 0.29^{+0.37}_{-0.33}. \end{cases}$$

Contrary to the previous model, here the fractional matter exhibits an increase and a diminution in the $r_d h$ parameter. For the free Linder model parameter $1/p_2$ this dataset alone recovers the ΛCDM model as $1/p_2 \to 0$ in 2σ limit. The Bayes factor must be compared to the tested one for the ΛCDM model which results in $\ln \mathcal{B}_{ij} = +0.33$, this favours the fit of the standard cosmological model. (ii) By using a prior on r_d from Planck 2018 [3] of $r_d = 147.09 \pm 0.87$[Mpc] it is possible to break the degeneracy with H_0. The results within 95% confidence interval for the ΛCDM model are:

$$\begin{cases} H_0 = 69.7 \pm 1.7 \text{[km/s/Mpc]}, \\ \Omega_m = 0.286 \pm 0.029, \end{cases} \text{DESI } + r_d \text{ CMB Planck}$$

Remarkably interesting, these results have a high H_0 value even though we are using a r_d from the Planck estimations. For the $f_1(T)$ model, the parameters are:

$$\begin{cases} H_0 = 69.4^{+3.0}_{-2.9} \text{[km/s/Mpc]}, \\ \Omega_m = 0.281^{+0.031}_{-0.034}, \\ p_1 = 0.05^{+0.45}_{-0.48}, \end{cases} \text{DESI } + r_d \text{ CMB Planck}$$

where the H_0 value shows a compatibility in 2σ with the value obtained by the SH0ES collaboration [1]. This model returns a confirmation of ΛCDM for the p_1 value with a significant systematic error, probably because this dataset alone can not constrain the parameter solely. In

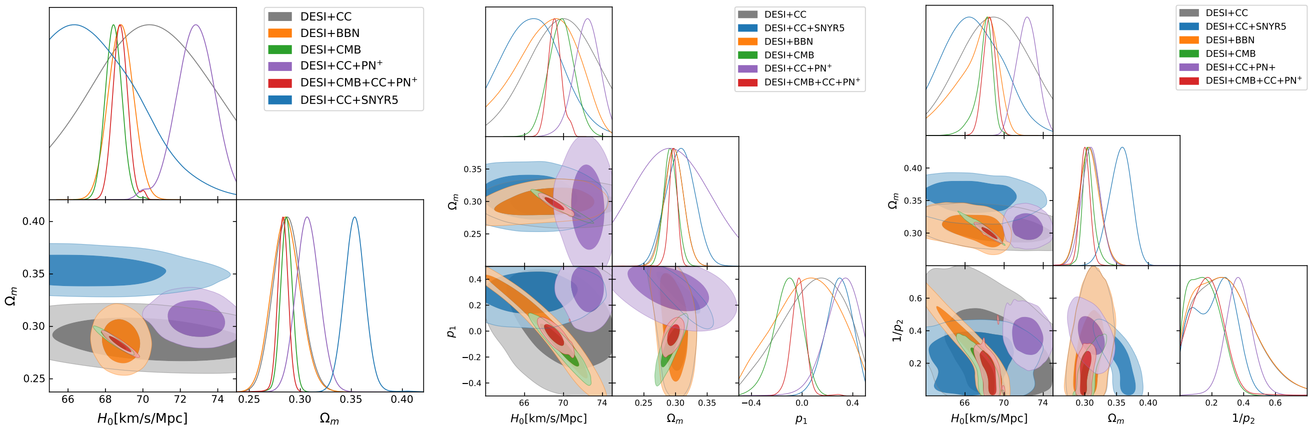

Figure 1. 1-2σ Confidence Levels (C.L) and posterior distributions including H_0 and $\Omega_{m,0}$. The baselines are indicated in colours for each case. *Left:* For the ΛCDM model. *Middle:* For the power law model f_1. *Right:* For the Linder model f_2.

this case the Bayes factor $\ln \mathcal{B}_{ij} = -0.23$ which suggests that using a r_d prior to the dataset and the power-law model have a better fit than the one obtained using the cosmological standard model. For the $f_2(T)$ model the results are:

$$\begin{cases} H_0 = 68.3^{+3.0}_{-3.5}[\mathrm{km/s/Mpc}], \\ \quad \Omega_m = 0.306^{+0.032}_{-0.029}, \qquad \text{DESI } +r_d \text{ CMB Planck} \\ \quad 1/p_2 = 0.29^{+0.36}_{-0.32}, \end{cases}$$

that, similarly to the previous model, confirm ΛCDM at 2σ level. In this case, the value of Ω_m presents a higher value that is a tendency using this specific model. This model presents a Bayes factor $\ln \mathcal{B}_{ij} = -0.19$ that, again, suggests that this dataset has a preference for $f(T)$ models over ΛCDM. (*iii*) Using a prior on $w_b = \Omega_b h^2$ using the results of BBN presented in [6] of $w_b = 0.02218 \pm 0.00055$ to break the degeneracy. In this case, we calculate r_d as a derived parameter. This analysis is presented in Tables I, II, III including DESI + BBN. (*iv*) Finally, since the systematics on this release are substantial we will consider other datasets without the necessity to introduce a prior on w_b as the baselines are sufficient enough to constraint the cosmological parameters. These results are reported in Tables I, II, III in combination with other baselines including DESI BNN measurements.

In conclusion, $f(T)$ cosmologies constrained by new BAO measurements from DESI 2024 can be a good alternative to explain the current H_0 tension as the results using this dataset alone show an improvement in the alleviation on the H_0 value closer to the SH0ES collaboration. Furthermore, it is important to note that for combinations of DESI BAO with other datasets such as CC, Pantheon+, and CMB distance priors, the statistics show a slight preference for the f_1 model. This preference is in addition to the aforementioned advantage of alleviating the Hubble tension.

New analyses will be conducted using the data released in the coming months, employing these extended gravity models. Finally, this result from DESI BAO 2024 measurements could be a hint that the cosmological tension needs new physics to be solved.

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